Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/151-5.3.5-u-a+b-arctan-c+d-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [70]. This is test number [151].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (70)	0.00 (0)
Maple	98.57 (69)	1.43 (1)
Mathematica	94.29 (66)	5.71 (4)
Maxima	52.86 (37)	47.14 (33)
Mupad	42.86 (30)	57.14 (40)
Fricas	40.00 (28)	60.00 (42)
Sympy	34.29 (24)	65.71 (46)
Giac	12.86 (9)	87.14 (61)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

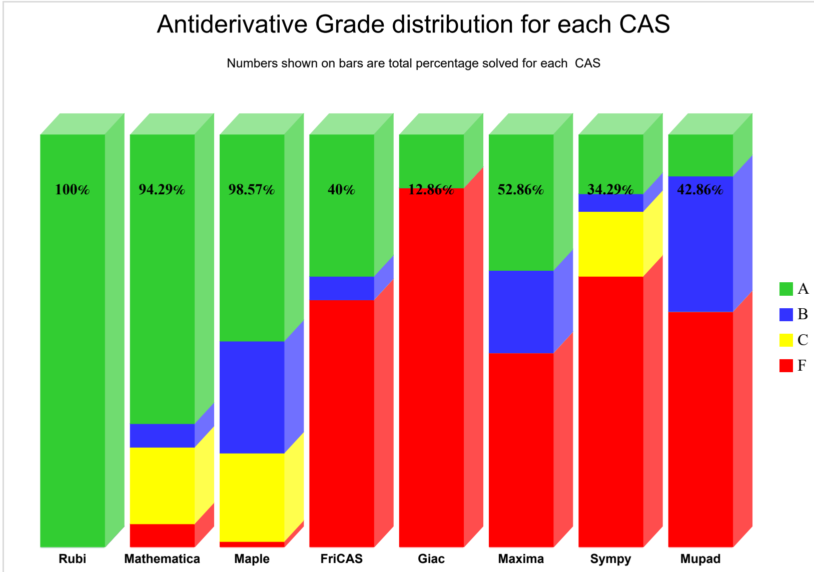
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

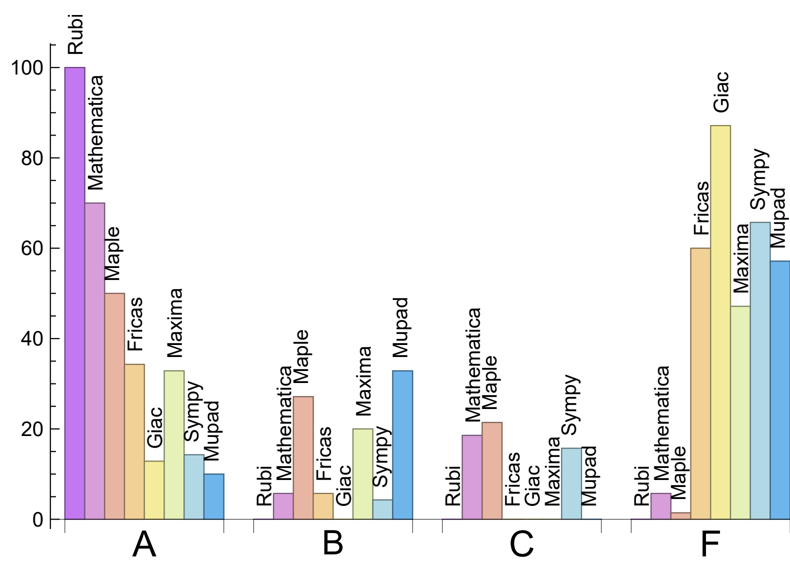
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.00	5.71	18.57	5.71
Maple	50.00	27.14	21.43	1.43
Fricas	34.29	5.71	0.00	60.00
Maxima	32.86	20.00	0.00	47.14
Sympy	14.29	4.29	15.71	65.71
Giac	12.86	0.00	0.00	87.14
Mupad	N/A	32.86	0.00	57.14

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	75.00 %	25.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	42	97.62 %	0.00 %	2.38 %
Giac	61	80.33 %	16.39 %	3.28 %
Maxima	33	93.94 %	0.00 %	6.06 %
Sympy	46	54.35 %	45.65 %	0.00 %
Mupad	40	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

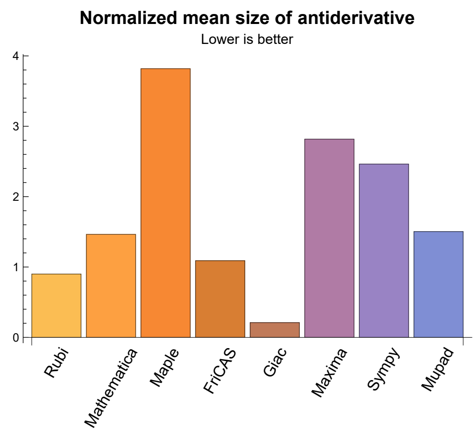
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

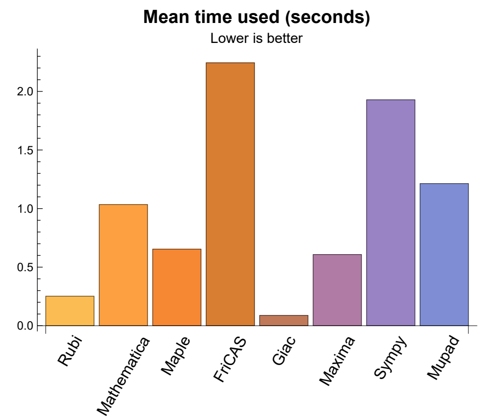
System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	228.04	0.90	159.50	1.00
Mathematica	1.03	261.82	1.46	167.50	1.01
Maple	0.65	1253.71	3.82	222.00	1.43
Maxima	0.61	973.46	2.82	124.00	1.28
Fricas	2.24	128.18	1.09	79.00	1.12
Sympy	1.93	248.08	2.46	172.50	2.20
Giac	0.09	7.44	0.21	0.00	0.00
Mupad	1.21	171.53	1.50	102.50	1.35

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.



Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad



Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{23, 42, 43, 65, 66, 69, 70\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica $\{65, 66, 69, 70\}$

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{36, 65, 66, 69, 70\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

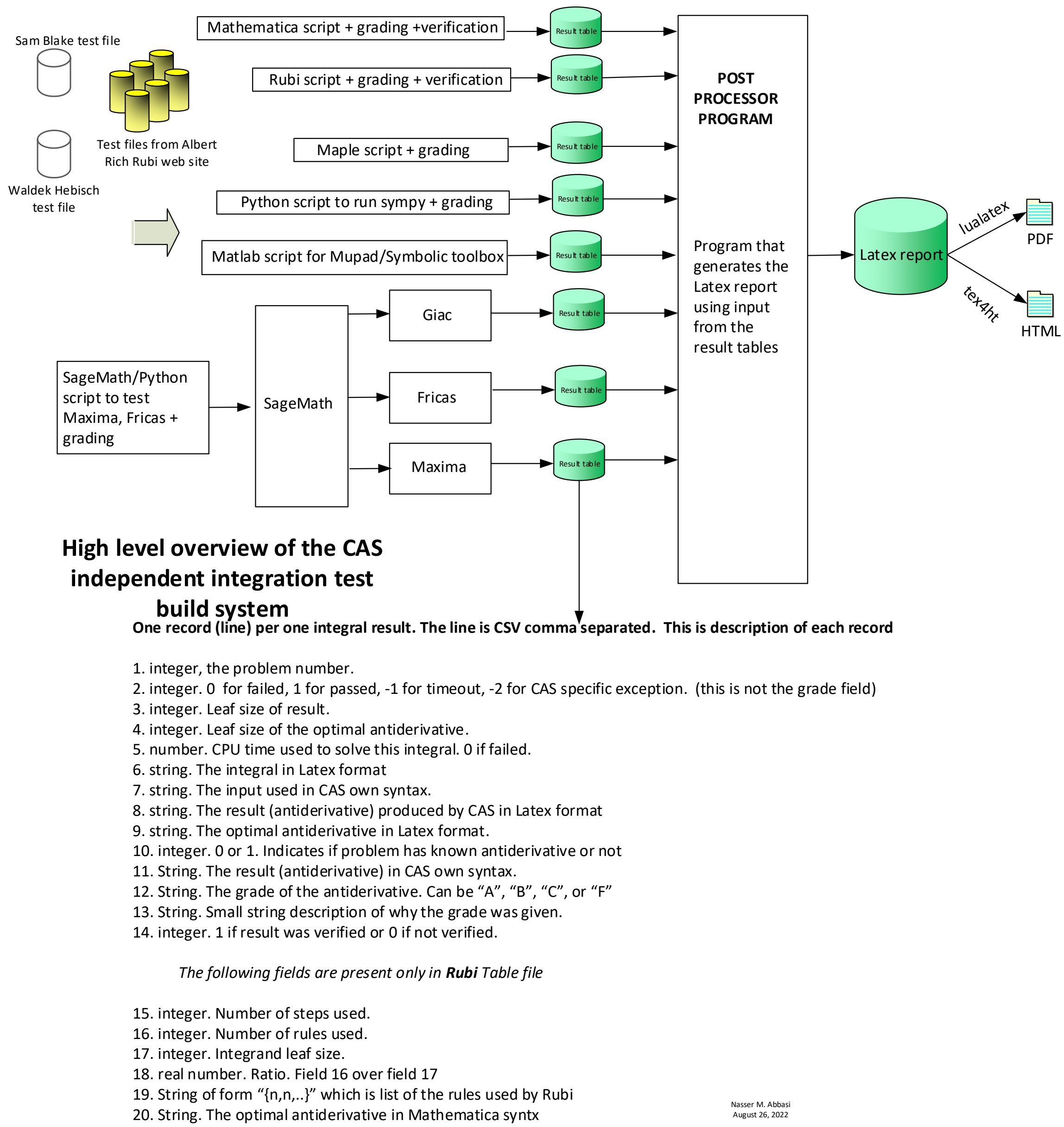
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 32, 33, 35, 37, 38, 41, 42, 43, 47, 48, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 4, 31, 36, 56 }

C grade: { 6, 24, 25, 26, 29, 30, 44, 45, 46, 49, 50, 51, 57 }

F grade: { 34, 39, 40, 62 }

2.1.3 Maple

A grade: { 1, 2, 3, 5, 6, 7, 9, 12, 14, 23, 26, 27, 28, 29, 33, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade: { 4, 8, 11, 13, 16, 19, 21, 22, 24, 25, 30, 31, 32, 35, 38, 53, 60, 61, 62 }

C grade: { 10, 15, 17, 18, 20, 34, 36, 37, 39, 40, 52, 56, 57, 58, 59 }

F grade: { 41 }

2.1.4 Maxima

A grade: { 5, 24, 25, 26, 27, 29, 30, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 60, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 6, 7, 9, 12, 14, 21, 22, 53, 54, 56, 61 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 57, 58, 59, 62, 63, 64, 67, 68 }

2.1.5 FriCAS

A grade: { 2, 3, 5, 6, 9, 12, 24, 25, 26, 27, 29, 42, 43, 44, 45, 46, 47, 49, 50, 51, 65, 66, 69, 70 }

B grade: { 1, 7, 14, 30 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.1.6 Sympy

A grade: { 23, 27, 44, 45, 46, 47, 65, 66, 69, 70 }

B grade: { 1, 3, 6 }

C grade: { 2, 5, 7, 9, 12, 24, 25, 26, 49, 50, 51 }

F grade: { 4, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.1.7 Giac

A grade: { 23, 27, 42, 43, 47, 65, 66, 69, 70 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.1.8 Mupad

A grade: { 23, 42, 43, 65, 66, 69, 70 }

B grade: { 1, 2, 3, 5, 6, 7, 9, 12, 14, 21, 24, 25, 26, 27, 29, 30, 44, 45, 46, 47, 49, 50, 51 }

C grade: { }

F grade: { 4, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	74	362	128	231	0	371
N.S.	1	1.00	0.78	1.03	5.03	1.78	3.21	0.00	5.15
time (sec)	N/A	0.040	0.015	0.158	0.496	2.422	1.120	0.000	0.654

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	67	232	113	182	0	144
N.S.	1	1.00	0.81	1.00	3.46	1.69	2.72	0.00	2.15
time (sec)	N/A	0.040	0.015	0.082	0.470	2.756	1.211	0.000	1.170

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	53	124	59	95	0	73
N.S.	1	1.00	0.83	1.10	2.58	1.23	1.98	0.00	1.52
time (sec)	N/A	0.022	0.012	0.072	0.486	2.605	0.606	0.000	1.454

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	189	118	0	0	0	0	-1
N.S.	1	1.00	3.00	1.87	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.074	0.078	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	65	86	71	238	0	88
N.S.	1	1.00	0.82	1.07	1.41	1.16	3.90	0.00	1.44
time (sec)	N/A	0.036	0.019	0.098	0.279	3.567	2.267	0.000	0.665

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	63	111	63	314	0	103
N.S.	1	1.00	0.81	1.00	1.76	1.00	4.98	0.00	1.63
time (sec)	N/A	0.035	0.017	0.111	0.510	3.471	3.724	0.000	0.754

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	216	192	579	298	583	0	633
N.S.	1	1.00	1.38	1.22	3.69	1.90	3.71	0.00	4.03
time (sec)	N/A	0.165	0.077	0.406	1.243	2.212	3.465	0.000	3.263

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	163	355	0	0	0	0	-1
N.S.	1	1.00	0.89	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.273	0.375	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	229	147	240	0	216
N.S.	1	1.00	1.13	1.31	2.41	1.55	2.53	0.00	2.27
time (sec)	N/A	0.086	0.040	0.155	1.152	3.946	1.375	0.000	1.611

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	170	1362	0	0	0	0	-1
N.S.	1	1.00	0.93	7.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.049	1.713	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	135	418	0	0	0	0	-1
N.S.	1	1.00	1.13	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.160	0.601	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	194	159	252	202	1107	0	232
N.S.	1	1.00	1.66	1.36	2.15	1.73	9.46	0.00	1.98
time (sec)	N/A	0.107	0.080	0.211	0.505	3.317	9.433	0.000	2.857

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	163	482	0	0	0	0	-1
N.S.	1	1.00	0.84	2.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.529	0.634	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	245	210	506	435	0	0	438
N.S.	1	1.00	1.44	1.24	2.98	2.56	0.00	0.00	2.58
time (sec)	N/A	0.186	0.215	0.282	0.567	2.708	0.000	0.000	3.645

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	349	2315	0	0	0	0	-1
N.S.	1	1.00	1.29	8.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	0.447	3.727	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	196	384	0	0	0	0	-1
N.S.	1	1.00	1.20	2.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.243	0.273	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	252	2757	0	0	0	0	-1
N.S.	1	1.00	0.90	9.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.079	1.139	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	263	2547	0	0	0	0	-1
N.S.	1	1.00	1.61	15.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.395	0.632	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	225	557	0	0	0	0	-1
N.S.	1	1.00	1.25	3.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.197	0.648	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	360	6805	0	0	0	0	-1
N.S.	1	1.00	1.25	23.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.875	4.127	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	68	44	0	0	0	25
N.S.	1	1.00	1.00	2.19	1.42	0.00	0.00	0.00	0.81
time (sec)	N/A	0.030	0.005	0.063	0.509	0.000	0.000	0.000	0.081

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	98	123	0	0	0	-1
N.S.	1	1.00	0.83	2.39	3.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.007	0.074	0.509	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.012	4.234	0.320	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	157	557	344	317	654	0	787
N.S.	1	1.00	0.67	2.39	1.48	1.36	2.81	0.00	3.38
time (sec)	N/A	0.281	0.189	0.533	0.481	2.039	12.058	0.000	1.019

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	118	303	220	199	376	0	411
N.S.	1	1.00	0.76	1.95	1.42	1.28	2.43	0.00	2.65
time (sec)	N/A	0.154	0.107	0.243	0.477	2.326	4.603	0.000	0.782

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	163	150	118	104	177	0	136
N.S.	1	1.00	1.68	1.55	1.22	1.07	1.82	0.00	1.40
time (sec)	N/A	0.087	0.048	0.080	0.466	2.523	1.822	0.000	1.804

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	49	42	36	48	51	36	49
N.S.	1	1.00	1.29	1.11	0.95	1.26	1.34	0.95	1.29
time (sec)	N/A	0.014	0.012	0.038	0.265	3.234	0.150	0.398	1.103

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	315	238	0	0	0	0	-1
N.S.	1	1.00	1.94	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.180	0.102	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	121	234	186	195	0	0	127
N.S.	1	1.00	0.80	1.55	1.23	1.29	0.00	0.00	0.84
time (sec)	N/A	0.092	0.158	0.155	0.466	2.200	0.000	0.000	1.829

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	175	468	436	679	0	0	399
N.S.	1	1.00	0.77	2.06	1.92	2.99	0.00	0.00	1.76
time (sec)	N/A	0.224	0.644	0.333	0.500	6.837	0.000	0.000	7.534

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	801	1608	0	0	0	0	-1
N.S.	1	1.00	2.10	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	2.663	0.430	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	264	701	0	0	0	0	-1
N.S.	1	1.00	1.19	3.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.265	0.204	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	146	0	0	0	0	-1
N.S.	1	1.00	1.02	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.134	0.145	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	0	2022	0	0	0	0	-1
N.S.	1	1.00	0.00	7.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.127	5.129	2.115	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	419	1159	0	0	0	0	-1
N.S.	1	1.00	0.74	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.029	4.999	1.579	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	1844	6607	0	0	0	0	-1
N.S.	1	1.00	3.27	11.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	8.568	10.244	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	592	16045	0	0	0	0	-1
N.S.	1	1.00	1.76	47.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.464	0.435	1.688	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	190	297	0	0	0	0	-1
N.S.	1	1.00	1.33	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.120	0.461	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	0	4124	0	0	0	0	-1
N.S.	1	1.00	0.00	11.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	56.242	1.812	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1233	1233	0	4691	0	0	0	0	-1
N.S.	1	1.00	0.00	3.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.665	52.865	1.925	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	162	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.284	0.106	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	3.351	0.093	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	0.193	0.096	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	157	104	87	155	0	133
N.S.	1	1.00	0.90	1.48	0.98	0.82	1.46	0.00	1.25
time (sec)	N/A	0.082	0.051	0.059	0.497	3.192	0.420	0.000	0.594

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	114	113	85	66	117	0	102
N.S.	1	1.00	1.44	1.43	1.08	0.84	1.48	0.00	1.29
time (sec)	N/A	0.062	0.038	0.046	0.509	1.965	0.300	0.000	0.860

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	90	63	68	52	78	0	61
N.S.	1	1.00	1.50	1.05	1.13	0.87	1.30	0.00	1.02
time (sec)	N/A	0.042	0.024	0.040	0.469	2.471	0.203	0.000	0.970

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	39	30	31	39	46	31	42
N.S.	1	1.00	1.18	0.91	0.94	1.18	1.39	0.94	1.27
time (sec)	N/A	0.009	0.011	0.033	0.268	2.447	0.145	0.394	0.446

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	171	106	134	0	0	0	-1
N.S.	1	1.00	1.42	0.88	1.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.007	0.046	0.511	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	61	77	57	168	0	63
N.S.	1	1.00	1.08	0.98	1.24	0.92	2.71	0.00	1.02
time (sec)	N/A	0.032	0.045	0.064	0.490	2.096	0.737	0.000	1.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	84	112	95	382	0	232
N.S.	1	1.00	0.96	0.88	1.17	0.99	3.98	0.00	2.42
time (sec)	N/A	0.069	0.076	0.086	0.470	2.297	1.018	0.000	1.220

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	128	115	165	135	760	0	288
N.S.	1	1.00	0.99	0.89	1.28	1.05	5.89	0.00	2.23
time (sec)	N/A	0.096	0.103	0.115	0.472	2.224	1.631	0.000	1.048

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	863	863	701	787	0	0	0	0	-1
N.S.	1	1.00	0.81	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.127	0.637	0.431	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2184	8520	0	0	0	-1
N.S.	1	1.00	0.75	4.02	15.69	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.247	0.436	5.417	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	231	186	284	0	0	0	-1
N.S.	1	1.00	1.52	1.22	1.87	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.017	0.076	0.562	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	405	321	284	0	0	0	-1
N.S.	1	1.00	1.66	1.32	1.16	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	7.669	0.060	0.573	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	1536	14058	8518	0	0	0	-1
N.S.	1	1.00	2.30	21.04	12.75	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	16.758	1.212	1.200	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	933	933	933	673	0	0	0	0	-1
N.S.	1	1.00	1.00	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.263	4.141	0.396	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	604	364	0	0	0	0	-1
N.S.	1	1.00	0.90	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	0.417	0.073	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	770	666	388	0	0	0	0	-1
N.S.	1	1.00	0.86	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.806	0.601	0.065	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	283	501	328	0	0	0	-1
N.S.	1	1.00	1.03	1.83	1.20	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.043	0.309	0.510	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	409	2184	13320	0	0	0	-1
N.S.	1	1.00	0.75	4.02	24.53	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	0.286	0.474	1.159	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	0	4753	0	0	0	0	-1
N.S.	1	1.00	0.00	12.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.534	180.015	0.677	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	97	135	0	0	0	0	-1
N.S.	1	1.00	0.73	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.083	0.161	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	125	176	0	0	0	0	-1
N.S.	1	1.00	0.58	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.047	0.204	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	163	0	0	0	0	0	-1
N.S.	1	0.00	7.09	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	0.307	0.081	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	165	0	0	0	0	0	-1
N.S.	1	0.00	6.60	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.123	0.077	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	145	180	0	0	0	0	-1
N.S.	1	1.00	0.78	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.468	0.458	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	189	222	0	0	0	0	-1
N.S.	1	1.00	0.67	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.100	0.483	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	181	0	0	0	0	0	-1
N.S.	1	0.00	6.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	4.932	0.347	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	225	0	0	0	0	0	-1
N.S.	1	0.00	7.03	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.126	0.633	0.346	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [68] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	21	0.238
2	A	6	5	1.00	21	0.238
3	A	5	5	1.00	19	0.263
4	A	5	4	1.00	21	0.190
5	A	7	7	1.00	21	0.333
6	A	5	5	1.00	21	0.238
7	A	13	9	1.00	23	0.391
8	A	11	10	1.00	23	0.435
9	A	8	7	1.00	21	0.333
10	A	8	7	1.00	23	0.304
11	A	6	6	1.00	23	0.261
12	A	10	9	1.00	23	0.391
13	A	10	9	1.00	23	0.391
14	A	15	10	1.00	23	0.435
15	A	14	11	1.00	23	0.478
16	A	10	10	1.00	21	0.476
17	A	10	8	1.00	23	0.348
18	A	7	8	1.00	23	0.348
19	A	9	8	1.00	23	0.348
20	A	16	13	1.00	23	0.565
21	A	5	4	1.00	12	0.333
22	A	5	4	1.00	19	0.210
23	A	0	0	0.00	0	0.000
24	A	7	6	1.00	18	0.333
25	A	7	6	1.00	18	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	6	1.00	16	0.375
27	A	4	3	1.00	10	0.300
28	A	5	5	1.00	18	0.278
29	A	8	8	1.00	18	0.444
30	A	9	8	1.00	18	0.444
31	A	16	13	1.00	20	0.650
32	A	13	10	1.00	18	0.556
33	A	6	6	1.00	12	0.500
34	A	2	2	1.00	20	0.100
35	A	25	25	1.00	20	1.250
36	A	21	14	1.00	20	0.700
37	A	15	11	1.00	18	0.611
38	A	6	7	1.00	12	0.583
39	A	2	2	1.00	20	0.100
40	A	35	22	1.00	20	1.100
41	A	6	4	1.00	18	0.222
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	7	6	1.00	10	0.600
45	A	7	6	1.00	10	0.600
46	A	7	6	1.00	8	0.750
47	A	3	3	1.00	6	0.500
48	A	5	5	1.00	10	0.500
49	A	7	7	1.00	10	0.700
50	A	8	7	1.00	10	0.700
51	A	8	7	1.00	10	0.700
52	A	23	5	1.00	16	0.312
53	A	17	5	1.00	16	0.312
54	A	5	5	1.00	14	0.357
55	A	15	7	1.00	16	0.438
56	A	25	7	1.00	16	0.438
57	A	31	7	1.00	16	0.438
58	A	31	13	1.00	18	0.722
59	A	37	16	1.00	18	0.889
60	A	17	5	1.00	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	17	5	1.00	16	0.312
62	A	12	8	1.00	19	0.421
63	A	2	2	1.00	28	0.071
64	A	3	3	1.00	33	0.091
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	4	4	1.00	35	0.114
68	A	5	5	1.00	40	0.125
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (ce+dex)^3(a+b\mathbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=72

$$\frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\text{ArcTan}(c+dx)}{4d}+\frac{e^3(c+dx)^4(a+b\text{ArcTan}(c+dx))}{4d}$$

```
[Out] 1/4*b*e^3*x-1/12*b*e^3*(d*x+c)^3/d-1/4*b*e^3*arctan(d*x+c)/d+1/4*e^3*(d*x+c
)^4*(a+b*arctan(d*x+c))/d
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {5151, 12, 4946, 308, 209}

$$\frac{e^3(c+dx)^4(a+b\text{ArcTan}(c+dx))}{4d}-\frac{be^3\text{ArcTan}(c+dx)}{4d}-\frac{be^3(c+dx)^3}{12d}+\frac{1}{4}be^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (b*e^3*x)/4 - (b*e^3*(c + d*x)^3)/(12*d) - (b*e^3*ArcTan[c + d*x])/(4*d) +
(e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^3\left(a+b\tan^{-1}(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int e^3x^3(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\ &= \frac{e^3\text{Subst}\left(\int x^3(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\ &= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx,x,c+\right.}{4d}\\ &= \frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{(be^3)\,\text{Subst}\left(\int \left(-1+x^2+\frac{}{1}\right.\right.}{4d}\\ &= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}+\frac{e^3(c+dx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4d}-\frac{(be}{}\\ &= \frac{1}{4}be^3x-\frac{be^3(c+dx)^3}{12d}-\frac{be^3\tan^{-1}(c+dx)}{4d}+\frac{e^3(c+dx)^4\left(a+b\,t\right.}{4d}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 56, normalized size = 0.78

$$\frac{e^3\left(-\frac{1}{4}b\left(-dx+\frac{1}{3}(c+dx)^3+\text{ArcTan}(c+dx)\right)+\frac{1}{4}(c+dx)^4(a+b\text{ArcTan}(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^3*(-1/4*(b*(-(d*x) + (c + d*x)^3/3 + ArcTan[c + d*x])) + ((c + d*x)^4*(a
 + b*ArcTan[c + d*x]))/4))/d
```

Maple [**A**]
time = 0.16, size = 74, normalized size = 1.03

method	result

derivativedivides	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{e^3 b(dx+c)^4 \arctan(dx+c)}{4}-\frac{e^3(dx+c)^3 b}{12}+\frac{e^3 b(dx+c)}{4}-\frac{e^3 b\arctan(dx+c)}{4}}{d}$
default	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{e^3 b(dx+c)^4 \arctan(dx+c)}{4}-\frac{e^3(dx+c)^3 b}{12}+\frac{e^3 b(dx+c)}{4}-\frac{e^3 b\arctan(dx+c)}{4}}{d}$
risch	$-\frac{ie^3(dx+c)^4 b\ln(1+i(dx+c))}{8d}+\frac{3ie^3 db\,c^2x^2\ln(1-i(dx+c))}{4}+\frac{ie^3 b\,c^4\ln(d^2x^2+2cdx+c^2+1)}{16d}+\frac{ie^3d^2bc\,x^3\ln(1-i(dx}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a+1/4*e^3*b*(d*x+c)^4*arctan(d*x+c)-1/12*e^3*(d*x+c)
^3*b+1/4*e^3*b*(d*x+c)-1/4*e^3*b*arctan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(60) = 120$.
time = 0.50, size = 362, normalized size = 5.03

$$\frac{1}{4}ad^3x^4e^3+acd^2x^3e^3+\frac{3}{2}ac^2dx^2e^3+\frac{3}{2}\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bc^2de^3+\frac{1}{2}\left(2x^3\arctan(dx+c)-d\left(\frac{dx^2-4cx}{d^3}-\frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3c^2-1)\log(d^2x^2+2cdx+c^2+1)}{d^4}\right)\right)bcd^2e^3+\frac{1}{12}\left(3x^4\arctan(dx+c)-d\left(\frac{d^2x^3-3cdx^2+3(3c^2-1)x}{d^4}+\frac{3(c^4-6c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5}-\frac{6(c^3-c)\log(d^2x^2+2cdx+c^2+1)}{d^5}\right)\right)bd^3e^3+ac^3xe^3+\frac{(2(dx+c)\arctan(dx+c)-\log((dx+c)^2+1))bc^3e^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4*e^3 + a*c*d^2*x^3*e^3 + 3/2*a*c^2*d*x^2*e^3 + 3/2*(x^2*arctan
(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)/d^3))*b*c^2*d*e^3 + 1/2*(2*x^3*arctan(d*x + c) - d*(
(d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 -
1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/12*(3*x^4*arctan(
d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2
+ 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^5))*b*d^3*e^3 + a*c^3*x*e^3 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log
((d*x + c)^2 + 1))*b*c^3*e^3/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.
time = 2.42, size = 128, normalized size = 1.78

$$\frac{3\left(bd^4x^4+4bcd^3x^3+6bc^2d^2x^2+4bc^3dx+bc^4-b\right)\arctan(dx+c)e^3+\left(3ad^4x^4+(12ac-b)d^3x^3+3(6ac^2-bc)d^2x^2+3(4ac^3-bc^2+b)dx\right)e^3}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4
- b)*arctan(d*x + c)*e^3 + (3*a*d^4*x^4 + (12*a*c - b)*d^3*x^3 + 3*(6*a*c^2
 - b*c)*d^2*x^2 + 3*(4*a*c^3 - b*c^2 + b)*d*x)*e^3)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(61) = 122$.
time = 1.12, size = 231, normalized size = 3.21

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{atan}(c+dx)}{4d} + bc^3e^3x\operatorname{atan}(c+dx) + \frac{3bc^2de^3x^2\operatorname{atan}(c+dx)}{2} - \frac{bc^2e^3x}{4} + bcd^2e^3x^3\operatorname{atan}(c+dx) - \frac{bcde^3x^2}{4} + \frac{bd^3e^3x^4\operatorname{atan}(c+dx)}{4} - \frac{bd^2e^3x^3}{12} + \frac{be^3x}{4} - \frac{be^3\operatorname{atan}(c+dx)}{4d} & \text{for } d \neq 0 \\ c^3e^3x(a + b\operatorname{atan}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*atan(c + d*x)/(4*d) + b*c**3*e**3*x*atan(c +
 d*x) + 3*b*c**2*d*e**3*x**2*atan(c + d*x)/2 - b*c**2*e**3*x/4 + b*c*d**2*e
**3*x**3*atan(c + d*x) - b*c*d*e**3*x**2/4 + b*d**3*e**3*x**4*atan(c + d*x)
/4 - b*d**2*e**3*x**3/12 + b*e**3*x/4 - b*e**3*atan(c + d*x)/(4*d), Ne(d, 0
)), (c**3*e**3*x*(a + b*atan(c)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.65, size = 371, normalized size = 5.15

$$\operatorname{atan}(c+dx)\left(b\,c^3\,e^3\,x+\frac{3\,b\,c^2\,d\,e^3\,x^2}{2}+b\,c\,d^2\,e^3\,x^3+\frac{b\,d^3\,e^3\,x^4}{4}\right)-x^3\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{12}+\frac{2\,a\,c\,d^2\,e^3}{3}\right)+x^2\left(\frac{c\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+\frac{d\,e^3\,(10\,a\,c^2-b\,c+a)}{2}-\frac{a\,d\,e^3\,(4\,c^2+4)}{8}\right)+x\left(\frac{c\,e^3\,(20\,a\,c^2-3\,b\,c+6\,a)}{2}+\frac{(4\,c^2+4)\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{4\,d^2}-\frac{2\,c\left(\frac{2\,c\left(\frac{d^2\,e^3\,(b-20\,a\,c)}{4}+2\,a\,c\,d^2\,e^3\right)}{d}+d\,e^3\,(10\,a\,c^2-b\,c+a)-\frac{a\,d\,e^3\,(4\,c^2+4)}{4}\right)}{d}\right)+\frac{a\,d^3\,e^3\,x^4}{4}-\frac{b\,e^3\operatorname{atan}\left(\frac{\frac{b\,c\,e^3\,(c^2+1)\,(c-1)\,(c+1)}{4}+\frac{b\,d\,e^3\,x\,(c^2+1)\,(c-1)\,(c+1)}{4}}{\frac{b\,e^3}{4}-\frac{b\,c^4\,e^3}{4}}\right)(c^2+1)\,(c-1)\,(c+1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*d^3*e^3*x^4)/4 + b*c^3*e^3*x + (3*b*c^2*d*e^3*x^2)/2 + b*
c*d^2*e^3*x^3) - x^3*((d^2*e^3*(b - 20*a*c))/12 + (2*a*c*d^2*e^3)/3) + x^2*
((c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + (d*e^3*(a - b*c + 10*a*
c^2))/2 - (a*d*e^3*(4*c^2 + 4))/8) + x*((c*e^3*(6*a - 3*b*c + 20*a*c^2))/2
+ ((4*c^2 + 4)*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/(4*d^2) - (2*c*(
(2*c*((d^2*e^3*(b - 20*a*c))/4 + 2*a*c*d^2*e^3))/d + d*e^3*(a - b*c + 10*a*
c^2) - (a*d*e^3*(4*c^2 + 4))/4))/d) + (a*d^3*e^3*x^4)/4 - (b*e^3*atan(((b*c
*e^3*(c^2 + 1)*(c - 1)*(c + 1))/4 + (b*d*e^3*x*(c^2 + 1)*(c - 1)*(c + 1))/4
)/((b*e^3)/4 - (b*c^4*e^3)/4))*(c^2 + 1)*(c - 1)*(c + 1))/(4*d)
```

3.2 $\int (ce+dex)^2(a+b\textbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=67

$$-\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3(a+b\text{ArcTan}(c+dx))}{3d}+\frac{be^2\log\left(1+(c+dx)^2\right)}{6d}$$

```
[Out] -1/6*b*e^2*(d*x+c)^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))/d+1/6*b*e^2*ln
(1+(d*x+c)^2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {5151, 12, 4946, 272, 45}

$$\frac{e^2(c+dx)^3(a+b\text{ArcTan}(c+dx))}{3d}-\frac{be^2(c+dx)^2}{6d}+\frac{be^2\log\left((c+dx)^2+1\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/6*(b*e^2*(c + d*x)^2)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(3*d
) + (b*e^2*Log[1 + (c + d*x)^2])/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^2\left(a+b\tan^{-1}(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int e^2x^2(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\ &= \frac{e^2\text{Subst}\left(\int x^2(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\ &= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{(be^2)\,\text{Subst}\left(\int\frac{x^3}{1+x^2}\,dx,x,c+\right.}{3d}\\ &= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{(be^2)\,\text{Subst}\left(\int\frac{x}{1+x}\,dx,x,(c+\right.}{6d}\\ &= \frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}-\frac{(be^2)\,\text{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)\,d\right.}{6d}\\ &= -\frac{be^2(c+dx)^2}{6d}+\frac{e^2(c+dx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3d}+\frac{be^2\log(1+}{6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.81

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3(a+b\text{ArcTan}(c+dx))-\frac{1}{6}b((c+dx)^2-\log\left(1+(c+dx)^2\right))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] (e^2*(((c + d*x)^3*(a + b*ArcTan[c + d*x]))/3 - (b*((c + d*x)^2 - Log[1 + (
c + d*x)^2]))/6))/d
```

Maple [A]
time = 0.08, size = 67, normalized size = 1.00

method	result

derivativedivides	$\frac{\frac{e^2(dx+c)^3 a}{3}+\frac{e^2 b(dx+c)^3 \arctan(dx+c)}{3}-\frac{e^2(dx+c)^2 b}{6}+\frac{e^2 b \ln\left(1+(dx+c)^2\right)}{6}}{d}$
default	$\frac{\frac{e^2(dx+c)^3 a}{3}+\frac{e^2 b(dx+c)^3 \arctan(dx+c)}{3}-\frac{e^2(dx+c)^2 b}{6}+\frac{e^2 b \ln\left(1+(dx+c)^2\right)}{6}}{d}$
risch	$-\frac{ie^2(dx+c)^3 b \ln(1+i(dx+c))}{6d}+\frac{ie^2 d^2 b x^3 \ln(1-i(dx+c))}{6}+\frac{ie^2 dbc x^2 \ln(1-i(dx+c))}{2}+\frac{ie^2 b c^2 x \ln(1-i(dx+c))}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a+1/3*e^2*b*(d*x+c)^3*arctan(d*x+c)-1/6*e^2*(d*x+c)^
2*b+1/6*e^2*b*ln(1+(d*x+c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(58) = 116$.
time = 0.47, size = 232, normalized size = 3.46

$$\frac{1}{3}\,ad^2x^3e^2 + acdx^2e^2 + \left(x^2\arctan\left(dx+c\right) - d\left(\frac{x}{d^2} + \frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3} - \frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bcde^2 + \frac{1}{6}\left(2\,x^3\arctan\left(dx+c\right) - d\left(\frac{dx^2-4\,cx}{d^3} - \frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4} + \frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4}\right)\right)bd^2e^2 + ac^2xe^2 + \frac{\left(2\,(dx+c)\arctan\left(dx+c\right) - \log\left((dx+c)^2+1\right)\right)bc^2e^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3*e^2 + a*c*d*x^2*e^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2
- 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))
*b*c*d*e^2 + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 -
 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)/d^4))*b*d^2*e^2 + a*c^2*x*e^2 + 1/2*(2*(d*x + c)*arctan(d*x + c) - lo
g((d*x + c)^2 + 1))*b*c^2*e^2/d
```

Fricas [A]
time = 2.76, size = 113, normalized size = 1.69

$$\frac{2\,(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3)\arctan\left(dx+c\right)e^2 + be^2\log\left(d^2x^2+2\,cdx+c^2+1\right) + \left(2\,ad^3x^3 + (6\,ac-b)d^2x^2 + 2\,(3\,ac^2-bc)dx\right)e^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*arctan(d*x + c)*e^
2 + b*e^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*a*d^3*x^3 + (6*a*c - b)*d^2
*x^2 + 2*(3*a*c^2 - b*c)*d*x)*e^2)/d
```

Sympy [C] Result contains complex when optimal does not.
time = 1.21, size = 182, normalized size = 2.72

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{atan}(c+dx)}{3d} + bc^2e^2x\operatorname{atan}\left(c+dx\right) + bcde^2x^2\operatorname{atan}\left(c+dx\right) - \frac{bce^2x}{3} + \frac{bd^2e^2x^3\operatorname{atan}(c+dx)}{3} - \frac{bde^2x^2}{6} + \frac{be^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{3d} - \frac{ibe^2\operatorname{atan}(c+dx)}{3d} & \text{for}\ d \neq 0 \\ c^2e^2x(a+b\operatorname{atan}\left(c\right)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*atan(c + d*x)/(3*d) + b*c**2*e**2*x*atan(c + d*x) + b*c*d*e**2*x**2*atan
(c + d*x) - b*c*e**2*x/3 + b*d**2*e**2*x**3*atan(c + d*x)/3 - b*d*e**2*x**2
/6 + b*e**2*log(c/d + x - I/d)/(3*d) - I*b*e**2*atan(c + d*x)/(3*d), Ne(d,
0)), (c**2*e**2*x*(a + b*atan(c)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.17, size = 144, normalized size = 2.15

$$\frac{a\,d^2\,e^2\,x^3}{3}-\frac{b\,c\,e^2\,x}{3}+\frac{b\,e^2\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{6\,d}+a\,c^2\,e^2\,x-\frac{b\,d\,e^2\,x^2}{6}+b\,c^2\,e^2\,x\,\mathrm{atan}(c+d\,x)+a\,c\,d\,e^2\,x^2+\frac{b\,c^3\,e^2\,\mathrm{atan}(c+d\,x)}{3\,d}+\frac{b\,d^2\,e^2\,x^3\,\mathrm{atan}(c+d\,x)}{3}+b\,c\,d\,e^2\,x^2\,\mathrm{atan}(c+d\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] (a*d^2*e^2*x^3)/3 - (b*c*e^2*x)/3 + (b*e^2*log(c^2 + d^2*x^2 + 2*c*d*x + 1)
)/(6*d) + a*c^2*e^2*x - (b*d*e^2*x^2)/6 + b*c^2*e^2*x*atan(c + d*x) + a*c*d
*e^2*x^2 + (b*c^3*e^2*atan(c + d*x))/(3*d) + (b*d^2*e^2*x^3*atan(c + d*x))/
3 + b*c*d*e^2*x^2*atan(c + d*x)
```

3.3 $\int (ce + dex)(a + b\mathbf{ArcTan}(c + dx))\,dx$

Optimal. Leaf size=48

$$-\frac{1}{2}bex + \frac{be\text{ArcTan}(c + dx)}{2d} + \frac{e(c + dx)^2(a + b\text{ArcTan}(c + dx))}{2d}$$

```
[Out] -1/2*b*x*e+1/2*b*e*arctan(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5151, 12, 4946, 327, 209}

$$\frac{e(c + dx)^2(a + b\text{ArcTan}(c + dx))}{2d} + \frac{be\text{ArcTan}(c + dx)}{2d} - \frac{bex}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/2*(b*e*x) + (b*e*ArcTan[c + d*x])/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c
 + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int ex(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int x(a+b\tan^{-1}(x))\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= -\frac{1}{2}bex + \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d} + \frac{(be)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right.}{2d}\\
&= -\frac{1}{2}bex + \frac{be\tan^{-1}(c+dx)}{2d} + \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.83

$$\frac{e(b(-dx+\text{ArcTan}(c+dx))+(c+dx)^2(a+b\text{ArcTan}(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x]),x]
[Out] (e*(b*(-(d*x) + ArcTan[c + d*x]) + (c + d*x)^2*(a + b*ArcTan[c + d*x])))/(2
*d)
```

Maple [A]
time = 0.07, size = 53, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2 a}{2}+\frac{be(dx+c)^2\arctan(dx+c)}{2}-\frac{e(dx+c)b}{2}+\frac{be\arctan(dx+c)}{2}}{d}$

default	$\frac{\frac{e(dx+c)^2 a}{2}+\frac{be(dx+c)^2 \arctan(dx+c)}{2}-\frac{e(dx+c)b}{2}+\frac{be \arctan(dx+c)}{2}}{d}$
risch	$-\frac{ieb(d\,x^2+2cx)\ln(1+i(dx+c))}{4}+\frac{iedb\,x^2\ln(1-i(dx+c))}{4}+\frac{iebcx\ln(1-i(dx+c))}{2}+\frac{ade\,x^2}{2}+\frac{e\arctan(dx+c)b\,c^2}{2d}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a+1/2*b*e*(d*x+c)^2*arctan(d*x+c)-1/2*e*(d*x+c)*b+1/2*
b*e*arctan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(45) = 90$.
time = 0.49, size = 124, normalized size = 2.58

$$\frac{1}{2}adx^2e+\frac{1}{2}\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bde+acxe+\frac{\left(2\left(dx+c\right)\arctan\left(dx+c\right)-\log\left(\left(dx+c\right)^2+1\right)\right)bce}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2*e + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2
*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*d*e + a*c*x*e
 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*c*e/d
```

Fricas [A]
time = 2.61, size = 59, normalized size = 1.23

$$\frac{\left(bd^2x^2+2\,bcdx+bc^2+b\right)\arctan\left(dx+c\right)e+\left(ad^2x^2+\left(2\,ac-b\right)dx\right)e}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c)*e + (a*d^2*x^2 + (
2*a*c - b)*d*x)*e)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.
time = 0.61, size = 95, normalized size = 1.98

$$\begin{cases} acex+\frac{adex^2}{2}+\frac{bc^2e\,\text{atan}\,(c+dx)}{2d}+bcex\,\text{atan}\,(c+dx)+\frac{bdex^2\,\text{atan}\,(c+dx)}{2}-\frac{bex}{2}+\frac{be\,\text{atan}\,(c+dx)}{2d} & \text{for}\ d\neq 0\\ cex(a+b\,\text{atan}\,(c)) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*atan(c + d*x)/(2*d) + b*c*e*x*
atan(c + d*x) + b*d*e*x**2*atan(c + d*x)/2 - b*e*x/2 + b*e*atan(c + d*x)/(2
*d), Ne(d, 0)), (c*e*x*(a + b*atan(c)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.45, size = 73, normalized size = 1.52

$$a\,c\,e\,x-\frac{b\,e\,x}{2}+\frac{b\,e\,\mathrm{atan}(c+d\,x)}{2\,d}+\frac{a\,d\,e\,x^2}{2}+\frac{b\,c^2\,e\,\mathrm{atan}(c+d\,x)}{2\,d}+b\,c\,e\,x\,\mathrm{atan}(c+d\,x)+\frac{b\,d\,e\,x^2\,\mathrm{atan}(c+d\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*c*e*x - (b*e*x)/2 + (b*e*atan(c + d*x))/(2*d) + (a*d*e*x^2)/2 + (b*c^2*e*
atan(c + d*x))/(2*d) + b*c*e*x*atan(c + d*x) + (b*d*e*x^2*atan(c + d*x))/2
```

3.4 $\int \frac{a+b\text{ArcTan}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=63

$$\frac{a\log(c+dx)}{de}+\frac{ib\text{PolyLog}(2,-i(c+dx))}{2de}-\frac{ib\text{PolyLog}(2,i(c+dx))}{2de}$$

```
[Out] a*ln(d*x+c)/d/e+1/2*I*b*polylog(2,-I*(d*x+c))/d/e-1/2*I*b*polylog(2,I*(d*x+
c))/d/e
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5151, 12, 4940, 2438}

$$\frac{a\log(c+dx)}{de}+\frac{ib\text{Li}_2(-i(c+dx))}{2de}-\frac{ib\text{Li}_2(i(c+dx))}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a*Log[c + d*x])/(d*e) + ((I/2)*b*PolyLog[2, (-I)*(c + d*x)])/(d*e) - ((I/2
)*b*PolyLog[2, I*(c + d*x)])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{ce+dex}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{ex}\,dx,x,c+dx\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x}\,dx,x,c+dx\right)}{de}\\
&= \frac{a\log(c+dx)}{de}+\frac{(ib)\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,c+dx\right)}{2de}-\frac{(ib)\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx\right.}{2de}\\
&= \frac{a\log(c+dx)}{de}+\frac{ib\operatorname{Li}_2(-i(c+dx))}{2de}-\frac{ib\operatorname{Li}_2(i(c+dx))}{2de}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 189 vs. $2(63) = 126$.
time = 0.07, size = 189, normalized size = 3.00

$$-\frac{ib\pi^2-4ib\pi\operatorname{ArcTan}(c+dx)+8ib\operatorname{ArcTan}(c+dx)^2+b\pi\log(16)-4b\pi\log\left(1+e^{-2i\operatorname{ArcTan}(c+dx)}\right)+8b\operatorname{ArcTan}(c+dx)\log\left(1+e^{-2i\operatorname{ArcTan}(c+dx)}\right)-8b\operatorname{ArcTan}(c+dx)\log\left(1-e^{2i\operatorname{ArcTan}(c+dx)}\right)-8a\log(c+dx)-2b\pi\log\left(1+c^2+2cdx+d^2x^2\right)+4ib\operatorname{PolyLog}\left(2,-e^{-2i\operatorname{ArcTan}(c+dx)}\right)+4ib\operatorname{PolyLog}\left(2,e^{2i\operatorname{ArcTan}(c+dx)}\right)}{8de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] -1/8*(I*b*Pi^2 - (4*I)*b*Pi*ArcTan[c + d*x] + (8*I)*b*ArcTan[c + d*x]^2 + b
*Pi*Log[16] - 4*b*Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] + 8*b*ArcTan[c + d
*x]*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 8*b*ArcTan[c + d*x]*Log[1 - E^((2
*I)*ArcTan[c + d*x])] - 8*a*Log[c + d*x] - 2*b*Pi*Log[1 + c^2 + 2*c*d*x + d
^2*x^2] + (4*I)*b*PolyLog[2, -E^((-2*I)*ArcTan[c + d*x])] + (4*I)*b*PolyLog
[2, E^((2*I)*ArcTan[c + d*x])])/(d*e)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(55) = 110$.
time = 0.08, size = 118, normalized size = 1.87

method	result
risch	$-\frac{ib\,\operatorname{dilog}(-idx-ic+1)}{2ed}+\frac{a\ln(-idx-ic)}{ed}+\frac{ib\,\operatorname{dilog}(idx+ic+1)}{2ed}$
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\ln(dx+c)\arctan(dx+c)}{e}+\frac{ib\ln(dx+c)\ln(1+i(dx+c))}{2e}-\frac{ib\ln(dx+c)\ln(1-i(dx+c))}{2e}+\frac{ib\,\operatorname{dilog}(1+i(dx+c))}{2e}-\frac{ib\,\operatorname{dilog}(1-i}{2e}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}+\frac{b\ln(dx+c)\arctan(dx+c)}{e}+\frac{ib\ln(dx+c)\ln(1+i(dx+c))}{2e}-\frac{ib\ln(dx+c)\ln(1-i(dx+c))}{2e}+\frac{ib\,\operatorname{dilog}(1+i(dx+c))}{2e}-\frac{ib\,\operatorname{dilog}(1-i}{2e}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/e*ln(d*x+c)+b/e*ln(d*x+c)*arctan(d*x+c)+1/2*I*b/e*ln(d*x+c)*ln(1+I*(
d*x+c))-1/2*I*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+1/2*I*b/e*dilog(1+I*(d*x+c))-1/
2*I*b/e*dilog(1-I*(d*x+c)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(d*x*e + c*e), x) + a*e^(-1)*log(d*x*e +
c*e)/d
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
[Out] integral((b*arctan(d*x + c) + a)*e^(-1)/(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx}\,dx + \int \frac{b\operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e),x)
[Out] (Integral(a/(c + d*x), x) + Integral(b*atan(c + d*x)/(c + d*x), x))/e
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{atan}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x),x)
[Out] int((a + b*atan(c + d*x))/(c*e + d*e*x), x)
```

3.5 $\int \frac{a+b\text{ArcTan}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{a+b\text{ArcTan}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1+(c+dx)^2\right)}{2de^2}$$

```
[Out] (-a-b*arctan(d*x+c))/d/e^2/(d*x+c)+b*ln(d*x+c)/d/e^2-1/2*b*ln(1+(d*x+c)^2)/
d/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5151, 12, 4946, 272, 36, 29, 31}

$$-\frac{a+b\text{ArcTan}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left((c+dx)^2+1\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcTan[c + d*x])/(d*e^2*(c + d*x))) + (b*Log[c + d*x])/(d*e^2) - (
b*Log[1 + (c + d*x)^2])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x(1+x^2)}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,(c+dx)^2\right)}{2de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x}\,dx,x,(c+dx)^2\right)}{2de^2}-\frac{b\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,(c\right.}{2de^2}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\log(c+dx)}{de^2}-\frac{b\log\left(1+(c+dx)^2\right)}{2de^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.82

$$\frac{-\frac{a+b\text{ArcTan}(c+dx)}{c+dx}+b\log(c+dx)-\frac{1}{2}b\log\left(1+(c+dx)^2\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(c + d*x)) + b*Log[c + d*x] - (b*Log[1 + (c + d*
x)^2])/2)/(d*e^2)
```

Maple [A]
time = 0.10, size = 65, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{a}{e^{2}(dx+c)}-\frac{b\arctan(dx+c)}{e^{2}(dx+c)}-\frac{b\ln\left(1+(dx+c)^{2}\right)}{2e^{2}}+\frac{b\ln(dx+c)}{e^{2}}}{d}$
default	$\frac{-\frac{a}{e^{2}(dx+c)}-\frac{b\arctan(dx+c)}{e^{2}(dx+c)}-\frac{b\ln\left(1+(dx+c)^{2}\right)}{2e^{2}}+\frac{b\ln(dx+c)}{e^{2}}}{d}$
risch	$\frac{ib\ln(1+i(dx+c))}{2d\,e^{2}(dx+c)}-\frac{-2\ln(-dx-c)bdx+\ln\left(-d^{2}x^{2}-2cdx-c^{2}-1\right)bdx+ib\ln(1-i(dx+c))-2\ln(-dx-c)bc+\ln\left(-d^{2}x^{2}-2cd\right.}{2e^{2}(dx+c)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)-b/e^2/(d*x+c)*arctan(d*x+c)-1/2*b/e^2*ln(1+(d*x+c)^2)+b
/e^2*ln(d*x+c))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.41

$$-\frac{1}{2}\left(d\left(\frac{e^{(-2)}\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)}{d^{2}}-\frac{2\,e^{(-2)}\log\left(dx+c\right)}{d^{2}}\right)+\frac{2\,\arctan\left(dx+c\right)}{d^{2}xe^{2}+cde^{2}}\right)b-\frac{a}{d^{2}xe^{2}+cde^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(e^(-2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2 - 2*e^(-2)*log(d*x + c
)/d^2) + 2*arctan(d*x + c)/(d^2*x*e^2 + c*d*e^2))*b - a/(d^2*x*e^2 + c*d*e^
2)
```

Fricas [A]
time = 3.57, size = 71, normalized size = 1.16

$$-\frac{\left(2\,b\arctan\left(dx+c\right)+\left(bdx+bc\right)\log\left(d^{2}x^{2}+2\,cdx+c^{2}+1\right)-2\,\left(bdx+bc\right)\log\left(dx+c\right)+2\,a\right)e^{(-2)}}{2\,\left(d^{2}x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*arctan(d*x + c) + (b*d*x + b*c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
- 2*(b*d*x + b*c)*log(d*x + c) + 2*a)*e^(-2)/(d^2*x + c*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.27, size = 238, normalized size = 3.90

$$\begin{cases} \frac{\tilde{\infty} a}{e^{2} x} & \text{for}\: c = 0 \wedge d = 0 \\ \frac{x \left(a + b \operatorname{atan}{\left(c \right)}\right)}{c^{2} e^{2}} & \text{for}\: d = 0 \\ \tilde{\infty} a x & \text{for}\: c = - d x \\ - \frac{a}{c d e^{2} + d^{2} e^{2} x} + \frac{b c \log{\left(\frac{c}{d} + x \right)}}{c d e^{2} + d^{2} e^{2} x} - \frac{b c \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{c d e^{2} + d^{2} e^{2} x} + \frac{i b c \operatorname{atan}{\left(c + d x \right)}}{c d e^{2} + d^{2} e^{2} x} + \frac{b d x \log{\left(\frac{c}{d} + x \right)}}{c d e^{2} + d^{2} e^{2} x} - \frac{b d x \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{c d e^{2} + d^{2} e^{2} x} + \frac{i b d x \operatorname{atan}{\left(c + d x \right)}}{c d e^{2} + d^{2} e^{2} x} - \frac{b \operatorname{atan}{\left(c + d x \right)}}{c d e^{2} + d^{2} e^{2} x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] Piecewise((zoo*a/(e**2*x), Eq(c, 0) & Eq(d, 0)), (x*(a + b*atan(c))/(c**2*e
**2), Eq(d, 0)), (zoo*a*x, Eq(c, -d*x)), (-a/(c*d*e**2 + d**2*e**2*x) + b*c
*log(c/d + x)/(c*d*e**2 + d**2*e**2*x) - b*c*log(c/d + x - I/d)/(c*d*e**2 +
 d**2*e**2*x) + I*b*c*atan(c + d*x)/(c*d*e**2 + d**2*e**2*x) + b*d*x*log(c/
d + x)/(c*d*e**2 + d**2*e**2*x) - b*d*x*log(c/d + x - I/d)/(c*d*e**2 + d**2
*e**2*x) + I*b*d*x*atan(c + d*x)/(c*d*e**2 + d**2*e**2*x) - b*atan(c + d*x)
/(c*d*e**2 + d**2*e**2*x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.67, size = 88, normalized size = 1.44

$$\frac{b\,\ln\left(c+d\,x\right)}{d\,e^2}-\frac{b\,\mathrm{atan}\left(c+d\,x\right)}{x\,d^2\,e^2+c\,d\,e^2}-\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d\,e^2}-\frac{a}{x\,d^2\,e^2+c\,d\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] (b*log(c + d*x))/(d*e^2) - (b*atan(c + d*x))/(d^2*e^2*x + c*d*e^2) - (b*log
(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d*e^2) - a/(d^2*e^2*x + c*d*e^2)
```

3.6 $\int \frac{a+b\mathbf{ArcTan}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=63

$$-\frac{b}{2de^3(c+dx)}-\frac{b\text{ArcTan}(c+dx)}{2de^3}-\frac{a+b\text{ArcTan}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] -1/2*b/d/e^3/(d*x+c)-1/2*b*arctan(d*x+c)/d/e^3+1/2*(-a-b*arctan(d*x+c))/d/e
^3/(d*x+c)^2
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5151, 12, 4946, 331, 209}

$$-\frac{a+b\text{ArcTan}(c+dx)}{2de^3(c+dx)^2}-\frac{b\text{ArcTan}(c+dx)}{2de^3}-\frac{b}{2de^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*b/(d*e^3*(c + d*x)) - (b*ArcTan[c + d*x])/(2*d*e^3) - (a + b*ArcTan[c
+ d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b}{2de^3(c+dx)}-\frac{b\tan^{-1}(c+dx)}{2de^3}-\frac{a+b\tan^{-1}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 51, normalized size = 0.81

$$-\frac{a+b\text{ArcTan}(c+dx)+b(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-(c+dx)^2\right)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a + b*ArcTan[c + d*x] + b*(c + d*x)*Hypergeometric2F1[-1/2, 1, 1/2, -
(c + d*x)^2])/(d*e^3*(c + d*x)^2)
```

Maple [A]
time = 0.11, size = 63, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}-\frac{b\arctan(dx+c)}{2e^3(dx+c)^2}-\frac{b}{2e^3(dx+c)}-\frac{b\arctan(dx+c)}{2e^3}}{d}$
default	$\frac{-\frac{a}{2e^3(dx+c)^2}-\frac{b\arctan(dx+c)}{2e^3(dx+c)^2}-\frac{b}{2e^3(dx+c)}-\frac{b\arctan(dx+c)}{2e^3}}{d}$
risch	$\frac{ib\ln(1+i(dx+c))}{4d\,e^3(dx+c)^2}-\frac{i\ln(-dx-c-i)b\,d^2x^2-i\ln(-dx-c+i)b\,d^2x^2+2i\ln(-dx-c-i)bcdx-2i\ln(-dx-c+i)bcdx+i\ln(-dx}{4e^3(dx+c)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2-1/2*b/e^3/(d*x+c)^2*arctan(d*x+c)-1/2*b/e^3/(d*x+
c)-1/2*b/e^3*arctan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.
time = 0.51, size = 111, normalized size = 1.76

$$-\frac{1}{2}\left(d\left(\frac{\arctan\left(\frac{d^2x+cd}{d}\right)e^{(-3)}}{d^2}+\frac{1}{d^3xe^3+cd^2e^3}\right)+\frac{\arctan\left(dx+c\right)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)b-\frac{a}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(arctan((d^2*x + c*d)/d)*e^(-3)/d^2 + 1/(d^3*x*e^3 + c*d^2*e^3)) +
arctan(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3))*b - 1/2*a/(d^3*x
^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [A]
time = 3.47, size = 63, normalized size = 1.00

$$-\frac{(bdx+bc+(bd^2x^2+2\,bcdx+bc^2+b)\arctan\left(dx+c\right)+a)e^{(-3)}}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x + b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + b)*arctan(d*x + c) + a
)*e^(-3)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(54) = 108$.
time = 3.72, size = 314, normalized size = 4.98

$$\begin{cases} -\frac{a}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{2bcdx\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bc}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bd^2x^2\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{bdx}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2}-\frac{b\operatorname{atan}(c+dx)}{2c^2de^3+4cd^2e^3x+2d^3e^3x^2} & \text{for}\ d\neq 0 \\ \frac{x(a+b\operatorname{atan}(c))}{c^3e^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*c**2
*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b*c
*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b
*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d**2*x**2*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*d*x/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b*atan(c + d*x)/(2*c**2*d
*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(c))/
(c**3*e**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.75, size = 103, normalized size = 1.63

$$-\frac{\frac{a+b\,c}{d}+b\,x}{2\,c^2\,e^3+4\,c\,d\,e^3\,x+2\,d^2\,e^3\,x^2}-\frac{b\,\mathrm{atan}\left(\frac{b\,c+b\,d\,x}{b}\right)}{2\,d\,e^3}-\frac{b\,\mathrm{atan}(c+d\,x)}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] - ((a + b*c)/d + b*x)/(2*c^2*e^3 + 2*d^2*e^3*x^2 + 4*c*d*e^3*x) - (b*atan((
b*c + b*d*x)/b))/(2*d*e^3) - (b*atan(c + d*x))/(2*d^3*e^3*(x^2 + c^2/d^2 +
(2*c*x)/d))
```

3.7 $\int (ce+dex)^3 (a+b\textbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)^2}{12d}+\frac{b^2e^3(c+dx)\text{ArcTan}(c+dx)}{2d}-\frac{be^3(c+dx)^3(a+b\text{ArcTan}(c+dx))}{6d}-\frac{e^3(a+b\text{ArcTan}}{4d}$$

```
[Out] 1/2*a*b*e^3*x+1/12*b^2*e^3*(d*x+c)^2/d+1/2*b^2*e^3*(d*x+c)*arctan(d*x+c)/d-
1/6*b*e^3*(d*x+c)^3*(a+b*arctan(d*x+c))/d-1/4*e^3*(a+b*arctan(d*x+c))^2/d+1
/4*e^3*(d*x+c)^4*(a+b*arctan(d*x+c))^2/d-1/3*b^2*e^3*ln(1+(d*x+c)^2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5151, 12, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{e^3(c+dx)^4(a+b\text{ArcTan}(c+dx))^2}{4d}-\frac{be^3(c+dx)^3(a+b\text{ArcTan}(c+dx))}{6d}-\frac{e^3(a+b\text{ArcTan}(c+dx))^2}{4d}+\frac{1}{2}abe^3x+\frac{b^2e^3(c+dx)\text{ArcTan}(c+dx)}{2d}+\frac{b^2e^3(c+dx)^2}{12d}-\frac{b^2e^3\log\left((c+dx)^2+1\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (a*b*e^3*x)/2 + (b^2*e^3*(c + d*x)^2)/(12*d) + (b^2*e^3*(c + d*x)*ArcTan[c
+ d*x])/(2*d) - (b*e^3*(c + d*x)^3*(a + b*ArcTan[c + d*x]))/(6*d) - (e^3*(a
 + b*ArcTan[c + d*x])^2)/(4*d) + (e^3*(c + d*x)^4*(a + b*ArcTan[c + d*x])^2
)/(4*d) - (b^2*e^3*Log[1 + (c + d*x)^2])/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int e^3x^3(a+b\tan^{-1}(x))^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3(a+b\tan^{-1}(x))^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\tan^{-1}(x)}{1+x^2}\right.}{2d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\tan^{-1}(c+dx)\right)^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int x^2(a+b\tan^{-}\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{6d} + \frac{e^3(c+dx)^4 \left(a+b\tan^{-1}(c\right.}{4d}$$

$$= \frac{1}{2}abe^3x - \frac{be^3(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)}{6d} - \frac{e^3(a+b\tan^{-1}(c-}{4d}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+dx)^3 \left(a+b\tan^{-1}\right.}{6d}$$

$$= \frac{1}{2}abe^3x + \frac{b^2e^3(c+dx)^2}{12d} + \frac{b^2e^3(c+dx)\tan^{-1}(c+dx)}{2d} - \frac{be^3(c+}{}$$

Mathematica [A]
time = 0.08, size = 216, normalized size = 1.38

$$\frac{e^3\left((c+dx)\left(b^2(c+dx)+3a^2(c+dx)^3-2ab(-3+c^2+2cdx+d^2x^2)\right)+2b\left(-b(-3c+c^3-3dx+3c^2dx+3cd^2x^2+d^3x^3)+3a(-1+c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4)\right)\text{ArcTan}(c+dx)+3b^2(-1+c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4)\text{ArcTan}(c+dx)^2-4b^2\log\left(1+(c+dx)^2\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^3*((c + d*x)*(b^2*(c + d*x) + 3*a^2*(c + d*x)^3 - 2*a*b*(-3 + c^2 + 2*c*
d*x + d^2*x^2)) + 2*b*(-(b*(-3*c + c^3 - 3*d*x + 3*c^2*d*x + 3*c*d^2*x^2 +
d^3*x^3)) + 3*a*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x
^4))*ArcTan[c + d*x] + 3*b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^
3*x^3 + d^4*x^4)*ArcTan[c + d*x]^2 - 4*b^2*Log[1 + (c + d*x)^2]))/(12*d)
```

Maple [A]
time = 0.41, size = 192, normalized size = 1.22

method	result
derivativedivides	$\frac{\frac{e^3(dx+c)^4a^2}{4}+\frac{e^3b^2(dx+c)^4\arctan(dx+c)^2}{4}-\frac{e^3b^2\arctan(dx+c)(dx+c)^3}{6}+\frac{e^3b^2\arctan(dx+c)(dx+c)}{2}-\frac{e^3b^2\arctan(dx+c)^2}{4}+\frac{e^3b^2(d}{12}}{d}$

default	$\frac{\frac{e^3(dx+c)^4a^2}{4}+\frac{e^3b^2(dx+c)^4\arctan(dx+c)^2}{4}-\frac{e^3b^2\arctan(dx+c)(dx+c)^3}{6}+\frac{e^3b^2\arctan(dx+c)(dx+c)}{2}-\frac{e^3b^2\arctan(dx+c)^2}{4}+\frac{e^3b^2(}{}}{d}$
risch	$\frac{3ie^3dab\,c^2x^2\ln(1-i(dx+c))}{2}+ie^3d^2abc\,x^3\ln(1-i(dx+c))+ie^3ab\,c^3x\ln(1-i(dx+c))+\frac{ie}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a^2+1/4*e^3*b^2*(d*x+c)^4*arctan(d*x+c)^2-1/6*e^3*b^
2*arctan(d*x+c)*(d*x+c)^3+1/2*e^3*b^2*arctan(d*x+c)*(d*x+c)-1/4*e^3*b^2*arc
tan(d*x+c)^2+1/12*e^3*b^2*(d*x+c)^2-1/3*e^3*b^2*ln(1+(d*x+c)^2)+1/2*e^3*a*b
*(d*x+c)^4*arctan(d*x+c)-1/6*e^3*(d*x+c)^3*a*b+1/2*e^3*a*b*(d*x+c)-1/2*e^3*
a*b*arctan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(136) = 272$.
time = 1.24, size = 579, normalized size = 3.69

$$\frac{1}{4}a^2d^3x^4e^3+a^2cd^2x^3e^3+\frac{3}{2}a^2c^2dx^2e^3+3\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)abc^2de^3+\left(2x^3\arctan(dx+c)-d\left(\frac{dx^2-4cx}{d^3}-\frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3c^2-1)\log(d^2x^2+2cdx+c^2+1)}{d^4}\right)\right)abcd^2e^3+\frac{1}{6}\left(3x^4\arctan(dx+c)-d\left(\frac{d^2x^3-3cdx^2+3(3c^2-1)x}{d^4}+\frac{3(c^4-6c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5}-\frac{6(c^3-c)\log(d^2x^2+2cdx+c^2+1)}{d^5}\right)\right)abd^3e^3+a^2c^3xe^3+\frac{(2(dx+c)\arctan(dx+c)-\log((dx+c)^2+1))abc^3e^3}{d}+\frac{b^2d^2x^2e^3+2b^2cdxe^3-4b^2e^3\log(d^2x^2+2cdx+c^2+1)+3(b^2d^4x^4e^3+4b^2cd^3x^3e^3+6b^2c^2d^2x^2e^3+4b^2c^3dxe^3+b^2c^4e^3-b^2e^3)\arctan(dx+c)^2-2(b^2d^3x^3e^3+3b^2cd^2x^2e^3+b^2c^3e^3-3b^2ce^3+3(b^2c^2e^3-b^2e^3)dx)\arctan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4*e^3 + a^2*c*d^2*x^3*e^3 + 3/2*a^2*c^2*d*x^2*e^3 + 3*(x^2*ar
ctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*c^2*d*e^3 + (2*x^3*arctan(d*x + c) - d
*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2
- 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/6*(3*x^4*arct
an(d*x + c) - d*((d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c
^2 + 1)*arctan((d^2*x + c*d)/d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)/d^5))*a*b*d^3*e^3 + a^2*c^3*x*e^3 + (2*(d*x + c)*arctan(d*x + c) -
log((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/12*(b^2*d^2*x^2*e^3 + 2*b^2*c*d*x*e
^3 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^2*d^4*x^4*e^3 + 4*b^
2*c*d^3*x^3*e^3 + 6*b^2*c^2*d^2*x^2*e^3 + 4*b^2*c^3*d*x*e^3 + b^2*c^4*e^3 -
 b^2*e^3)*arctan(d*x + c)^2 - 2*(b^2*d^3*x^3*e^3 + 3*b^2*c*d^2*x^2*e^3 + b^
2*c^3*e^3 - 3*b^2*c*e^3 + 3*(b^2*c^2*e^3 - b^2*e^3)*d*x)*arctan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(136) = 272$.
time = 2.21, size = 298, normalized size = 1.90

$$\frac{3(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4-b^2)\arctan(dx+c)^2e^3-4b^2e^3\log(d^2x^2+2cdx+c^2+1)+2(3abd^4x^4+(12abc-b^2)d^3x^3+3abc^4-b^2c^3+3(6abc^2-b^2c)d^2x^2+3b^2c+3(4abc^3-b^2c^2+b^2)dx-3ab)\arctan(dx+c)e^3+(3a^2d^4x^4+2(6a^2c-ab)d^3x^3+(18a^2c^2-6abc+b^2)d^2x^2+2(6a^2c^3-3abc^2+b^2c+3ab)dx)e^3}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x
+ b^2*c^4 - b^2)*arctan(d*x + c)^2*e^3 - 4*b^2*e^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1) + 2*(3*a*b*d^4*x^4 + (12*a*b*c - b^2)*d^3*x^3 + 3*a*b*c^4 - b^2*c^
3 + 3*(6*a*b*c^2 - b^2*c)*d^2*x^2 + 3*b^2*c + 3*(4*a*b*c^3 - b^2*c^2 + b^2)
*d*x - 3*a*b)*arctan(d*x + c)*e^3 + (3*a^2*d^4*x^4 + 2*(6*a^2*c - a*b)*d^3*
x^3 + (18*a^2*c^2 - 6*a*b*c + b^2)*d^2*x^2 + 2*(6*a^2*c^3 - 3*a*b*c^2 + b^2
*c + 3*a*b)*d*x)*e^3)/d
```

Sympy [C] Result contains complex when optimal does not.
time = 3.46, size = 583, normalized size = 3.71

$$\begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\operatorname{atan}(c+dx)}{2d} + 2abc^3e^3x\operatorname{atan}(c+dx) + 3abc^2de^3x^2\operatorname{atan}(c+dx) - \frac{abc^2e^3x}{2} + 2abcd^2e^3x^3\operatorname{atan}(c+dx) - \frac{abcde^3x^2}{2} + \frac{abd^3e^3x^4\operatorname{atan}(c+dx)}{2} - \frac{abd^2e^3x^3}{6} + \frac{abe^3x}{2} - \frac{abe^3\operatorname{atan}(c+dx)}{2d} + \frac{b^2c^4e^3\operatorname{atan}^2(c+dx)}{4d} + b^2c^3e^3x\operatorname{atan}^2(c+dx) - \frac{b^2c^3e^3\operatorname{atan}(c+dx)}{6d} + \frac{3b^2c^2de^3x^2\operatorname{atan}^2(c+dx)}{2} - \frac{b^2c^2e^3x\operatorname{atan}(c+dx)}{2} + b^2cd^2e^3x^3\operatorname{atan}^2(c+dx) - \frac{b^2cde^3x^2\operatorname{atan}(c+dx)}{2} + \frac{b^2ce^3x}{6} + \frac{b^2ce^3\operatorname{atan}(c+dx)}{2d} + \frac{b^2d^3e^3x^4\operatorname{atan}^2(c+dx)}{4} - \frac{b^2d^2e^3x^3\operatorname{atan}(c+dx)}{6} + \frac{b^2de^3x^2}{12} + \frac{b^2e^3x\operatorname{atan}(c+dx)}{2} - \frac{2b^2e^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{3d} - \frac{b^2e^3\operatorname{atan}^2(c+dx)}{4d} + \frac{2ib^2e^3\operatorname{atan}(c+dx)}{3d} & \text{for } d \neq 0 \\ c^3e^3x(a+b\operatorname{atan}(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*atan(c + d*x)/(2*d) + 2*a*b*c*
*3*e**3*x*atan(c + d*x) + 3*a*b*c**2*d*e**3*x**2*atan(c + d*x) - a*b*c**2*e
**3*x/2 + 2*a*b*c*d**2*e**3*x**3*atan(c + d*x) - a*b*c*d*e**3*x**2/2 + a*b*
d**3*e**3*x**4*atan(c + d*x)/2 - a*b*d**2*e**3*x**3/6 + a*b*e**3*x/2 - a*b*
e**3*atan(c + d*x)/(2*d) + b**2*c**4*e**3*atan(c + d*x)**2/(4*d) + b**2*c**
3*e**3*x*atan(c + d*x)**2 - b**2*c**3*e**3*atan(c + d*x)/(6*d) + 3*b**2*c**
2*d*e**3*x**2*atan(c + d*x)**2/2 - b**2*c**2*e**3*x*atan(c + d*x)/2 + b**2*
c*d**2*e**3*x**3*atan(c + d*x)**2 - b**2*c*d*e**3*x**2*atan(c + d*x)/2 + b*
*2*c*e**3*x/6 + b**2*c*e**3*atan(c + d*x)/(2*d) + b**2*d**3*e**3*x**4*atan(
c + d*x)**2/4 - b**2*d**2*e**3*x**3*atan(c + d*x)/6 + b**2*d*e**3*x**2/12 +
 b**2*e**3*x*atan(c + d*x)/2 - 2*b**2*e**3*log(c/d + x - I/d)/(3*d) - b**2*
e**3*atan(c + d*x)**2/(4*d) + 2*I*b**2*e**3*atan(c + d*x)/(3*d), Ne(d, 0)),
 (c**3*e**3*x*(a + b*atan(c))**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 3.26, size = 633, normalized size = 4.03

$$x\left(\frac{c\,e^3\,\left(20\,a^2\,c^2+6\,a^2-6\,a\,b\,c+b^2\right)}{2}+\frac{\left(6\,c^2+6\right)\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{6\,d^2}-\frac{2\,c\,\left(\frac{2\,c\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{d}+\frac{d\,e^3\,\left(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2\right)}{6}-\frac{a^2\,d\,e^3\,\left(6\,c^2+6\right)}{6}\right)}{d}\right)+x^2\,\left(\frac{c\,\left(2\,a^2\,c\,d^2\,e^3+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{2}\right)}{d}+\frac{d\,e^3\,\left(60\,a^2\,c^2+6\,a^2-12\,a\,b\,c+b^2\right)}{12}-\frac{a^2\,d\,e^3\,\left(6\,c^2+6\right)}{12}\right)-x^3\,\left(\frac{2\,a^2\,c\,d^2\,e^3}{3}+\frac{a\,d^2\,e^3\,\left(b-10\,a\,c\right)}{6}\right)+{\mathrm{atan}\left(c+d\,x\right)}^2\,\left(b^2\,c^3\,e^3\,x-\frac{b^2\,e^3-b^2\,c^4\,e^3}{4\,d}+\frac{b^2\,d^3\,e^3\,x^4}{4}+\frac{3\,b^2\,c^2\,d\,e^3\,x^2}{2}+b^2\,c\,d^2\,e^3\,x^3\right)-d^2\,\mathrm{atan}\left(c+d\,x\right)\,\left(x^3\,\left(\frac{b^2\,e^3}{6}-2\,a\,b\,c\,e^3\right)-\frac{x\,\left(-b^2\,c^2\,e^3+b^2\,e^3+4\,a\,b\,c^3\,e^3\right)}{2\,d^2}+\frac{x^2\,\left(b^2\,c\,e^3-6\,a\,b\,c^2\,e^3\right)}{2\,d}-\frac{a\,b\,d\,e^3\,x^4}{2}\right)+\frac{a^2\,d^3\,e^3\,x^4}{4}-\frac{b^2\,e^3\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{3\,d}+\frac{b\,e^3\,\mathrm{atan}\left(\frac{\frac{b\,c\,e^3\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}+\frac{b\,d\,e^3\,x\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6}}{-\frac{b^2\,c^3\,e^3}{6}+\frac{b^2\,c\,e^3}{2}+\frac{a\,b\,c^4\,e^3}{2}-\frac{a\,b\,e^3}{2}}\right)\,\left(-3\,a\,c^4+b\,c^3-3\,b\,c+3\,a\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*atan(c + d*x))^2,x)

[Out] x*((c*e^3*(6*a^2 + b^2 + 20*a^2*c^2 - 6*a*b*c))/2 + ((6*c^2 + 6)*(2*a^2*c*d
^2*e^3 + (a*d^2*e^3*(b - 10*a*c))/2))/(6*d^2) - (2*c*((2*c*(2*a^2*c*d^2*e^3
 + (a*d^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a
*b*c))/6 - (a^2*d*e^3*(6*c^2 + 6))/6))/d) + x^2*((c*(2*a^2*c*d^2*e^3 + (a*d
^2*e^3*(b - 10*a*c))/2))/d + (d*e^3*(6*a^2 + b^2 + 60*a^2*c^2 - 12*a*b*c))/
12 - (a^2*d*e^3*(6*c^2 + 6))/12) - x^3*((2*a^2*c*d^2*e^3)/3 + (a*d^2*e^3*(b
 - 10*a*c))/6) + atan(c + d*x)^2*(b^2*c^3*e^3*x - (b^2*e^3 - b^2*c^4*e^3)/(
4*d) + (b^2*d^3*e^3*x^4)/4 + (3*b^2*c^2*d*e^3*x^2)/2 + b^2*c*d^2*e^3*x^3) -
 d^2*atan(c + d*x)*(x^3*((b^2*e^3)/6 - 2*a*b*c*e^3) - (x*(b^2*e^3 - b^2*c^2
*e^3 + 4*a*b*c^3*e^3))/(2*d^2) + (x^2*(b^2*c*e^3 - 6*a*b*c^2*e^3))/(2*d) -
(a*b*d*e^3*x^4)/2) + (a^2*d^3*e^3*x^4)/4 - (b^2*e^3*log(c^2 + d^2*x^2 + 2*c
*d*x + 1))/(3*d) + (b*e^3*atan(((b*c*e^3*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6
 + (b*d*e^3*x*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/6)/((b^2*c*e^3)/2 - (b^2*c^3
*e^3)/6 - (a*b*e^3)/2 + (a*b*c^4*e^3)/2))*(3*a - 3*b*c - 3*a*c^4 + b*c^3))/
(6*d)
```

3.8 $\int (ce+dex)^2(a+b\textbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=183

$$\frac{1}{3}b^2e^2x-\frac{b^2e^2\text{ArcTan}(c+dx)}{3d}-\frac{be^2(c+dx)^2(a+b\text{ArcTan}(c+dx))}{3d}-\frac{ie^2(a+b\text{ArcTan}(c+dx))^2}{3d}+\frac{e^2(c+dx)}{}$$

```
[Out] 1/3*b^2*e^2*x-1/3*b^2*e^2*arctan(d*x+c)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arctan(d
*x+c))/d-1/3*I*e^2*(a+b*arctan(d*x+c))^2/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*
x+c))^2/d-2/3*b*e^2*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-1/3*I*b^2*e^2
*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A]
time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5151, 12, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$\frac{e^2(c+dx)^3(a+b\text{ArcTan}(c+dx))^2}{3d}-\frac{be^2(c+dx)^2(a+b\text{ArcTan}(c+dx))}{3d}-\frac{ie^2(a+b\text{ArcTan}(c+dx))^2}{3d}-\frac{2be^2\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{3d}-\frac{b^2e^2\text{ArcTan}(c+dx)}{3d}-\frac{ib^2e^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{3d}+\frac{1}{3}b^2e^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*e^2*x)/3 - (b^2*e^2*ArcTan[c + d*x])/(3*d) - (b*e^2*(c + d*x)^2*(a + b
*ArcTan[c + d*x]))/(3*d) - ((I/3)*e^2*(a + b*ArcTan[c + d*x])^2)/d + (e^2*(
c + d*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*d) - (2*b*e^2*(a + b*ArcTan[c + d*
x])*Log[2/(1 + I*(c + d*x))])/(3*d) - ((I/3)*b^2*e^2*PolyLog[2, 1 - 2/(1 +
I*(c + d*x))])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 \, dx &= \frac{\text{Subst}\left(\int e^2x^2(a+b\tan^{-1}(x))^2 \, dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int x^2(a+b\tan^{-1}(x))^2 \, dx, x, c+dx\right)}{d} \\
&= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\tan^{-1}(x}{1+x^2}\right.}{3d} \\
&= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int x(a+b\tan^{-}\right.}{3d} \\
&= -\frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2(a+b\tan^{-1}(c+dx))^2}{3d} \\
&= \frac{1}{3}b^2e^2x - \frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right)}{3d} - \frac{ie^2(a+b\tan^{-1}(c}{3d} \\
&= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right.}{3d} \\
&= \frac{1}{3}b^2e^2x - \frac{b^2e^2\tan^{-1}(c+dx)}{3d} - \frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right.}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 163, normalized size = 0.89

$$
\frac{e^2\left(a^2(c+dx)^3+ab\left(-(c+dx)^2+2(c+dx)^3\text{ArcTan}(c+dx)+\log\left(1+(c+dx)^2\right)\right)+b^2\left(c+dx-\text{ArcTan}(c+dx)-(c+dx)^2\text{ArcTan}(c+dx)+i\text{ArcTan}(c+dx)^2+(c+dx)^3\text{ArcTan}(c+dx)^2-2\text{ArcTan}(c+dx)\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)+i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)\right)\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (e^2*(a^2*(c + d*x)^3 + a*b*(-(c + d*x)^2 + 2*(c + d*x)^3*ArcTan[c + d*x] +
 Log[1 + (c + d*x)^2]) + b^2*(c + d*x - ArcTan[c + d*x] - (c + d*x)^2*ArcTa
n[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan[c + d*x]^2 - 2*ArcTan
[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*PolyLog[2, -E^((2*I)*ArcTa
n[c + d*x])])))/(3*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(165) = 330$.
time = 0.38, size = 355, normalized size = 1.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a^2+1/3*b^2*e^2*(d*x+c)^3*arctan(d*x+c)^2-1/3*b^2*e^
2*arctan(d*x+c)*(d*x+c)^2+1/3*b^2*e^2*arctan(d*x+c)*ln(1+(d*x+c)^2)+1/3*b^2
*e^2*(d*x+c)-1/3*b^2*e^2*arctan(d*x+c)+1/6*I*b^2*e^2*ln(d*x+c+I)*ln(1/2*I*(
d*x+c-I))-1/6*I*b^2*e^2*dilog(-1/2*I*(d*x+c+I))+1/12*I*b^2*e^2*ln(d*x+c+I)^
2+1/6*I*b^2*e^2*dilog(1/2*I*(d*x+c-I))-1/6*I*b^2*e^2*ln(d*x+c+I)*ln(1+(d*x+
c)^2)-1/12*I*b^2*e^2*ln(d*x+c-I)^2+1/6*I*b^2*e^2*ln(d*x+c-I)*ln(1+(d*x+c)^2
)-1/6*I*b^2*e^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+2/3*e^2*a*b*(d*x+c)^3*arct
an(d*x+c)-1/3*e^2*(d*x+c)^2*a*b+1/3*e^2*a*b*ln(1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^4*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d - 1/4*(3*arctan
(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^
4*e^2 + 1/3*a^2*d^2*x^3*e^2 + 36*b^2*d^4*e^2*integrate(1/48*x^4*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^4*e^2*integrate(1/48*x^4
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 144
*b^2*c*d^3*e^2*integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^2*d^4*e^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d^3*e^2*integrate(1/48*x^3
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 216
*b^2*c^2*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 16*b^2*c*d^3*e^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^2*c^2*d^2*e^2*integrate(1
/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 144*b^2*c^3*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 24*b^2*c^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^3*d*e^2*integrat
e(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^2*c^3*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^4*e^2*integrate(1/48*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + a^2*c*d*x^2*e^2 +
 3/4*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d - 8*b^2*d^3*e^
2*integrate(1/48*x^3*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24
*b^2*c*d^2*e^2*integrate(1/48*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) - 24*b^2*c^2*d*e^2*integrate(1/48*x*arctan(d*x + c)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d -
arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*e^2 + 2*(x^2*arctan(d*x + c) - d*(x/d^
2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
```

```
 1)/d^3))*a*b*c*d*e^2 + 1/3*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3
 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*x*e^2 + 36*b^2*d^2*e^2*integra
te(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2
*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 72*b^2*c*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*integrate(1/48*x*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*int
egrate(1/48*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c^2*e^2/d
+ 1/12*(b^2*d^2*x^3*e^2 + 3*b^2*c*d*x^2*e^2 + 3*b^2*c^2*x*e^2)*arctan(d*x +
 c)^2 - 1/48*(b^2*d^2*x^3*e^2 + 3*b^2*c*d*x^2*e^2 + 3*b^2*c^2*x*e^2)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*arctan(d*x + c)^2*e^2 + 2*(a
*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*arctan(d*x + c)*e^2 + (a^2*d^2*x^2 + 2*
a^2*c*d*x + a^2*c^2)*e^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int a^{2}c^{2}\,dx+\int a^{2}d^{2}x^{2}\,dx+\int b^{2}c^{2}\operatorname{atan}^{2}{\left(c+dx\right)}\,dx+\int 2abc^{2}\operatorname{atan}{\left(c+dx\right)}\,dx+\int 2a^{2}cdx\,dx+\int b^{2}d^{2}x^{2}\operatorname{atan}^{2}{\left(c+dx\right)}\,dx+\int 2abd^{2}x^{2}\operatorname{atan}{\left(c+dx\right)}\,dx+\int 2b^{2}cdx\operatorname{atan}^{2}{\left(c+dx\right)}\,dx+\int 4abcdx\operatorname{atan}{\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(a**2*c**2, x) + Integral(a**2*d**2*x**2, x) + Integral(b**2*
c**2*atan(c + d*x)**2, x) + Integral(2*a*b*c**2*atan(c + d*x), x) + Integra
l(2*a**2*c*d*x, x) + Integral(b**2*d**2*x**2*atan(c + d*x)**2, x) + Integra
l(2*a*b*d**2*x**2*atan(c + d*x), x) + Integral(2*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(4*a*b*c*d*x*atan(c + d*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{atan}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.9 $\int (ce+dex)(a+b\mathbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=95

$$-abex-\frac{b^2e(c+dx)\mathrm{ArcTan}(c+dx)}{d}+\frac{e(a+b\mathrm{ArcTan}(c+dx))^2}{2d}+\frac{e(c+dx)^2(a+b\mathrm{ArcTan}(c+dx))^2}{2d}+\frac{b^2e\log$$

```
[Out] -a*b*e*x-b^2*e*(d*x+c)*arctan(d*x+c)/d+1/2*e*(a+b*arctan(d*x+c))^2/d+1/2*e*
(d*x+c)^2*(a+b*arctan(d*x+c))^2/d+1/2*b^2*e*ln(1+(d*x+c)^2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5151, 12, 4946, 5036, 4930, 266, 5004}

$$\frac{e(c+dx)^2(a+b\mathrm{ArcTan}(c+dx))^2}{2d}+\frac{e(a+b\mathrm{ArcTan}(c+dx))^2}{2d}-abex-\frac{b^2e(c+dx)\mathrm{ArcTan}(c+dx)}{d}+\frac{b^2e\log\left((c+dx)^2+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -(a*b*e*x) - (b^2*e*(c + d*x)*ArcTan[c + d*x])/d + (e*(a + b*ArcTan[c + d*x
])^2)/(2*d) + (e*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) + (b^2*e*Log[
1 + (c + d*x)^2])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int ex(a+b\tan^{-1}(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int x(a+b\tan^{-1}(x))^2\,dx,x,c+dx\right)}{d}\\
&= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2(a+b\tan^{-1}(x))}{1+x^2}\,d\right.}{d}\\
&= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{(be)\text{Subst}\left(\int (a+b\tan^{-1}(x)\right.}{d}\\
&= -abex + \frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+\right.}{2d}\\
&= -abex - \frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d} + \frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} +\\
&= -abex - \frac{b^2e(c+dx)\tan^{-1}(c+dx)}{d} + \frac{e\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} +
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 107, normalized size = 1.13

$$\frac{e\left(a(c+dx)(-2b+ac+adx)+2b(-b(c+dx)+a(1+c^2+2cdx+d^2x^2))\operatorname{ArcTan}(c+dx)+b^2(1+c^2+2cdx+d^2x^2)\operatorname{ArcTan}(c+dx)^2+b^2\log\left(1+(c+dx)^2\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^2,x]

[Out] (e*(a*(c + d*x)*(-2*b + a*c + a*d*x) + 2*b*(-(b*(c + d*x)) + a*(1 + c^2 + 2*c*d*x + d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^2 + b^2*Log[1 + (c + d*x)^2]))/(2*d)

Maple [A]
time = 0.16, size = 124, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a^2}{2}+\frac{e\,b^2(dx+c)^2\arctan(dx+c)^2}{2}+\frac{e\,b^2\arctan(dx+c)^2}{2}-e\,b^2\arctan(dx+c)(dx+c)+\frac{e\,b^2\ln\left(1+(dx+c)^2\right)}{2}+eab(dx+c)^2\operatorname{ar}}{d}$
default	$\frac{\frac{e(dx+c)^2a^2}{2}+\frac{e\,b^2(dx+c)^2\arctan(dx+c)^2}{2}+\frac{e\,b^2\arctan(dx+c)^2}{2}-e\,b^2\arctan(dx+c)(dx+c)+\frac{e\,b^2\ln\left(1+(dx+c)^2\right)}{2}+eab(dx+c)^2\operatorname{ar}}{d}$
risch	$-\frac{e\,b^2(d^2x^2+2cdx+c^2+1)\ln(1+i(dx+c))^2}{8d}+\frac{be(-2ia\,d^2x^2+b\,d^2x^2\ln(1-i(dx+c))-4iacxd+2bcdx\ln(1-i(dx+c))+2ib}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2*e*(d*x+c)^2*a^2+1/2*e*b^2*(d*x+c)^2*arctan(d*x+c)^2+1/2*e*b^2*arctan(d*x+c)^2-e*b^2*arctan(d*x+c)*(d*x+c)+1/2*e*b^2*ln(1+(d*x+c)^2)+e*a*b*(d*x+c)^2*arctan(d*x+c)+e*a*b*arctan(d*x+c)-e*(d*x+c)*a*b)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(94) = 188.
time = 1.15, size = 229, normalized size = 2.41

$$\frac{1}{2}\,a^2dx^2e+\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)abde+a^2cxe+\frac{(2\,(dx+c)\arctan\left(dx+c\right)-\log\left((dx+c)^2+1\right))abce}{d}+\frac{b^2e\log\left(d^2x^2+2\,cdx+c^2+1\right)+(b^2d^2x^2e+2\,b^2cdxe+b^2c^2e+b^2e)\arctan\left(dx+c\right)^2-2\,(b^2dxe+b^2ce)\arctan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*a^2*d*x^2*e + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*d*e + a^2*c*x*e + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*c*e/d + 1/2*(b^2*e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^2*x^2*e + 2*b^2*c*d*x*e + b^2*c^2*e + b^2*e)*arctan(d*x + c)^2 - 2*(b^2*d*x*e + b^2*c*e)*arctan(d*x + c))/d

Fricas [A]
time = 3.95, size = 147, normalized size = 1.55

$$\frac{\left(b^{2} d^{2} x^{2}+2 b^{2} c d x+b^{2} c^{2}+b^{2}\right) \arctan \left(d x+c\right)^{2} e+b^{2} e \log \left(d^{2} x^{2}+2 c d x+c^{2}+1\right)+2 \left(a b d^{2} x^{2}+a b c^{2}-b^{2} c+\left(2 a b c-b^{2}\right) d x+a b\right) \arctan \left(d x+c\right) e+\left(a^{2} d^{2} x^{2}+2 \left(a^{2} c-a b\right) d x\right) e}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arctan(d*x + c)^2*e + b^2*
e*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b*d^2*x^2 + a*b*c^2 - b^2*c + (2*
a*b*c - b^2)*d*x + a*b)*arctan(d*x + c)*e + (a^2*d^2*x^2 + 2*(a^2*c - a*b)*
d*x)*e)/d
```

Sympy [C] Result contains complex when optimal does not.
time = 1.38, size = 240, normalized size = 2.53

$$\begin{cases} a^{2} c e x + \frac{a^{2} d e x^{2}}{2} + \frac{a b c^{2} e \operatorname{atan}{\left(c + d x \right)}}{d} + 2 a b c e x \operatorname{atan}{\left(c + d x \right)} + a b d e x^{2} \operatorname{atan}{\left(c + d x \right)} - a b e x + \frac{a b e \operatorname{atan}{\left(c + d x \right)}}{d} + \frac{b^{2} c^{2} e \operatorname{atan}^{2}{\left(c + d x \right)}}{2 d} + b^{2} c e x \operatorname{atan}^{2}{\left(c + d x \right)} - \frac{b^{2} c e \operatorname{atan}{\left(c + d x \right)}}{d} + \frac{b^{2} d e x^{2} \operatorname{atan}^{2}{\left(c + d x \right)}}{2} - b^{2} e x \operatorname{atan}{\left(c + d x \right)} + \frac{b^{2} e \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{d} + \frac{b^{2} e \operatorname{atan}^{2}{\left(c + d x \right)}}{2 d} - \frac{i b^{2} e \operatorname{atan}{\left(c + d x \right)}}{d} & \text{for}\: d \neq 0 \\c e x \left(a + b \operatorname{atan}{\left(c \right)}\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*atan(c + d*x)/d + 2*a*
b*c*e*x*atan(c + d*x) + a*b*d*e*x**2*atan(c + d*x) - a*b*e*x + a*b*e*atan(c
 + d*x)/d + b**2*c**2*e*atan(c + d*x)**2/(2*d) + b**2*c*e*x*atan(c + d*x)**
2 - b**2*c*e*atan(c + d*x)/d + b**2*d*e*x**2*atan(c + d*x)**2/2 - b**2*e*x*
atan(c + d*x) + b**2*e*log(c/d + x - I/d)/d + b**2*e*atan(c + d*x)**2/(2*d)
 - I*b**2*e*atan(c + d*x)/d, Ne(d, 0)), (c*e*x*(a + b*atan(c))**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.61, size = 216, normalized size = 2.27

$$\mathrm{atan}\left(c+d\,x\right)^2\,\left(\frac{e\,b^2\,c^2+e\,b^2}{2\,d}+b^2\,c\,e\,x+\frac{b^2\,d\,e\,x^2}{2}\right)-x\,\left(a\,e\,\left(b-3\,a\,c\right)+2\,a^2\,c\,e\right)-d^2\,\mathrm{atan}\left(c+d\,x\right)\,\left(\frac{x\,\left(b^2\,e-2\,a\,b\,c\,e\right)}{d^2}-\frac{a\,b\,e\,x^2}{d}\right)+\frac{b^2\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{a^2\,d\,e\,x^2}{2}+\frac{b\,e\,\mathrm{atan}\left(\frac{b\,c\,e\,\left(a\,c^2-b\,c+a\right)+b\,d\,e\,x\,\left(a\,c^2-b\,c+a\right)}{-e\,b^2\,c+a\,e\,b\,c^2+a\,e\,b}\right)\,\left(a\,c^2-b\,c+a\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^2,x)
```

```
[Out] atan(c + d*x)^2*((b^2*e + b^2*c^2*e)/(2*d) + b^2*c*e*x + (b^2*d*e*x^2)/2) -
 x*(a*e*(b - 3*a*c) + 2*a^2*c*e) - d^2*atan(c + d*x)*((x*(b^2*e - 2*a*b*c*e
))/d^2 - (a*b*e*x^2)/d) + (b^2*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) +
(a^2*d*e*x^2)/2 + (b*e*atan((b*c*e*(a - b*c + a*c^2) + b*d*e*x*(a - b*c + a
*c^2))/(a*b*e - b^2*c*e + a*b*c^2*e))*(a - b*c + a*c^2))/d
```

3.10 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{ce+dex}\,dx$

Optimal. Leaf size=183

$$\frac{2(a+b\text{ArcTan}(c+dx))^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de}-\frac{ib(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{de}+\frac{ib(a}{}$$

```
[Out] -2*(a+b*arctan(d*x+c))^2*arctanh(-1+2/(1+I*(d*x+c)))/d/e-I*b*(a+b*arctan(d*
x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d/e+I*b*(a+b*arctan(d*x+c))*polylog(2,-1
+2/(1+I*(d*x+c)))/d/e-1/2*b^2*polylog(3,1-2/(1+I*(d*x+c)))/d/e+1/2*b^2*poly
log(3,-1+2/(1+I*(d*x+c)))/d/e
```

Rubi [A]
time = 0.22, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5151, 12, 4942, 5108, 5004, 5114, 6745}

$$-\frac{ib\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))}{de}+\frac{ib\text{Li}_2\left(\frac{2}{i(c+dx)+1}-1\right)(a+b\text{ArcTan}(c+dx))}{de}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{de}-\frac{b^2\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2de}+\frac{b^2\text{Li}_3\left(\frac{2}{i(c+dx)+1}-1\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^2*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*e) + (I*b*
(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 + I*(c + d*x))])/(d*e) - (b^2*
PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*d*e) + (b^2*PolyLog[3, -1 + 2/(1 +
I*(c + d*x))])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan^{-1}(c+dx))^2}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x}\,dx, x, c+dx\right)}{de} \\
&= \frac{2(a+b\tan^{-1}(c+dx))^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(4b)\text{Subst}\Big(\int \frac{(a+b\tan^{-1}(x))\,\text{ta}}{1+}}{c} \\
&= \frac{2(a+b\tan^{-1}(c+dx))^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(2b)\text{Subst}\Big(\int \frac{(a+b\tan^{-1}(x))\,\text{lo}}{1+x^2}}{de} \\
&= \frac{2(a+b\tan^{-1}(c+dx))^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib(a+b\tan^{-1}(c+dx))\,\text{Li}_2}{de} \\
&= \frac{2(a+b\tan^{-1}(c+dx))^2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{ib(a+b\tan^{-1}(c+dx))\,\text{Li}_2}{de}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 170, normalized size = 0.93

$$
\frac{4(a+b\text{ArcTan}(c+dx))^2\tanh^{-1}\left(\frac{i+c+dx}{-i+c+dx}\right)+2ib(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(2,-\frac{i+c+dx}{-i+c+dx}\right)-2ib(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(2,\frac{i+c+dx}{-i+c+dx}\right)+b^2\text{PolyLog}\left(3,-\frac{i+c+dx}{-i+c+dx}\right)-b^2\text{PolyLog}\left(3,\frac{i+c+dx}{-i+c+dx}\right)}{2de}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (4*(a + b*ArcTan[c + d*x])^2*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (2*I)*
b*(a + b*ArcTan[c + d*x])*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (2*
I)*b*(a + b*ArcTan[c + d*x])*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] + b^2
*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))] - b^2*PolyLog[3, (I + c + d*x)
/(-I + c + d*x)])/(2*d*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.71, size = 1362, normalized size = 7.44

method	result	size
derivativedivides	Expression too large to display	1362
default	Expression too large to display	1362

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)-I*a*b/e*ln(d*x+c)*ln(1-I*(d*x+c))+b^2/e*arctan(d*x+c)^
2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+b^2/e*ln(d*x+c)*arctan(d*x+c)^2-b
^2/e*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+b^2/e*arctan(d*x+c
)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+2*a*b/e*ln(d*x+c)*arctan(d*x+c)
+I*a*b/e*dilog(1+I*(d*x+c))-I*a*b/e*dilog(1-I*(d*x+c))+I*b^2/e*arctan(d*x+c
)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-2*I*b^2/e*arctan(d*x+c)*polylog
(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-2*I*b^2/e*arctan(d*x+c)*polylog(2,-(1
+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*b^2/e*Pi*arctan(d*x+c)^2-1/2*I*b^2/e
*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))^2*arctan(d*x+c)^2-1/2*I*b^2/e*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))^2*arctan(d*x+c)^2+1/2*I*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arctan(d*x+c)^2-1/2*I*b^2/e*Pi*csgn(I
*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2-1/2*b^2/e*polylog(3,
-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+2*b^2/e*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)
^2)^(1/2))+2*b^2/e*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/2*I*b^2/e
*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)
))^2*arctan(d*x+c)^2+I*a*b/e*ln(d*x+c)*ln(1+I*(d*x+c))+1/2*I*b^2/e*Pi*csgn(
I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*ar
ctan(d*x+c)^2+1/2*I*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I
*(d*x+c))^2/(1+(d*x+c)^2)))^3*arctan(d*x+c)^2+1/2*I*b^2/e*Pi*csgn(I*((1+I*(
d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(
I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arct
an(d*x+c)^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(d*x*e + c*e)/d + integrate(1/16*(12*b^2*arctan(d*x + c)^2 +
b^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(d*x*e + c
*e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*e^(-1)/(d*x
+ c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2\operatorname{atan}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab\operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*atan(c + d*x)**2/(c + d*x), x)
 + Integral(2*a*b*atan(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atan}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x), x)
```

3.11 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{(ce+dex)^2} dx$

Optimal. Leaf size=119

$$-\frac{i(a+b\text{ArcTan}(c+dx))^2}{de^2}-\frac{(a+b\text{ArcTan}(c+dx))^2}{de^2(c+dx)}+\frac{2b(a+b\text{ArcTan}(c+dx))\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2}-\frac{ib^2\text{Po}}{}$$

[Out] -I*(a+b*arctan(d*x+c))^2/d/e^2-(a+b*arctan(d*x+c))^2/d/e^2/(d*x+c)+2*b*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^2-I*b^2*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^2

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$,
Rules used = {5151, 12, 4946, 5044, 4988, 2497}

$$-\frac{(a+b\text{ArcTan}(c+dx))^2}{de^2(c+dx)}-\frac{i(a+b\text{ArcTan}(c+dx))^2}{de^2}+\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{de^2}-\frac{ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)}{de^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]

[Out] ((-I)*(a + b*ArcTan[c + d*x])^2)/(d*e^2) - (a + b*ArcTan[c + d*x])^2/(d*e^2*(c + d*x)) + (2*b*(a + b*ArcTan[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^2) - (I*b^2*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(d*e^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^2}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{(2ib)\text{Subst}\left(\int \frac{a+b\tan^{-1}(}{x(i+x)}\right.}{de^2}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^2}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{2b(a+b\tan^{-1}(c+dx)}{de^}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^2}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^2}{de^2(c+dx)} + \frac{2b(a+b\tan^{-1}(c+dx)}{de^}$$

Mathematica [A]

time = 0.16, size = 135, normalized size = 1.13

$$\frac{-ib^2(-i+c+dx)\text{ArcTan}(c+dx)^2+2b\text{ArcTan}(c+dx)\left(-a+b(c+dx)\log\left(1-e^{2i\text{ArcTan}(c+dx)}\right)\right)+a\left(-a+2b(c+dx)\log\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}\right)\right)-ib^2(c+dx)\text{PolyLog}\left(2,e^{2i\text{ArcTan}(c+dx)}\right)}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*b^2*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*b*ArcTan[c + d*x]*(-a + b*(c
 + d*x)*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + a*(-a + 2*b*(c + d*x)*Log[(c
+ d*x)/Sqrt[1 + (c + d*x)^2]]) - I*b^2*(c + d*x)*PolyLog[2, E^((2*I)*ArcTan
[c + d*x])])/(d*e^2*(c + d*x))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(115) = 230$.
time = 0.60, size = 418, normalized size = 3.51

method	result
derivativedivides	$-\frac{a^2}{e^2(dx+c)}-\frac{b^2\arctan(dx+c)^2}{e^2(dx+c)}-\frac{b^2\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{e^2}+\frac{2b^2\ln(dx+c)\arctan(dx+c)}{e^2}-\frac{ib^2\ln(dx+c+i)^2}{4e^2}-\frac{ib^2\operatorname{dilog}\left(\frac{i(dx}{}\right.}{2e^2}$
default	$-\frac{a^2}{e^2(dx+c)}-\frac{b^2\arctan(dx+c)^2}{e^2(dx+c)}-\frac{b^2\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{e^2}+\frac{2b^2\ln(dx+c)\arctan(dx+c)}{e^2}-\frac{ib^2\ln(dx+c+i)^2}{4e^2}-\frac{ib^2\operatorname{dilog}\left(\frac{i(dx}{}\right.}{2e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)-b^2/e^2/(d*x+c)*arctan(d*x+c)^2-b^2/e^2*arctan(d*x+c)
*ln(1+(d*x+c)^2)+2*b^2/e^2*ln(d*x+c)*arctan(d*x+c)-1/4*I*b^2/e^2*ln(d*x+c+I
)^2-1/2*I*b^2/e^2*dilog(1/2*I*(d*x+c-I))-I*b^2/e^2*dilog(1-I*(d*x+c))+1/4*I
*b^2/e^2*ln(d*x+c-I)^2-I*b^2/e^2*ln(d*x+c)*ln(1-I*(d*x+c))-1/2*I*b^2/e^2*ln
(d*x+c+I)*ln(1/2*I*(d*x+c-I))+I*b^2/e^2*ln(d*x+c)*ln(1+I*(d*x+c))+1/2*I*b^2
/e^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+1/2*I*b^2/e^2*dilog(-1/2*I*(d*x+c+I))
-1/2*I*b^2/e^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)+I*b^2/e^2*dilog(1+I*(d*x+c))+1/2
*I*b^2/e^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)-2*a*b/e^2/(d*x+c)*arctan(d*x+c)-a*b/
e^2*ln(1+(d*x+c)^2)+2*a*b/e^2*ln(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(e^(-2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2 - 2*e^(-2)*log(d*x + c)/d^
2) + 2*arctan(d*x + c)/(d^2*x*e^2 + c*d*e^2))*a*b - 1/16*(4*arctan(d*x + c)
^2 - 16*(d^2*x*e^2 + c*d*e^2)*integrate(1/16*(12*(d^2*x^2 + 2*c*d*x + c^2 +
 1)*arctan(d*x + c)^2 + (d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 8*(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2)*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*e^
2 + e^2)*d^2*x^2 + c^4*e^2 + 2*(2*c^3*e^2 + c*e^2)*d*x + c^2*e^2), x) - log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)*b^2/(d^2*x*e^2 + c*d*e^2) - a^2/(d^2*x*e^2
 + c*d*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*e^(-2)/(d^2*
x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*atan(c + d*
x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*atan(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2,x)
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.12 $\int \frac{(a+b\mathbf{ArcTan}(c+dx))^2}{(ce+dex)^3} \, dx$

Optimal. Leaf size=117

$$-\frac{b(a+b\text{ArcTan}(c+dx))}{de^3(c+dx)}-\frac{(a+b\text{ArcTan}(c+dx))^2}{2de^3}-\frac{(a+b\text{ArcTan}(c+dx))^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left(1+(\right.}{2de}$$

```
[Out] -b*(a+b*arctan(d*x+c))/d/e^3/(d*x+c)-1/2*(a+b*arctan(d*x+c))^2/d/e^3-1/2*(a
+b*arctan(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-1/2*b^2*ln(1+(d*x+c
)^2)/d/e^3
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5151, 12, 4946, 5038, 272, 36, 29, 31, 5004}

$$-\frac{b(a+b\text{ArcTan}(c+dx))}{de^3(c+dx)}-\frac{(a+b\text{ArcTan}(c+dx))^2}{2de^3(c+dx)^2}-\frac{(a+b\text{ArcTan}(c+dx))^2}{2de^3}+\frac{b^2\log(c+dx)}{de^3}-\frac{b^2\log\left((c+dx)^2+1\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*(a + b*ArcTan[c + d*x]))/(d*e^3*(c + d*x))) - (a + b*ArcTan[c + d*x])^
2/(2*d*e^3) - (a + b*ArcTan[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c
+ d*x])/(d*e^3) - (b^2*Log[1 + (c + d*x)^2])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2(1+x^2)}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2} + \frac{b\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\,dx, x, c+dx\right)}{de^3} - \frac{b\text{Subst}\left(\int \right.}{}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)^2}$$

Mathematica [A]
time = 0.08, size = 194, normalized size = 1.66

$$-\frac{a^2+2abc+2abdx+2b(b(c+dx)+a(1+c^2+2cdx+d^2x^2))\,\text{ArcTan}(c+dx)+b^2(1+c^2+2cdx+d^2x^2)\,\text{ArcTan}(c+dx)^2-2b^2(c+dx)^2\log(c+dx)+b^2c^2\log\left(1+c^2+2cdx+d^2x^2\right)+2b^2cdx\log\left(1+c^2+2cdx+d^2x^2\right)+b^2d^2x^2\log\left(1+c^2+2cdx+d^2x^2\right)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c + 2*a*b*d*x + 2*b*(b*(c + d*x) + a*(1 + c^2 + 2*c*d*x +
 d^2*x^2))*ArcTan[c + d*x] + b^2*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d
*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x] + b^2*c^2*Log[1 + c^2 + 2*c*d*x + d^
2*x^2] + 2*b^2*c*d*x*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + b^2*d^2*x^2*Log[1 +
 c^2 + 2*c*d*x + d^2*x^2])/(d*e^3*(c + d*x)^2)
```

Maple [A]
time = 0.21, size = 159, normalized size = 1.36

method	result

derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2\arctan(dx+c)^2}{2e^3(dx+c)^2}-\frac{b^2\arctan(dx+c)}{e^3(dx+c)}-\frac{b^2\arctan(dx+c)^2}{2e^3}-\frac{b^2\ln\left(1+(dx+c)^2\right)}{2e^3}+\frac{b^2\ln(dx+c)}{e^3}-\frac{ab\arctan(dx+c)}{e^3(dx+c)^2}-\frac{}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2\arctan(dx+c)^2}{2e^3(dx+c)^2}-\frac{b^2\arctan(dx+c)}{e^3(dx+c)}-\frac{b^2\arctan(dx+c)^2}{2e^3}-\frac{b^2\ln\left(1+(dx+c)^2\right)}{2e^3}+\frac{b^2\ln(dx+c)}{e^3}-\frac{ab\arctan(dx+c)}{e^3(dx+c)^2}-\frac{}{e^3}}{d}$
risch	$\frac{b^2(d^2x^2+2cdx+c^2+1)\ln(1+i(dx+c))^2}{8e^3(dx+c)^2d}-\frac{b(b\,d^2x^2\ln(1-i(dx+c))+2bcdx\ln(1-i(dx+c))-2ibdx+\ln(1-i(dx+c))b\,c^2-}{4e^3(dx+c)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2-1/2*b^2/e^3/(d*x+c)^2*arctan(d*x+c)^2-b^2/e^3*a
rctan(d*x+c)/(d*x+c)-1/2*b^2/e^3*arctan(d*x+c)^2-1/2*b^2/e^3*ln(1+(d*x+c)^2
)+b^2/e^3*ln(d*x+c)-a*b/e^3/(d*x+c)^2*arctan(d*x+c)-a*b/e^3/(d*x+c)-a*b/e^3
*arctan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(106) = 212$.
time = 0.50, size = 252, normalized size = 2.15

$$-\left(d\left(\frac{\arctan\left(\frac{d^2x+cd}{d}\right)e^{(-3)}}{d^2}+\frac{1}{d^3xe^3+cd^2e^3}\right)+\frac{\arctan(dx+c)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)ab-\frac{1}{2}\left(2\,d\left(\frac{\arctan\left(\frac{d^2x+cd}{d}\right)e^{(-3)}}{d^2}+\frac{1}{d^3xe^3+cd^2e^3}\right)\arctan(dx+c)-\frac{\left(\arctan(dx+c)^2-\log(d^2x^2+2\,cdx+c^2+1)+2\,\log(dx+c)\right)e^{(-3)}}{d}\right)b^2-\frac{b^2\arctan(dx+c)^2}{2\,(d^3x^2e^3+2\,cd^2xe^3+c^2de^3)}-\frac{a^2}{2\,(d^3x^2e^3+2\,cd^2xe^3+c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(d*(arctan((d^2*x + c*d)/d)*e^(-3)/d^2 + 1/(d^3*x*e^3 + c*d^2*e^3)) + arct
an(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3))*a*b - 1/2*(2*d*(arct
an((d^2*x + c*d)/d)*e^(-3)/d^2 + 1/(d^3*x*e^3 + c*d^2*e^3))*arctan(d*x + c)
 - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x + c))*
e^(-3)/d)*b^2 - 1/2*b^2*arctan(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^
2*d*e^3) - 1/2*a^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [A]
time = 3.32, size = 202, normalized size = 1.73

$$-\frac{\left(2\,abdx+2\,abc+(b^2d^2x^2+2\,b^2cdx+b^2c^2+b^2)\arctan(dx+c)^2+a^2+2\,(abd^2x^2+abc^2+b^2c+(2\,abc+b^2)dx+ab)\arctan(dx+c)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(d^2x^2+2\,cdx+c^2+1)-2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(dx+c)\right)e^{(-3)}}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*d*x + 2*a*b*c + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + b^2)*arc
tan(d*x + c)^2 + a^2 + 2*(a*b*d^2*x^2 + a*b*c^2 + b^2*c + (2*a*b*c + b^2)*d
*x + a*b)*arctan(d*x + c) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d^2*x
^2 + 2*c*d*x + c^2 + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x +
 c))*e^(-3)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.43, size = 1107, normalized size = 9.46

$$\begin{cases} - \frac{a^{2}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 a b c^{2} \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{4 a b c d x \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 a b c}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 a b d^{2} x^{2} \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 a b d x}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 a b \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{2 b^{2} c^{2} \log{\left(\frac{c}{d} + x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 b^{2} c^{2} \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{b^{2} c^{2} \operatorname{atan}^{2}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{2 i b^{2} c^{2} \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{4 b^{2} c d x \log{\left(\frac{c}{d} + x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{4 b^{2} c d x \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 b^{2} c d x \operatorname{atan}^{2}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{4 i b^{2} c d x \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 b^{2} c \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{2 b^{2} d^{2} x^{2} \log{\left(\frac{c}{d} + x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 b^{2} d^{2} x^{2} \log{\left(\frac{c}{d} + x - \frac{i}{d} \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{b^{2} d^{2} x^{2} \operatorname{atan}^{2}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} + \frac{2 i b^{2} d^{2} x^{2} \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{2 b^{2} d x \operatorname{atan}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} - \frac{b^{2} \operatorname{atan}^{2}{\left(c + d x \right)}}{2 c^{2} d e^{3} + 4 c d^{2} e^{3} x + 2 d^{3} e^{3} x^{2}} & \text{for}\: d \neq 0 \\ \frac{x \left(a + b \operatorname{atan}{\left(c \right)}\right)^{2}}{c^{3} e^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] Piecewise((-a**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a
*b*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 4*a*b*c*d*x*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*
x**2) - 2*a*b*c/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*d**2*x**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x*
*2) - 2*a*b*d*x/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*a*
b*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b*
*2*c**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) -
 2*b**2*c**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e
**3*x**2) - b**2*c**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2
*d**3*e**3*x**2) + 2*I*b**2*c**2*atan(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e*
*3*x + 2*d**3*e**3*x**2) + 4*b**2*c*d*x*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d
**2*e**3*x + 2*d**3*e**3*x**2) - 4*b**2*c*d*x*log(c/d + x - I/d)/(2*c**2*d*
e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*d*x*atan(c + d*x)**2/
(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 4*I*b**2*c*d*x*atan(
c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*c*at
an(c + d*x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) + 2*b**2*d
**2*x**2*log(c/d + x)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2)
- 2*b**2*d**2*x**2*log(c/d + x - I/d)/(2*c**2*d*e**3 + 4*c*d**2*e**3*x + 2*
d**3*e**3*x**2) - b**2*d**2*x**2*atan(c + d*x)**2/(2*c**2*d*e**3 + 4*c*d**2
*e**3*x + 2*d**3*e**3*x**2) + 2*I*b**2*d**2*x**2*atan(c + d*x)/(2*c**2*d*e*
*3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - 2*b**2*d*x*atan(c + d*x)/(2*c**2
*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2) - b**2*atan(c + d*x)**2/(2*c*
*2*d*e**3 + 4*c*d**2*e**3*x + 2*d**3*e**3*x**2), Ne(d, 0)), (x*(a + b*atan(
c))**2/(c**3*e**3), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.86, size = 232, normalized size = 1.98

$$\frac{b^2\,\ln\left(c+d\,x\right)}{d\,e^3}-\frac{\frac{a^2+2\,b\,c\,a}{2\,d}+a\,b\,x}{c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2}-\frac{\mathrm{atan}\left(c+d\,x\right)\,\left(\frac{b^2\,c}{d^3\,e^3}+\frac{b^2\,x}{d^2\,e^3}+\frac{a\,b}{d^3\,e^3}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}}-{\mathrm{atan}\left(c+d\,x\right)}^2\,\left(\frac{b^2}{2\,d\,e^3}+\frac{b^2}{2\,d^3\,e^3\,\left(x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}\right)}\right)+\frac{\ln\left(c+d\,x-\mathrm{i}\right)\,\left(-\frac{b^2}{2}+\frac{a\,b\,1{}\mathrm{i}}{2}\right)}{d\,e^3}-\frac{\ln\left(c+d\,x+1{}\mathrm{i}\right)\,\left(\frac{b^2}{2}+\frac{1{}\mathrm{i}\,a\,b}{2}\right)}{d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/(d*e^3) - ((a^2 + 2*a*b*c)/(2*d) + a*b*x)/(c^2*e^3 + d^2
*e^3*x^2 + 2*c*d*e^3*x) - (atan(c + d*x)*((b^2*c)/(d^3*e^3) + (b^2*x)/(d^2*
e^3) + (a*b)/(d^3*e^3)))/(x^2 + c^2/d^2 + (2*c*x)/d) - atan(c + d*x)^2*(b^2
/(2*d*e^3) + b^2/(2*d^3*e^3*(x^2 + c^2/d^2 + (2*c*x)/d))) + (log(c + d*x -
1i)*((a*b*1i)/2 - b^2/2))/(d*e^3) - (log(c + d*x + 1i)*((a*b*1i)/2 + b^2/2)
)/(d*e^3)
```

3.13 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=194

$$-\frac{b^2}{3de^4(c+dx)}-\frac{b^2\text{ArcTan}(c+dx)}{3de^4}-\frac{b(a+b\text{ArcTan}(c+dx))}{3de^4(c+dx)^2}+\frac{i(a+b\text{ArcTan}(c+dx))^2}{3de^4}-\frac{(a+b\text{ArcTan}(c+dx))^2}{3de^4(c+dx)^3}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*b^2*arctan(d*x+c)/d/e^4-1/3*b*(a+b*arctan(d*x+c)
)/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^2/d/e^4-1/3*(a+b*arctan(d*x+c))
^2/d/e^4/(d*x+c)^3-2/3*b*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^4+1/
3*I*b^2*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A]
time = 0.19, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5151, 12, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$-\frac{b(a+b\text{ArcTan}(c+dx))}{3de^4(c+dx)^2}-\frac{(a+b\text{ArcTan}(c+dx))^2}{3de^4(c+dx)^3}+\frac{i(a+b\text{ArcTan}(c+dx))^2}{3de^4}-\frac{2b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{3de^4}-\frac{b^2\text{ArcTan}(c+dx)}{3de^4}+\frac{ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)}{3de^4}-\frac{b^2}{3de^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*b^2/(d*e^4*(c + d*x)) - (b^2*ArcTan[c + d*x])/(3*d*e^4) - (b*(a + b*Ar
cTan[c + d*x]))/(3*d*e^4*(c + d*x)^2) + ((I/3)*(a + b*ArcTan[c + d*x])^2)/(
d*e^4) - (a + b*ArcTan[c + d*x])^2/(3*d*e^4*(c + d*x)^3) - (2*b*(a + b*ArcT
an[c + d*x])*Log[2 - 2/(1 - I*(c + d*x))])/(3*d*e^4) + ((I/3)*b^2*PolyLog[2
, -1 + 2/(1 - I*(c + d*x))])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3(1+x^2)}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{3de^4} - \frac{(2b)\text{Sub}}{}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\tan^{-1}(c+dx))^2}{3de^4} - \frac{(a+b\tan^{-1}(c+dx))^2}{3de^4(c+dx)^3}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b(a+b\tan^{-1}(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\tan^{-1}(c+dx))^2}{3de^4} - \frac{(a+b}{3d}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b^2\tan^{-1}(c+dx)}{3de^4} - \frac{b(a+b\tan^{-1}(c+dx))}{3de^4(c+dx)^2} + \frac{i(a+b\tan^{-1}}{3de}$$

Mathematica [A]
time = 0.53, size = 163, normalized size = 0.84

$$-\frac{ab+\frac{a^2}{(c+dx)^3}+\frac{ab}{(c+dx)^2}+\frac{b^2}{c+dx}+b^2\left(-i+\frac{1}{(c+dx)^3}\right)\text{ArcTan}(c+dx)^2+b\text{ArcTan}(c+dx)\left(b+\frac{2a}{(c+dx)^3}+\frac{b}{(c+dx)^2}+2b\log\left(1-e^{2i\text{ArcTan}(c+dx)}\right)\right)+2ab\log\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}\right)-ib^2\text{PolyLog}\left(2,e^{2i\text{ArcTan}(c+dx)}\right)}{3de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*(a*b + a^2/(c + d*x)^3 + (a*b)/(c + d*x)^2 + b^2/(c + d*x) + b^2*(-I +
 (c + d*x)^(-3))*ArcTan[c + d*x]^2 + b*ArcTan[c + d*x]*(b + (2*a)/(c + d*x)
^3 + b/(c + d*x)^2 + 2*b*Log[1 - E^((2*I)*ArcTan[c + d*x])]) + 2*a*b*Log[(c
 + d*x)/Sqrt[1 + (c + d*x)^2]] - I*b^2*PolyLog[2, E^((2*I)*ArcTan[c + d*x])
])/(d*e^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(176) = 352.
time = 0.63, size = 482, normalized size = 2.48

method	result

derivativedivides	$\frac{-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\arctan(dx+c)^2}{3e^4(dx+c)^3}+\frac{b^2\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3e^4}-\frac{b^2\arctan(dx+c)}{3e^4(dx+c)^2}-\frac{2b^2\ln(dx+c)\arctan(dx+c)}{3e^4}-\frac{ib^2\,\mathrm{dilog}(1+}{3e^4}}{}$
default	$\frac{-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\arctan(dx+c)^2}{3e^4(dx+c)^3}+\frac{b^2\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{3e^4}-\frac{b^2\arctan(dx+c)}{3e^4(dx+c)^2}-\frac{2b^2\ln(dx+c)\arctan(dx+c)}{3e^4}-\frac{ib^2\,\mathrm{dilog}(1+}{3e^4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3-1/3*b^2/e^4/(d*x+c)^3*arctan(d*x+c)^2+1/3*b^2/e
^4*arctan(d*x+c)*ln(1+(d*x+c)^2)-1/3*b^2/e^4*arctan(d*x+c)/(d*x+c)^2-2/3*b^
2/e^4*ln(d*x+c)*arctan(d*x+c)+1/6*I*b^2/e^4*dilog(1/2*I*(d*x+c-I))-1/6*I*b^
2/e^4*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-1/3*I*b^2/e^4*ln(d*x+c)*ln(1+I*(d*x+
c))-1/12*I*b^2/e^4*ln(d*x+c-I)^2+1/6*I*b^2/e^4*ln(d*x+c+I)*ln(1/2*I*(d*x+c-
I))+1/6*I*b^2/e^4*ln(d*x+c-I)*ln(1+(d*x+c)^2)+1/3*I*b^2/e^4*dilog(1-I*(d*x+
c))+1/3*I*b^2/e^4*ln(d*x+c)*ln(1-I*(d*x+c))-1/3*b^2/e^4/(d*x+c)-1/3*b^2/e^4
*arctan(d*x+c)+1/12*I*b^2/e^4*ln(d*x+c+I)^2-1/6*I*b^2/e^4*ln(d*x+c+I)*ln(1+
(d*x+c)^2)-1/3*I*b^2/e^4*dilog(1+I*(d*x+c))-1/6*I*b^2/e^4*dilog(-1/2*I*(d*x
+c+I))-2/3*a*b/e^4/(d*x+c)^3*arctan(d*x+c)+1/3*a*b/e^4*ln(1+(d*x+c)^2)-1/3*
a*b/e^4/(d*x+c)^2-2/3*a*b/e^4*ln(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(d*(e^(-4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2 - 2*e^(-4)*log(d*x + c)
/d^2 - 1/(d^4*x^2*e^4 + 2*c*d^3*x*e^4 + c^2*d^2*e^4)) - 2*arctan(d*x + c)/(
d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4))*a*b - 1/48*(4
*arctan(d*x + c)^2 - 48*(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 +
c^3*d*e^4)*integrate(1/48*(36*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)
^2 + 3*(d^2*x^2 + 2*c*d*x + c^2 + 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 8
*(d*x + c)*arctan(d*x + c) - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*e^4 + e^4)*d^4*x^4
 + 4*(5*c^3*e^4 + c*e^4)*d^3*x^3 + c^6*e^4 + 3*(5*c^4*e^4 + 2*c^2*e^4)*d^2*
x^2 + c^4*e^4 + 2*(3*c^5*e^4 + 2*c^3*e^4)*d*x), x) - log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)^2)*b^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*
e^4) - 1/3*a^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*e^(-4)/(d^4*
x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2ab\operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*atan(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*atan(c + d*x)/(c**
4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.14 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{(ce+dex)^5}\,dx$

Optimal. Leaf size=170

$$-\frac{b^2}{12de^5(c+dx)^2}-\frac{b(a+b\text{ArcTan}(c+dx))}{6de^5(c+dx)^3}+\frac{b(a+b\text{ArcTan}(c+dx))}{2de^5(c+dx)}+\frac{(a+b\text{ArcTan}(c+dx))^2}{4de^5}-\frac{(a+b\text{Arc}}{4de}$$

```
[Out] -1/12*b^2/d/e^5/(d*x+c)^2-1/6*b*(a+b*arctan(d*x+c))/d/e^5/(d*x+c)^3+1/2*b*(
a+b*arctan(d*x+c))/d/e^5/(d*x+c)+1/4*(a+b*arctan(d*x+c))^2/d/e^5-1/4*(a+b*a
rctan(d*x+c))^2/d/e^5/(d*x+c)^4-2/3*b^2*ln(d*x+c)/d/e^5+1/3*b^2*ln(1+(d*x+c
)^2)/d/e^5
```

Rubi [A]
time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5151, 12, 4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\frac{b(a+b\text{ArcTan}(c+dx))}{2de^5(c+dx)}-\frac{b(a+b\text{ArcTan}(c+dx))}{6de^5(c+dx)^3}-\frac{(a+b\text{ArcTan}(c+dx))^2}{4de^5(c+dx)^4}+\frac{(a+b\text{ArcTan}(c+dx))^2}{4de^5}-\frac{b^2}{12de^5(c+dx)^2}-\frac{2b^2\log(c+dx)}{3de^5}+\frac{b^2\log\left((c+dx)^2+1\right)}{3de^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*b^2/(d*e^5*(c + d*x)^2) - (b*(a + b*ArcTan[c + d*x]))/(6*d*e^5*(c + d
*x)^3) + (b*(a + b*ArcTan[c + d*x]))/(2*d*e^5*(c + d*x)) + (a + b*ArcTan[c
+ d*x])^2/(4*d*e^5) - (a + b*ArcTan[c + d*x])^2/(4*d*e^5*(c + d*x)^4) - (2*
b^2*Log[c + d*x])/(3*d*e^5) + (b^2*Log[1 + (c + d*x)^2])/(3*d*e^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{e^5x^5}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^5}\,dx, x, c+dx\right)}{de^5}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4(1+x^2)}\,dx, x, c+dx\right)}{2de^5}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{2de^5} - \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{6de^5(c+dx)^3} - \frac{(a+b\tan^{-1}(c+dx))^2}{4de^5(c+dx)^4} - \frac{b\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{x^2}\right.}{2de^5}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\tan^{-1}(c+dx))}{2de^5(c+dx)} + \frac{(a+b\tan^{-1}(c+dx))^2}{4de^5}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\tan^{-1}(c+dx))}{2de^5(c+dx)} + \frac{(a+b\tan^{-1}(c+dx))^2}{4de^5}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b(a+b\tan^{-1}(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\tan^{-1}(c+dx))}{2de^5(c+dx)} + \frac{(a-}{}$$

$$= -\frac{b^2}{12de^5(c+dx)^2} - \frac{b(a+b\tan^{-1}(c+dx))}{6de^5(c+dx)^3} + \frac{b(a+b\tan^{-1}(c+dx))}{2de^5(c+dx)} + \frac{(a-}{}$$

Mathematica [A]
time = 0.21, size = 245, normalized size = 1.44

$$-\frac{3a^2+2ab(c+dx)+b^2(c+dx)^2-6ab(c+dx)^3-2b(b(-c+3c^3-dx+9c^2dx+9cd^2x^2+3d^3x^3)+3a(-1+c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4))\text{ArcTan}(c+dx)-3b^2(-1+c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4)\text{ArcTan}(c+dx)^2+8b^2(c+dx)^4\log(c+dx)-4b^2(c+dx)^4\log(1+c^2+2cdx+d^2x^2)}{12de^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(3*a^2 + 2*a*b*(c + d*x) + b^2*(c + d*x)^2 - 6*a*b*(c + d*x)^3 - 2*b*
(b*(-c + 3*c^3 - d*x + 9*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3) + 3*a*(-1 + c^4
 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4))*ArcTan[c + d*x] - 3*
b^2*(-1 + c^4 + 4*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)*ArcTan[c
 + d*x]^2 + 8*b^2*(c + d*x)^4*Log[c + d*x] - 4*b^2*(c + d*x)^4*Log[1 + c^2
+ 2*c*d*x + d^2*x^2])/(d*e^5*(c + d*x)^4)
```

Maple [A]
time = 0.28, size = 210, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{a^2}{4e^5(dx+c)^4}-\frac{b^2\arctan(dx+c)^2}{4e^5(dx+c)^4}+\frac{b^2\arctan(dx+c)^2}{4e^5}-\frac{b^2\arctan(dx+c)}{6e^5(dx+c)^3}+\frac{b^2\arctan(dx+c)}{2e^5(dx+c)}+\frac{b^2\ln\left(1+(dx+c)^2\right)}{3e^5}-\frac{b^2}{12e^5(dx+c)^2}-2b^2}{d}$
default	$\frac{-\frac{a^2}{4e^5(dx+c)^4}-\frac{b^2\arctan(dx+c)^2}{4e^5(dx+c)^4}+\frac{b^2\arctan(dx+c)^2}{4e^5}-\frac{b^2\arctan(dx+c)}{6e^5(dx+c)^3}+\frac{b^2\arctan(dx+c)}{2e^5(dx+c)}+\frac{b^2\ln\left(1+(dx+c)^2\right)}{3e^5}-\frac{b^2}{12e^5(dx+c)^2}-2b^2}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a^2/e^5/(d*x+c)^4-1/4*b^2/e^5/(d*x+c)^4*arctan(d*x+c)^2+1/4*b^2/e
^5*arctan(d*x+c)^2-1/6*b^2/e^5*arctan(d*x+c)/(d*x+c)^3+1/2*b^2/e^5*arctan(d
*x+c)/(d*x+c)+1/3*b^2/e^5*ln(1+(d*x+c)^2)-1/12*b^2/e^5/(d*x+c)^2-2/3*b^2/e^
5*ln(d*x+c)-1/2*a*b/e^5/(d*x+c)^4*arctan(d*x+c)+1/2*a*b/e^5*arctan(d*x+c)-1
/6*a*b/e^5/(d*x+c)^3+1/2*a*b/e^5/(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(149) = 298$.
time = 0.57, size = 506, normalized size = 2.98

$$\frac{1}{6}\left(d\left(\frac{3d^2x^2+6cdx+3c^2-1}{d^5x^3e^5+3cd^4x^2e^5+3c^2d^3xe^5+c^3d^2e^5}+\frac{3\arctan\left(\frac{d^2x+cd}{d}\right)e^{(-5)}}{d^2}\right)-\frac{3\arctan(dx+c)}{d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5}\right)ab+\frac{1}{12}\left(2d\left(\frac{3d^2x^2+6cdx+3c^2-1}{d^5x^3e^5+3cd^4x^2e^5+3c^2d^3xe^5+c^3d^2e^5}+\frac{3\arctan\left(\frac{d^2x+cd}{d}\right)e^{(-5)}}{d^2}\right)\arctan(dx+c)-\frac{\left(3\left(d^2x^2+2cdx+c^2\right)\arctan(dx+c)^2-4\left(d^2x^2+2cdx+c^2\right)\log\left(d^2x^2+2cdx+c^2+1\right)+8\left(d^2x^2+2cdx+c^2\right)\log(dx+c)+1\right)d^2}{d^5x^2e^5+2cd^4xe^5+c^2d^3e^5}\right)b^2-\frac{b^2\arctan(dx+c)^2}{4\left(d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5\right)}-\frac{a^2}{4\left(d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/(d^5*x^3*e^5 + 3*c*d^4*x^2*e^5 +
3*c^2*d^3*x*e^5 + c^3*d^2*e^5) + 3*arctan((d^2*x + c*d)/d)*e^(-5)/d^2) - 3*
arctan(d*x + c)/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*
d^2*x*e^5 + c^4*d*e^5))*a*b + 1/12*(2*d*((3*d^2*x^2 + 6*c*d*x + 3*c^2 - 1)/
(d^5*x^3*e^5 + 3*c*d^4*x^2*e^5 + 3*c^2*d^3*x*e^5 + c^3*d^2*e^5) + 3*arctan(
(d^2*x + c*d)/d)*e^(-5)/d^2)*arctan(d*x + c) - (3*(d^2*x^2 + 2*c*d*x + c^2)
*arctan(d*x + c)^2 - 4*(d^2*x^2 + 2*c*d*x + c^2)*log(d^2*x^2 + 2*c*d*x + c^
2 + 1) + 8*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c) + 1)*d^2/(d^5*x^2*e^5 + 2
*c*d^4*x*e^5 + c^2*d^3*e^5))*b^2 - 1/4*b^2*arctan(d*x + c)^2/(d^5*x^4*e^5 +
 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c^4*d*e^5) - 1/4*a
^2/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c
^4*d*e^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(149) = 298$.
time = 2.71, size = 435, normalized size = 2.56

$$\frac{\left(6abd^3x^3+6abc^3+\left(18abc-b^2\right)d^2x^2-b^2c^2-2abc+2\left(9abc^2-b^2c-ab\right)dx+3\left(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4-b^2\right)\arctan(dx+c)^2-3a^2+2\left(3abd^4x^4+3\left(4abc+b^2\right)d^3x^3+3abc^4+3b^2c^3+9\left(2abc^2+b^2c\right)d^2x^2-b^2c+\left(12abc^3+9b^2c^2-b^2\right)dx-3ab\right)\arctan(dx+c)+4\left(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4\right)\log\left(d^2x^2+2cdx+c^2+1\right)-8\left(b^2d^4x^4+4b^2cd^3x^3+6b^2c^2d^2x^2+4b^2c^3dx+b^2c^4\right)\log(dx+c)\right)e^{(-5)}}{12\left(d^5x^4+4cd^4x^3+6c^2d^3x^2+4c^3d^2x+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="fricas")

[Out] 1/12*(6*a*b*d^3*x^3 + 6*a*b*c^3 + (18*a*b*c - b^2)*d^2*x^2 - b^2*c^2 - 2*a*b*c + 2*(9*a*b*c^2 - b^2*c - a*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - b^2)*arctan(d*x + c)^2 - 3*a^2 + 2*(3*a*b*d^4*x^4 + 3*(4*a*b*c + b^2)*d^3*x^3 + 3*a*b*c^4 + 3*b^2*c^3 + 9*(2*a*b*c^2 + b^2*c)*d^2*x^2 - b^2*c + (12*a*b*c^3 + 9*b^2*c^2 - b^2)*d*x - 3*a*b)*arctan(d*x + c) + 4*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 8*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(d*x + c))*e^(-5)/(d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*atan(d*x+c))**2/(d*e*x+c*e)**5,x)

[Out] Timed out

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arctan(d*x+c))^2/(d*e*x+c*e)^5,x, algorithm="giac")

[Out] Timed out

Mupad [**B**]
time = 3.65, size = 438, normalized size = 2.58

$$\mathrm{atan}(c+d\,x)^2\left(\frac{b^2}{4\,d\,e^5}-\frac{b^2}{4\,d^3\,e^5\,\left(\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3\right)}\right)-\frac{x^2\left(\frac{b^2\,d}{2}-9\,a\,b\,c\,d\right)+x\,\left(b^2\,c-9\,a\,b\,c^2+a\,b\right)+\frac{3\,a^2-6\,a\,b\,c^3+2\,a\,b\,c+b^2\,c^2}{2\,d}-3\,a\,b\,d^2\,x^3}{6\,c^4\,e^5+24\,c^3\,d\,e^5\,x+36\,c^2\,d^2\,e^5\,x^2+24\,c\,d^3\,e^5\,x^3+6\,d^4\,e^5\,x^4}+\frac{\mathrm{atan}(c+d\,x)\,\left(\frac{b^2\,x^3}{2\,e^5}-\frac{a\,b}{2\,d^3\,e^5}+\frac{b^2\,c\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)}{2\,d\,e^5}+\frac{b^2\,x\,\left(d\,\left(\frac{c^2-1}{3\,d^2}+\frac{2\,c^2}{3\,d^2}\right)+\frac{2\,c^2}{d}\right)}{2\,d\,e^5}+\frac{3\,b^2\,c\,x^2}{2\,d\,e^5}\right)}{\frac{c^4}{d^2}+6\,c^2\,x^2+d^2\,x^4+\frac{4\,c^3\,x}{d}+4\,c\,d\,x^3}-\frac{2\,b^2\,\ln\left(c+d\,x\right)}{3\,d\,e^5}-\frac{\ln\left(c+d\,x-\mathrm{i}\right)\,\left(-\frac{b^2}{3}+\frac{a\,b\,1{}\mathrm{i}}{4}\right)}{d\,e^5}+\frac{\ln\left(c+d\,x+1{}\mathrm{i}\right)\,\left(\frac{b^2}{3}+\frac{1{}\mathrm{i}\,a\,b}{4}\right)}{d\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*atan(c + d*x))^2/(c*e + d*e*x)^5,x)

[Out] atan(c + d*x)^2*(b^2/(4*d*e^5) - b^2/(4*d^3*e^5*(c^4/d^2 + 6*c^2*x^2 + d^2*x^4 + (4*c^3*x)/d + 4*c*d*x^3))) - (x^2*((b^2*d)/2 - 9*a*b*c*d) + x*(a*b + b^2*c - 9*a*b*c^2) + (3*a^2 + b^2*c^2 + 2*a*b*c - 6*a*b*c^3)/(2*d) - 3*a*b*d^2*x^3)/(6*c^4*e^5 + 6*d^4*e^5*x^4 + 24*c*d^3*e^5*x^3 + 36*c^2*d^2*e^5*x^2

```
 + 24*c^3*d*e^5*x) + (atan(c + d*x)*((b^2*x^3)/(2*e^5) - (a*b)/(2*d^3*e^5)
+ (b^2*c*((c^2 - 1)/(3*d^2) + (2*c^2)/(3*d^2)))/(2*d*e^5) + (b^2*x*(d*((c^2
 - 1)/(3*d^2) + (2*c^2)/(3*d^2)) + (2*c^2)/d))/(2*d*e^5) + (3*b^2*c*x^2)/(2
*d*e^5)))/(c^4/d^2 + 6*c^2*x^2 + d^2*x^4 + (4*c^3*x)/d + 4*c*d*x^3) - (2*b^
2*log(c + d*x))/(3*d*e^5) - (log(c + d*x - 1i)*((a*b*1i)/4 - b^2/3))/(d*e^5
) + (log(c + d*x + 1i)*((a*b*1i)/4 + b^2/3))/(d*e^5)
```

3.15 $\int (ce+dex)^2(a+b\mathbf{ArcTan}(c+dx))^3\,dx$

Optimal. Leaf size=271

$$ab^2e^2x+\frac{b^3e^2(c+dx)\text{ArcTan}(c+dx)}{d}-\frac{be^2(a+b\text{ArcTan}(c+dx))^2}{2d}-\frac{be^2(c+dx)^2(a+b\text{ArcTan}(c+dx))^2}{2d}-$$

```
[Out] a*b^2*e^2*x+b^3*e^2*(d*x+c)*arctan(d*x+c)/d-1/2*b*e^2*(a+b*arctan(d*x+c))^2
/d-1/2*b*e^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d-1/3*I*e^2*(a+b*arctan(d*x+c)
)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arctan(d*x+c))^3/d-b*e^2*(a+b*arctan(d*x+c))^2
*ln(2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*ln(1+(d*x+c)^2)/d-I*b^2*e^2*(a+b*arctan(
d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d-1/2*b^3*e^2*polylog(3,1-2/(1+I*(d*x+
c)))/d
```

Rubi [A]
time = 0.30, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.478$, Rules used = {5151, 12, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$-\frac{ib^2e^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))}{d}-\frac{be^2(a+b\text{ArcTan}(c+dx))^2}{2d}-\frac{be^2(c+dx)^2(a+b\text{ArcTan}(c+dx))^2}{2d}+\frac{e^2(c+dx)^3(a+b\text{ArcTan}(c+dx))^3}{3d}-\frac{ie^2(a+b\text{ArcTan}(c+dx))^3}{3d}-\frac{be^2\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{d}+ab^2e^2x+\frac{b^3e^2(c+dx)\text{ArcTan}(c+dx)}{d}-\frac{b^3e^2\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d}-\frac{b^3e^2\log\left((c+dx)^2+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] a*b^2*e^2*x + (b^3*e^2*(c + d*x)*ArcTan[c + d*x])/d - (b*e^2*(a + b*ArcTan[
c + d*x])^2)/(2*d) - (b*e^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d) -
((I/3)*e^2*(a + b*ArcTan[c + d*x])^3)/d + (e^2*(c + d*x)^3*(a + b*ArcTan[c
+ d*x])^3)/(3*d) - (b*e^2*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))
])/d - (b^3*e^2*Log[1 + (c + d*x)^2])/(2*d) - (I*b^2*e^2*(a + b*ArcTan[c +
d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d - (b^3*e^2*PolyLog[3, 1 - 2/(1
 + I*(c + d*x))])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int e^2x^2(a+b\tan^{-1}(x))^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2(a+b\tan^{-1}(x))^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\tan^{-1}(x}{1+x^2}\right.}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3d} - \frac{(be^2)\,\text{Subst}\left(\int x(a+b\tan\right.}{d}$$

$$= -\frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{ie^2(a+b\tan^{-1}(c+dx)}{3d}$$

$$= -\frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{ie^2(a+b\tan^{-1}(c+dx)}{3d}$$

$$= ab^2e^2x - \frac{be^2(a+b\tan^{-1}(c+dx))^2}{2d} - \frac{be^2(c+dx)^2 \left(a+b\tan^{-1}(\right.}{2d}$$

$$= ab^2e^2x + \frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d} - \frac{be^2(a+b\tan^{-1}(c+dx)}{2d}$$

$$= ab^2e^2x + \frac{b^3e^2(c+dx)\tan^{-1}(c+dx)}{d} - \frac{be^2(a+b\tan^{-1}(c+dx)}{2d}$$

Mathematica [A]
time = 0.45, size = 349, normalized size = 1.29

$$\frac{e^2\left(-3a^2b(c+dx)^2+2a^3(c+dx)^3+6a^2b(c+dx)^3\text{ArcTan}(c+dx)+3a^2b\log\left(1+(c+dx)^2\right)+6ab^2\left(c+dx-\text{ArcTan}(c+dx)-(c+dx)^2\text{ArcTan}(c+dx)+i\text{ArcTan}(c+dx)^2+(c+dx)^3\text{ArcTan}(c+dx)^2-2\text{ArcTan}(c+dx)\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)+i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)\right)+b^3\left(6(c+dx)\text{ArcTan}(c+dx)-3\left(1+(c+dx)^2\right)\text{ArcTan}(c+dx)^2+2i\text{ArcTan}(c+dx)^3-2(c+dx)\text{ArcTan}(c+dx)^3+2(c+dx)\left(1+(c+dx)^2\right)\text{ArcTan}(c+dx)^3-6\text{ArcTan}(c+dx)^2\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)+6\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)+6i\text{ArcTan}(c+dx)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)-3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(c+dx)}\right)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e^2*(-3*a^2*b*(c + d*x)^2 + 2*a^3*(c + d*x)^3 + 6*a^2*b*(c + d*x)^3*ArcTan
[c + d*x] + 3*a^2*b*Log[1 + (c + d*x)^2] + 6*a*b^2*(c + d*x - ArcTan[c + d*
x] - (c + d*x)^2*ArcTan[c + d*x] + I*ArcTan[c + d*x]^2 + (c + d*x)^3*ArcTan
[c + d*x]^2 - 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + I*Poly
Log[2, -E^((2*I)*ArcTan[c + d*x])]) + b^3*(6*(c + d*x)*ArcTan[c + d*x] - 3*
(1 + (c + d*x)^2)*ArcTan[c + d*x]^2 + (2*I)*ArcTan[c + d*x]^3 - 2*(c + d*x)
*ArcTan[c + d*x]^3 + 2*(c + d*x)*(1 + (c + d*x)^2)*ArcTan[c + d*x]^3 - 6*Ar
cTan[c + d*x]^2*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*Log[1/Sqrt[1 + (c +
d*x)^2]] + (6*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] - 3
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])))/(6*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.73, size = 2315, normalized size = 8.54

method	result	size
derivativedivides	Expression too large to display	2315
default	Expression too large to display	2315

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a^3-1/8*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d
*x+c))^2/(1+(d*x+c)^2))^2)^3*(d*x+c)+1/4*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(
I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3-1/8*
I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+
I*(d*x+c))^2/(1+(d*x+c)^2)+I)^3+1/2*I*e^2*b^2*a*ln(d*x+c-I)*ln(1+(d*x+c)^2)
-1/2*I*e^2*b^2*a*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-1/2*I*e^2*b^2*a*ln(d*x+c+
I)*ln(1+(d*x+c)^2)+1/2*I*e^2*b^2*a*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))-1/2*e^2*
b^3*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+e^2*b^3*ln(1+(1+I*(d*x+c))^2/
(1+(d*x+c)^2))-1/2*e^2*b^3*arctan(d*x+c)^2+1/4*I*e^2*b^3*arctan(d*x+c)^2*Pi
*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)
^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2))^2)-1/2*I*e^2*b^2*a*dilog(-1/2*I*(d*x+c+I))-1/4*I*e^2*b^2*a*ln(d*x+c-I)^
2+1/2*I*e^2*b^2*a*dilog(1/2*I*(d*x+c-I))+1/4*I*e^2*b^2*a*ln(d*x+c+I)^2+e^2*
b^2*a*(d*x+c)^3*arctan(d*x+c)^2-e^2*b^2*a*(d*x+c)^2*arctan(d*x+c)+e^2*b^2*a
*arctan(d*x+c)*ln(1+(d*x+c)^2)+e^2*a^2*b*(d*x+c)^3*arctan(d*x+c)+I*e^2*b^3*
arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+e^2*b^2*a*(d*x+c)-e
^2*b^2*a*arctan(d*x+c)-1/2*e^2*a^2*b*(d*x+c)^2+1/2*e^2*a^2*b*ln(1+(d*x+c)^2
)-e^2*b^3*arctan(d*x+c)^2*ln(2)+e^2*b^3*arctan(d*x+c)*(d*x+c)+1/3*e^2*b^3*(
d*x+c)^3*arctan(d*x+c)^3-1/2*e^2*b^3*(d*x+c)^2*arctan(d*x+c)^2+1/2*e^2*b^3*
arctan(d*x+c)^2*ln(1+(d*x+c)^2)-e^2*b^3*arctan(d*x+c)^2*ln((1+I*(d*x+c))/(1
```

```
+(d*x+c)^2)^(1/2))-I*e^2*b^3*arctan(d*x+c)+1/3*I*e^2*b^3*arctan(d*x+c)^3+1/
4*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^
2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))+1/8*e^2*b^3*arctan(d*x+c)^2*Pi*
csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)
^3*(d*x+c)-1/8*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d
*x+c)^2))^2)^3+1/4*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(
d*x+c)^2))^3-1/8*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+
(d*x+c)^2))^2)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+1/4*I*e^2*b^3*ar
ctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d
*x+c))^2/(1+(d*x+c)^2))-1/2*I*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c
))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-1/4*I*e^2*b
^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-1/4*I*e^2*b^3*
arctan(d*x+c)^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+1/4*I*e^2*b
^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+
c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)-1/8*I*e^2*
b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2+1/4*e^2*b
^3*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2*csgn(I*
(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*(d*x+c)-1/8*e^2*b^3*arctan(d*x+c)^2*Pi*c
sgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*
x+c)^2)))^2*(d*x+c)-1/4*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^4/(
1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)^2*csgn(I*(1+I*(d*x+c))^
2/(1+(d*x+c)^2)+I)*(d*x+c)+1/8*e^2*b^3*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+
c))^4/(1+(d*x+c)^2)^2+2*I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+I)*csgn(I*(1+I*(d*x
+c))^2/(1+(d*x+c)^2)+I)^2*(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^4*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e^2/d + 3*a*b^2*c^4*a
rctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d - (3*arctan(d*x + c)*arctan(
(d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^4*e^2 - 7/32*(6
*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((
d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^4*e^2 + 1/3*a^3*d^
2*x^3*e^2 + 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e^2/d + 2
8*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 3*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^4*e^2*i
```

ntegrate(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112
*b^3*c*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 4*b^3*d^4*e^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d^3*e^2*in
tegrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d^3*e^2*integrate(1/32*x^3*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 168*b^3*c^2*d^2*e^2*integrat
e(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*b^3*c*d
^3*e^2*integrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 18*b^3*c^2*d^2*e^2*integrate(1/32*x^2*a
rctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 576*a*b^2*c^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*
x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c^2*d^2*e^2*integrate(1
/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c
^3*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^3*c^3*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^4*e^2*integrate(1/
32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + a^3*c*d*x^2*e^2 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*
x + c*d)/d)*e^2/d - 4*b^3*d^3*e^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d^3*e^2*integrate(1/32*x^3*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c*d^2*e^2
*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3
*b^3*c*d^2*e^2*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*c^2*d*e^2*integrate(1/32*x*arctan(d*x
+ c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*c^2*d*e^2*integrate(1/32*x
*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*
arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*
a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*a
rctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b
^3*c^2*e^2 + 3*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x +
c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*c*d*e^2 + 1/2*
(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2
*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b
*d^2*e^2 + a^3*c^2*x*e^2 + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*e^2*integrate(1/32*x^2*ar
ctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^3/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*e^2*integrate(1/32*x*arctan(d
*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x

```
 + c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)
*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/24*(b^3*d^2*x^
3*e^2 + 3*b^3*c*d*x^2*e^2 + 3*b^3*c^2*x*e^2)*arctan(d*x + c)^3 - 1/32*(b^3*
d^2*x^3*e^2 + 3*b^3*c*d*x^2*e^2 + 3*b^3*c^2*x*e^2)*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)^3*e^2 + 3*(a
*b^2*d^2*x^2 + 2*a*b^2*c*d*x + a*b^2*c^2)*arctan(d*x + c)^2*e^2 + 3*(a^2*b*
d^2*x^2 + 2*a^2*b*c*d*x + a^2*b*c^2)*arctan(d*x + c)*e^2 + (a^3*d^2*x^2 + 2
*a^3*c*d*x + a^3*c^2)*e^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int a^{3}c^{2}\, dx + \int a^{3}d^{2}x^{2}\, dx + \int b^{3}c^{2}\operatorname{atan}^{3}{\left(c + dx \right)}\, dx + \int 3ab^{2}c^{2}\operatorname{atan}^{2}{\left(c + dx \right)}\, dx + \int 3a^{2}bc^{2}\operatorname{atan}{\left(c + dx \right)}\, dx + \int 2a^{3}cdx\, dx + \int b^{3}d^{2}x^{2}\operatorname{atan}^{3}{\left(c + dx \right)}\, dx + \int 3ab^{2}d^{2}x^{2}\operatorname{atan}^{2}{\left(c + dx \right)}\, dx + \int 3a^{2}bd^{2}x^{2}\operatorname{atan}{\left(c + dx \right)}\, dx + \int 2b^{3}cdx\operatorname{atan}^{3}{\left(c + dx \right)}\, dx + \int 6ab^{2}cdx\operatorname{atan}^{2}{\left(c + dx \right)}\, dx + \int 6a^{2}bcdx\operatorname{atan}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(a**3*c**2, x) + Integral(a**3*d**2*x**2, x) + Integral(b**3*
c**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*c**2*atan(c + d*x)**2, x) + I
ntegral(3*a**2*b*c**2*atan(c + d*x), x) + Integral(2*a**3*c*d*x, x) + Integ
ral(b**3*d**2*x**2*atan(c + d*x)**3, x) + Integral(3*a*b**2*d**2*x**2*atan(
c + d*x)**2, x) + Integral(3*a**2*b*d**2*x**2*atan(c + d*x), x) + Integral(
2*b**3*c*d*x*atan(c + d*x)**3, x) + Integral(6*a*b**2*c*d*x*atan(c + d*x)**
2, x) + Integral(6*a**2*b*c*d*x*atan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{atan}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.16 $\int (ce + dex)(a + b\mathbf{ArcTan}(c + dx))^3 \, dx$

Optimal. Leaf size=164

$$-\frac{3ibe(a + b\text{ArcTan}(c + dx))^2}{2d} - \frac{3be(c + dx)(a + b\text{ArcTan}(c + dx))^2}{2d} + \frac{e(a + b\text{ArcTan}(c + dx))^3}{2d} + \frac{e(c + dx)}{}$$

```
[Out] -3/2*I*b*e*(a+b*arctan(d*x+c))^2/d-3/2*b*e*(d*x+c)*(a+b*arctan(d*x+c))^2/d+
1/2*e*(a+b*arctan(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arctan(d*x+c))^3/d-3*b^2
*e*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d-3/2*I*b^3*e*polylog(2,1-2/(1+I
*(d*x+c)))/d
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5151, 12, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$-\frac{3b^2 e \log\left(\frac{2}{1+i(c+dx)}\right)(a + b\text{ArcTan}(c + dx))}{d} - \frac{3ibe(a + b\text{ArcTan}(c + dx))^2}{2d} - \frac{3be(c + dx)(a + b\text{ArcTan}(c + dx))^2}{2d} + \frac{e(c + dx)^2(a + b\text{ArcTan}(c + dx))^3}{2d} + \frac{e(a + b\text{ArcTan}(c + dx))^3}{2d} - \frac{3ib^3 e\text{Li}_2\left(1 - \frac{2}{i(c+dx)+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c + d*x])^2)/d - (3*b*e*(c + d*x)*(a + b*ArcT
an[c + d*x])^2)/(2*d) + (e*(a + b*ArcTan[c + d*x])^3)/(2*d) + (e*(c + d*x)^
2*(a + b*ArcTan[c + d*x])^3)/(2*d) - (3*b^2*e*(a + b*ArcTan[c + d*x])*Log[2
/(1 + I*(c + d*x))])/d - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*(c + d*x)
)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (ce+dex)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int ex(a+b\tan^{-1}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x(a+b\tan^{-1}(x))^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\tan^{-1}(x))}{1+x^2}\right.}{2d}$$

$$= \frac{e(c+dx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\text{Subst}\left(\int (a+b\tan^{-1}(\right.}{2d}$$

$$= -\frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} + \frac{e\left(a+b\tan^{-1}(c+dx)\right)^3}{2d}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(c+dx)\right)^2}{2d} - \frac{3be(c+dx)\left(a+b\tan^{-1}(c+dx)\right)}{2d}$$

Mathematica [A]
time = 0.24, size = 196, normalized size = 1.20

$$\frac{e\left(3b^2(-i+c+dx)(-b+a(i+c+dx))\text{ArcTan}(c+dx)^2+b^3(1+c^2+2cdx+d^2x^2)\text{ArcTan}(c+dx)^3+3b\text{ArcTan}(c+dx)\left(a(-2b(c+dx)+a(1+c^2+2cdx+d^2x^2))-2b^2\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)\right)+a\left(a(c+dx)(-3b+ac+adx)-6b^2\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)\right)+3ib^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (e*(3*b^2*(-I + c + d*x)*(-b + a*(I + c + d*x))*ArcTan[c + d*x]^2 + b^3*(1
+ c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*b*ArcTan[c + d*x]*(a*(-2*b
*(c + d*x) + a*(1 + c^2 + 2*c*d*x + d^2*x^2)) - 2*b^2*Log[1 + E^((2*I)*ArcT
an[c + d*x])]) + a*(a*(c + d*x)*(-3*b + a*c + a*d*x) - 6*b^2*Log[1/Sqrt[1 +
 (c + d*x)^2]]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/(2*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(150) = 300.
time = 0.27, size = 384, normalized size = 2.34

method	result
derivativedivides	$\frac{e(dx+c)^2 a^3}{2}+\frac{e\,b^3(dx+c)^2\arctan(dx+c)^3}{2}+\frac{e\,b^3\arctan(dx+c)^3}{2}-\frac{3e\,b^3\arctan(dx+c)^2(dx+c)}{2}+\frac{3e\,b^3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}-$
default	$\frac{e(dx+c)^2 a^3}{2}+\frac{e\,b^3(dx+c)^2\arctan(dx+c)^3}{2}+\frac{e\,b^3\arctan(dx+c)^3}{2}-\frac{3e\,b^3\arctan(dx+c)^2(dx+c)}{2}+\frac{3e\,b^3\arctan(dx+c)\ln\left(1+(dx+c)^2\right)}{2}-$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^3+1/2*e*b^3*(d*x+c)^2*arctan(d*x+c)^3+1/2*e*b^3*arct
an(d*x+c)^3-3/2*e*b^3*arctan(d*x+c)^2*(d*x+c)+3/2*e*b^3*arctan(d*x+c)*ln(1+
(d*x+c)^2)-3/4*I*e*b^3*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+3/4*I*e*b^3*dilog(1
/2*I*(d*x+c-I))-3/4*I*e*b^3*dilog(-1/2*I*(d*x+c+I))+3/4*I*e*b^3*ln(d*x+c+I)
*ln(1/2*I*(d*x+c-I))+3/8*I*e*b^3*ln(d*x+c+I)^2-3/4*I*e*b^3*ln(d*x+c+I)*ln(1
+(d*x+c)^2)-3/8*I*e*b^3*ln(d*x+c-I)^2+3/4*I*e*b^3*ln(d*x+c-I)*ln(1+(d*x+c)^
2)+3/2*e*a*b^2*(d*x+c)^2*arctan(d*x+c)^2+3/2*e*a*b^2*arctan(d*x+c)^2-3*e*a*
b^2*(d*x+c)*arctan(d*x+c)+3/2*e*a*b^2*ln(1+(d*x+c)^2)+3/2*e*a^2*b*(d*x+c)^2
*arctan(d*x+c)-3/2*e*(d*x+c)*a^2*b+3/2*e*a^2*b*arctan(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d
^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a^2*b*d*e + a
^3*c*x*e + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b*c
*e/d + 1/32*(8*(b^3*d^2*x^2*e + 2*b^3*c*d*x*e + b^3*c^2*e + b^3*e)*arctan(d
*x + c)^3 + 12*(a*b^2*d^2*x^2*e + (2*a*b^2*c*e - b^3*e)*d*x)*arctan(d*x + c
)^2 - 3*(a*b^2*d^2*x^2*e + (2*a*b^2*c*e - b^3*e)*d*x)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 4*(4*b^3*c^3*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e/d
+ 18*a*b^2*c^3*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d - 6*(3*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c
^3*e - (6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)
*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^3*e - 3*b
^3*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d + 4*b^3*c*arctan(d*x +
 c)^3*arctan((d^2*x + c*d)/d)*e/d + 128*b^3*d^3*e*integrate(1/32*x^3*arctan
(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d^3*e*integrate(1
/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c*d^2
```

```
*e*integrate(1/32*x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
 48*a*b^2*d^3*e*integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c*d^2*e*integrate(1/32*x^2*arctan
(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*b^3*c^2*d*e*integrate(1
/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^3*e*
integrate(1/32*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 144*a*b^2*c*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c
^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1728*a*b^2*c^2*d*e*integrate(
1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 288*a*b^2*c*d^
2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 144*a*b^2*c^2*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c^2*d*e*integrate
(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
+ 48*a*b^2*c^3*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d
- arctan((d^2*x + c*d)/d)^3/d)*b^3*c^2*e + 18*a*b^2*c*arctan(d*x + c)^2*arc
tan((d^2*x + c*d)/d)*e/d - 96*b^3*d^2*e*integrate(1/32*x^2*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 24*b^3*d^2*e*integrate(1/32*x^2*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 192*a*b^2
*d^2*e*integrate(1/32*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 - 192*b^3*c*d*e*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) - 96*b^3*d^2*e*integrate(1/32*x^2*log(d^2*x^2 + 2*c*d*x + c^2 +
1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 48*b^3*c*d*e*integrate(1/32*x*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 384*a*b^2*
c*d*e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) -
96*b^3*c*d*e*integrate(1/32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) - 24*b^3*c^2*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 6*(3*arctan(d*x + c)*arctan
((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c*e - (6*arctan(
d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x +
c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c*e - 3*b^3*arctan(d*x + c)^
2*arctan((d^2*x + c*d)/d)*e/d + 128*b^3*d*e*integrate(1/32*x*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 576*a*b^2*d*e*integrate(1/32*x*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 48*a*b^2*d*e*integrate(1/
32*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
 192*b^3*d*e*integrate(1/32*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 48*a*b^2*c*e*integrate(1/32*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2
/d - arctan((d^2*x + c*d)/d)^3/d)*b^3*e - 24*b^3*e*integrate(1/32*log(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x))*d)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d*x + b^3*c)*arctan(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c)*ar
ctan(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arctan(d*x + c)*e + (a^3*d*x +
a^3*c)*e, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^3c\,dx + \int a^3dx\,dx + \int b^3c\,\mathrm{atan}^3\,(c+dx)\,dx + \int 3ab^2c\,\mathrm{atan}^2\,(c+dx)\,dx + \int 3a^2bc\,\mathrm{atan}\,(c+dx)\,dx + \int b^3dx\,\mathrm{atan}^3\,(c+dx)\,dx + \int 3ab^2dx\,\mathrm{atan}^2\,(c+dx)\,dx + \int 3a^2bdx\,\mathrm{atan}\,(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*atan(d*x+c))**3,x)
```

```
[Out] e*(Integral(a**3*c, x) + Integral(a**3*d*x, x) + Integral(b**3*c*atan(c + d
*x)**3, x) + Integral(3*a*b**2*c*atan(c + d*x)**2, x) + Integral(3*a**2*b*c
*atan(c + d*x), x) + Integral(b**3*d*x*atan(c + d*x)**3, x) + Integral(3*a*
b**2*d*x*atan(c + d*x)**2, x) + Integral(3*a**2*b*d*x*atan(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*atan(c + d*x))^3, x)
```

3.17 $\int \frac{(a+b\textbf{ArcTan}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=279

$$\frac{2(a+b\text{ArcTan}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de}-\frac{3ib(a+b\text{ArcTan}(c+dx))^2\text{PolyLog}\left(2,1-\frac{2}{1+i(c+dx)}\right)}{2de}+\text{-}$$

```
[Out] -2*(a+b*arctan(d*x+c))^3*arctanh(-1+2/(1+I*(d*x+c)))/d/e-3/2*I*b*(a+b*arcta
n(d*x+c))^2*polylog(2,1-2/(1+I*(d*x+c)))/d/e+3/2*I*b*(a+b*arctan(d*x+c))^2*
polylog(2,-1+2/(1+I*(d*x+c)))/d/e-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1+I*(d*x+c)))/d/e+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,-1+2/(1+I*(d*x+c)
))/d/e+3/4*I*b^3*polylog(4,1-2/(1+I*(d*x+c)))/d/e-3/4*I*b^3*polylog(4,-1+2/
(1+I*(d*x+c)))/d/e
```

Rubi [A]
time = 0.36, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5151, 12, 4942, 5108, 5004, 5114, 5118, 6745}

$$-\frac{3b^2\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))}{2de}+\frac{3b^2\text{Li}_3\left(\frac{2}{i(c+dx)+1}-1\right)(a+b\text{ArcTan}(c+dx))}{2de}-\frac{3ib\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))^2}{2de}+\frac{3ib\text{Li}_2\left(\frac{2}{i(c+dx)+1}-1\right)(a+b\text{ArcTan}(c+dx))^2}{2de}+\frac{2\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^3}{de}+\frac{3ib^3\text{Li}_4\left(1-\frac{2}{i(c+dx)+1}\right)}{4de}-\frac{3ib^3\text{Li}_4\left(\frac{2}{i(c+dx)+1}-1\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (2*(a + b*ArcTan[c + d*x])^3*ArcTanh[1 - 2/(1 + I*(c + d*x))])/(d*e) - (((3
*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d*
e) + (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -1 + 2/(1 + I*(c + d
*x))])/(d*e) - (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, 1 - 2/(1 + I*(c +
d*x))])/(2*d*e) + (3*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -1 + 2/(1 + I*(
c + d*x))])/(2*d*e) + (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*(c + d*x))])/(
d*e) - (((3*I)/4)*b^3*PolyLog[4, -1 + 2/(1 + I*(c + d*x))])/(d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2(a+b\tan^{-1}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(6b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))}{}\right.}{}$$

$$= \frac{2(a+b\tan^{-1}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))}{1+}\right.}{}$$

$$= \frac{2(a+b\tan^{-1}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de}$$

$$= \frac{2(a+b\tan^{-1}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de}$$

$$= \frac{2(a+b\tan^{-1}(c+dx))^3\tanh^{-1}\left(1-\frac{2}{1+i(c+dx)}\right)}{de} - \frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de}$$

Mathematica [A]
time = 0.08, size = 252, normalized size = 0.90

$$\frac{8(a+b\text{ArcTan}(c+dx))^3\tanh^{-1}\left(\frac{i+c+dx}{-i+c+dx}\right)+6ib(a+b\text{ArcTan}(c+dx))^2\text{PolyLog}\left(2,-\frac{i+c+dx}{-i+c+dx}\right)-6ib(a+b\text{ArcTan}(c+dx))^2\text{PolyLog}\left(2,\frac{i+c+dx}{-i+c+dx}\right)+6b^2(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(3,-\frac{i+c+dx}{-i+c+dx}\right)-6b^2(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(3,\frac{i+c+dx}{-i+c+dx}\right)-3ib^3\text{PolyLog}\left(4,-\frac{i+c+dx}{-i+c+dx}\right)+3ib^3\text{PolyLog}\left(4,\frac{i+c+dx}{-i+c+dx}\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (8*(a + b*ArcTan[c + d*x])^3*ArcTanh[(I + c + d*x)/(-I + c + d*x)] + (6*I)*
b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, -((I + c + d*x)/(-I + c + d*x))] - (
6*I)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, (I + c + d*x)/(-I + c + d*x)] +
 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, -((I + c + d*x)/(-I + c + d*x))]
- 6*b^2*(a + b*ArcTan[c + d*x])*PolyLog[3, (I + c + d*x)/(-I + c + d*x)] -
(3*I)*b^3*PolyLog[4, -((I + c + d*x)/(-I + c + d*x))] + (3*I)*b^3*PolyLog[4
, (I + c + d*x)/(-I + c + d*x)])/(4*d*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.14, size = 2757, normalized size = 9.88

method	result	size

derivativedivides	Expression too large to display	2757
default	Expression too large to display	2757

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3/e*ln(d*x+c)+1/2*I*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*arctan(d*x+c)^3+3/2*I*a*b^2/e*Pi*csg
n(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)
^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c
)^2)))*arctan(d*x+c)^2+1/2*I*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)
-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-
1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arctan(d*x+c)^3-1/2*I*b^3/e*Pi*csgn(I
((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(
((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arct
an(d*x+c)^3-1/2*I*b^3/e*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I
((1+I(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arc
tan(d*x+c)^3-1/2*I*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn(
I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*ar
ctan(d*x+c)^3-3/2*I*a*b^2/e*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1
+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2+3/2*I*a*b^2/e*Pi*csgn(((1+I
*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*arctan(d*
x+c)^2+3/2*I*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d
*x+c))^2/(1+(d*x+c)^2)))^3*arctan(d*x+c)^2-3/2*a*b^2/e*polylog(3,-(1+I*(d*x
+c))^2/(1+(d*x+c)^2))+6*a*b^2/e*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2
))+6*a*b^2/e*polylog(3,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+b^3/e*ln(d*x+c)*a
rctan(d*x+c)^3-b^3/e*arctan(d*x+c)^3*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+b^
3/e*arctan(d*x+c)^3*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*b^3/e*arctan(
d*x+c)*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+b^3/e*arctan(d*x+c)^3*
ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*b^3/e*arctan(d*x+c)*polylog(3,(1+
I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-3/2*b^3/e*arctan(d*x+c)*polylog(3,-(1+I*(d*
x+c))^2/(1+(d*x+c)^2))-3/4*I*b^3/e*polylog(4,-(1+I*(d*x+c))^2/(1+(d*x+c)^2)
)+6*I*b^3/e*polylog(4,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+6*I*b^3/e*polylog
(4,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-1/2*I*b^3/e*Pi*csgn(((1+I*(d*x+c))^2/
(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^3+3*I*a
*b^2/e*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-6*I*a*b^2/e*
arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))-6*I*a*b^2/e*arc
tan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3/2*I*a*b^2/e*Pi*ar
ctan(d*x+c)^2+3/2*I*a^2*b/e*ln(d*x+c)*ln(1+I*(d*x+c))-3/2*I*a^2*b/e*ln(d*x+
c)*ln(1-I*(d*x+c))-3/2*I*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1
)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)
/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2+3/2*I*a*b^2/e*Pi*csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csg

```
n(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arct
an(d*x+c)^2-3/2*I*a*b^2/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csgn
(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*a
rctan(d*x+c)^2+1/2*I*b^3/e*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))*csg
n(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2
)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arctan(d*x+c)^3-3/2*I*a*b^2/e*Pi*cs
gn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^
2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2+1/2*I*b^3/e*Pi*c
sgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*
arctan(d*x+c)^3+3*a^2*b/e*ln(d*x+c)*arctan(d*x+c)+3/2*I*a^2*b/e*dilog(1+I*(
d*x+c))-3/2*I*a^2*b/e*dilog(1-I*(d*x+c))+3*a*b^2/e*ln(d*x+c)*arctan(d*x+c)^
2-3*a*b^2/e*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)+3*a*b^2/e*a
rctan(d*x+c)^2*ln(1+(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*a*b^2/e*arctan(d*x
+c)^2*ln(1-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+1/2*I*b^3/e*Pi*arctan(d*x+c)^
3-3*I*b^3/e*arctan(d*x+c)^2*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3
/2*I*b^3/e*arctan(d*x+c)^2*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I*b^
3/e*arctan(d*x+c)^2*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*e^(-1)*log(d*x*e + c*e)/d + integrate(1/32*(28*b^3*arctan(d*x + c)^3 +
3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(
d*x + c)^2 + 96*a^2*b*arctan(d*x + c))/(d*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{atan}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{atan}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2 b \operatorname{atan}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*atan(c + d*x)**3/(c + d*x), x)
 + Integral(3*a*b**2*atan(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*ata
n(c + d*x)/(c + d*x), x))/e
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x), x)
```

3.18 $\int \frac{(a+b\textbf{ArcTan}(c+dx))^3}{(ce+dex)^2} \, dx$

Optimal. Leaf size=163

$$-\frac{i(a+b\text{ArcTan}(c+dx))^3}{de^2}-\frac{(a+b\text{ArcTan}(c+dx))^3}{de^2(c+dx)}+\frac{3b(a+b\text{ArcTan}(c+dx))^2\log\left(2-\frac{2}{1-i(c+dx)}\right)}{de^2}-\frac{3ib^2}{}$$

```
[Out] -I*(a+b*arctan(d*x+c))^3/d/e^2-(a+b*arctan(d*x+c))^3/d/e^2/(d*x+c)+3*b*(a+b
*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^2-3*I*b^2*(a+b*arctan(d*x+c))*p
olylog(2,-1+2/(1-I*(d*x+c)))/d/e^2+3/2*b^3*polylog(3,-1+2/(1-I*(d*x+c)))/d/
e^2
```

Rubi [A]
time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5151, 12, 4946, 5044, 4988, 5004, 5112, 6745}

$$-\frac{3ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)(a+b\text{ArcTan}(c+dx))}{de^2}-\frac{(a+b\text{ArcTan}(c+dx))^3}{de^2(c+dx)}-\frac{i(a+b\text{ArcTan}(c+dx))^3}{de^2}+\frac{3b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{de^2}+\frac{3b^3\text{Li}_3\left(\frac{2}{1-i(c+dx)}-1\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-I)*(a + b*ArcTan[c + d*x])^3)/(d*e^2) - (a + b*ArcTan[c + d*x])^3/(d*e^2
*(c + d*x)) + (3*b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/
(d*e^2) - ((3*I)*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c +
d*x))])/(d*e^2) + (3*b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
```

```
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x(1+x^2)}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^3}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{(3ib)\text{Subst}\left(\int \frac{(a+b\tan^{-1}}{x(i+}\right.}{de}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^3}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\tan^{-1}(c+dx}{d}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^3}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\tan^{-1}(c+dx}{d}$$

$$= -\frac{i(a+b\tan^{-1}(c+dx))^3}{de^2} - \frac{(a+b\tan^{-1}(c+dx))^3}{de^2(c+dx)} + \frac{3b(a+b\tan^{-1}(c+dx}{d}$$

Mathematica [A]
time = 0.39, size = 263, normalized size = 1.61

$$\frac{-\frac{2a^3}{c+dx} - \frac{6a^2b\text{ArcTan}(c+dx)}{c+dx} + 6a^2b\log(c+dx) - 3a^2b\log\left(1+c^2+2cdx+d^2x^2\right) + 6ab^2\left(\text{ArcTan}(c+dx)\left(\left(-i-\frac{1}{c+dx}\right)\text{ArcTan}(c+dx) + 2\log\left(1-e^{2i\text{ArcTan}(c+dx)}\right)\right) - i\text{PolyLog}\left(2, e^{2i\text{ArcTan}(c+dx)}\right)\right) + 2b^3\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(c+dx)^3 - \frac{\text{ArcTan}(c+dx)^3}{c+dx} + 3\text{ArcTan}(c+dx)^2\log\left(1-e^{-2i\text{ArcTan}(c+dx)}\right) + 3i\text{ArcTan}(c+dx)\text{PolyLog}\left(2, e^{-2i\text{ArcTan}(c+dx)}\right) + \frac{3}{2}\text{PolyLog}\left(3, e^{-2i\text{ArcTan}(c+dx)}\right)\right)}{2de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^3)/(c + d*x) - (6*a^2*b*ArcTan[c + d*x])/(c + d*x) + 6*a^2*b*Log[c +
 d*x] - 3*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2*x^2] + 6*a*b^2*(ArcTan[c + d*x]
*((-I - (c + d*x)^(-1))*ArcTan[c + d*x] + 2*Log[1 - E^((2*I)*ArcTan[c + d*x
])]) - I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*((-1/8*I)*Pi^3 + I*
ArcTan[c + d*x]^3 - ArcTan[c + d*x]^3/(c + d*x) + 3*ArcTan[c + d*x]^2*Log[1
 - E^((-2*I)*ArcTan[c + d*x])] + (3*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)
*ArcTan[c + d*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])])/2))/(2*d*e^
2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 2547, normalized size = 15.63

method	result	size
derivativedivides	Expression too large to display	2547
default	Expression too large to display	2547

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/e^2/(d*x+c)-3*I*a*b^2/e^2*ln(d*x+c)*ln(1-I*(d*x+c))-b^3/e^2/(d*x+
c)*arctan(d*x+c)^3+3*b^3/e^2*ln(d*x+c)*arctan(d*x+c)^2-3/2*b^3/e^2*arctan(d
*x+c)^2*ln(1+(d*x+c)^2)+3*b^3/e^2*arctan(d*x+c)^2*ln(2)+3*b^3/e^2*arctan(d*
x+c)^2*ln((1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*b^3/e^2*arctan(d*x+c)^2*ln(1
-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))+3*b^3/e^2*arctan(d*x+c)^2*ln(1+(1+I*(d*
x+c))/(1+(d*x+c)^2)^(1/2))-3*b^3/e^2*arctan(d*x+c)^2*ln((1+I*(d*x+c))^2/(1+
(d*x+c)^2)-1)-I*b^3/e^2*arctan(d*x+c)^3-3/2*a^2*b/e^2*ln(1+(d*x+c)^2)+3*a^2
*b/e^2*ln(d*x+c)-3/2*I*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1))
*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
)^2*arctan(d*x+c)^2-3/2*I*b^3/e^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^
2)))^2*arctan(d*x+c)^2+3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x
+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)+3/2*I*
b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I
*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2+3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(
1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*
(d*x+c))^2/(1+(d*x+c)^2))^2)^2+3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I/(1+(
1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1
+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(
1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3
/2*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*c
sgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+3/2*I*b^3/e^2*Pi*csgn(I*((1+I*
(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(((1+I*(
d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arctan(d*x+c)
^2-3/2*I*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c
))^2/(1+(d*x+c)^2)))*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2+3/2*I*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-1))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*((1+
I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*arctan(d*x
+c)^2-3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)
^2))^2)*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)-3/2*I*a*b^2/e^2*ln(d*x+c-I)*ln
(1+(d*x+c)^2)+3/2*I*a*b^2/e^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))+3/2*I*a*b^2/
e^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)-3/2*I*a*b^2/e^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c
-I))+3*I*a*b^2/e^2*ln(d*x+c)*ln(1+I*(d*x+c))-3/4*I*b^3/e^2*arctan(d*x+c)^2*
Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3+3/4*I*b^3/e^2*arctan(d*x+c)^2*Pi
```

```
*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^3-3/4*I*b^3/e^2*arctan(d*x+c)^
2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))
^2)^3-3/2*I*b^3/e^2*Pi*csgn(((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2)))^2*arctan(d*x+c)^2+3/2*I*b^3/e^2*Pi*csgn(((1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*arctan(d*x+c)^2+3
/2*I*b^3/e^2*Pi*csgn(I*((1+I*(d*x+c))^2/(1+(d*x+c)^2)-1)/(1+(1+I*(d*x+c))^2
/(1+(d*x+c)^2)))^3*arctan(d*x+c)^2-3/4*I*a*b^2/e^2*ln(d*x+c+I)^2+3*I*a*b^2/
e^2*dilog(1+I*(d*x+c))-3*I*a*b^2/e^2*dilog(1-I*(d*x+c))-3*a^2*b/e^2/(d*x+c)
*arctan(d*x+c)-6*I*b^3/e^2*arctan(d*x+c)*polylog(2,(1+I*(d*x+c))/(1+(d*x+c)
^2)^(1/2))-6*I*b^3/e^2*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))/(1+(d*x+c)^2)
^(1/2))+3/2*I*b^3/e^2*Pi*arctan(d*x+c)^2-3*a*b^2/e^2/(d*x+c)*arctan(d*x+c)^
2-3*a*b^2/e^2*arctan(d*x+c)*ln(1+(d*x+c)^2)+6*a*b^2/e^2*arctan(d*x+c)*ln(d*
x+c)+3/2*I*a*b^2/e^2*dilog(-1/2*I*(d*x+c+I))+3/4*I*a*b^2/e^2*ln(d*x+c-I)^2-
3/2*I*a*b^2/e^2*dilog(1/2*I*(d*x+c-I))+6*b^3/e^2*polylog(3,(1+I*(d*x+c))/(1
+(d*x+c)^2)^(1/2))+6*b^3/e^2*polylog(3,-(1+I*(d*x+c))/(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(e^(-2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2 - 2*e^(-2)*log(d*x + c
)/d^2) + 2*arctan(d*x + c)/(d^2*x*e^2 + c*d*e^2))*a^2*b - a^3/(d^2*x*e^2 +
c*d*e^2) - 1/32*(4*b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)^2 - 32*(d^2*x*e^2 + c*d*e^2)*integrate(1/32*(28*(b^3
*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2
*x^2 + 8*a*b^2*c^2 + b^3*c + 8*a*b^2 + (16*a*b^2*c + b^3)*d*x)*arctan(d*x +
 c)^2 - 12*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arctan(d*x + c)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*x + b^3*c - (b^3*d^2*x^2 + 2*b^3*c*d*x +
b^3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/(d^4*x^
4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*e^2 + e^2)*d^2*x^2 + c^4*e^2 + 2*(2*c^3*e^
2 + c*e^2)*d*x + c^2*e^2), x))/(d^2*x*e^2 + c*d*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*e^(-2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*atan(c + d*
x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*atan(c + d*x)**2
/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*atan(c + d*x)/(c**2 +
 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atan}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.19 $\int \frac{(a+b\mathbf{ArcTan}(c+dx))^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{3ib(a+b\text{ArcTan}(c+dx))^2}{2de^3}-\frac{3b(a+b\text{ArcTan}(c+dx))^2}{2de^3(c+dx)}-\frac{(a+b\text{ArcTan}(c+dx))^3}{2de^3}-\frac{(a+b\text{ArcTan}(c+dx}{2de^3(c+dx)^2}$$

```
[Out] -3/2*I*b*(a+b*arctan(d*x+c))^2/d/e^3-3/2*b*(a+b*arctan(d*x+c))^2/d/e^3/(d*x
+c)-1/2*(a+b*arctan(d*x+c))^3/d/e^3-1/2*(a+b*arctan(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arctan(d*x+c))*ln(2-2/(1-I*(d*x+c)))/d/e^3-3/2*I*b^3*polylog(
2,-1+2/(1-I*(d*x+c)))/d/e^3
```

Rubi [A]
time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5151, 12, 4946, 5038, 5044, 4988, 2497, 5004}

$$\frac{3b^2\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{de^3}-\frac{3b(a+b\text{ArcTan}(c+dx))^2}{2de^3(c+dx)}-\frac{3ib(a+b\text{ArcTan}(c+dx))^2}{2de^3}-\frac{(a+b\text{ArcTan}(c+dx))^3}{2de^3(c+dx)^2}-\frac{(a+b\text{ArcTan}(c+dx))^3}{2de^3}-\frac{3ib^3\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c + d*x])^2)/(d*e^3) - (3*b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcTan[c + d*x])^3/(2*d*e^3) - (a + b*
ArcTan[c + d*x])^3/(2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcTan[c + d*x])*L
og[2 - 2/(1 - I*(c + d*x))])/(d*e^3) - (((3*I)/2)*b^3*PolyLog[2, -1 + 2/(1
- I*(c + d*x))])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^2(1+x^2)}\,dx, x, c+dx\right)}{2de^3}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^2}\,dx, x, c+dx\right)}{2de^3} - \frac{(3b)}{}$$

$$= -\frac{3b(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+dx))^3}{2de^3} - \frac{(a+b\tan^{-1}(c+dx}{2de^3(c+dx)^2}$$

$$= -\frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{3b(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+}{2de^3}$$

$$= -\frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{3b(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+}{2de^3}$$

$$= -\frac{3ib(a+b\tan^{-1}(c+dx))^2}{2de^3} - \frac{3b(a+b\tan^{-1}(c+dx))^2}{2de^3(c+dx)} - \frac{(a+b\tan^{-1}(c+}{2de^3}$$

Mathematica [A]
time = 0.20, size = 225, normalized size = 1.25

$$-\frac{a^3+b^3(1+c^2+2cdx+d^2x^2)\text{ArcTan}(c+dx)^3+3a^2b(c+dx+(1+(c+dx)^2)\text{ArcTan}(c+dx))+3ab^2\left(2(c+dx)\text{ArcTan}(c+dx)+(1+(c+dx)^2)\text{ArcTan}(c+dx)^2-2(c+dx)^2\log\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}\right)\right)+3b^3(c+dx)\left(\text{ArcTan}(c+dx)^2-2(c+dx)\text{ArcTan}(c+dx)\log\left(1-e^{2i\text{ArcTan}(c+dx)}\right)+i(c+dx)\left(\text{ArcTan}(c+dx)^2+\text{PolyLog}(2,e^{2i\text{ArcTan}(c+dx)})\right)\right)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a^3 + b^3*(1 + c^2 + 2*c*d*x + d^2*x^2)*ArcTan[c + d*x]^3 + 3*a^2*b*(
c + d*x + (1 + (c + d*x)^2)*ArcTan[c + d*x]) + 3*a*b^2*(2*(c + d*x)*ArcTan[
c + d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 2*(c + d*x)^2*Log[(c + d*x
)/Sqrt[1 + (c + d*x)^2]]) + 3*b^3*(c + d*x)*(ArcTan[c + d*x]^2 - 2*(c + d*x
)*ArcTan[c + d*x]*Log[1 - E^((2*I)*ArcTan[c + d*x])] + I*(c + d*x)*(ArcTan[
c + d*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c + d*x])])))/(d*e^3*(c + d*x)^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(166) = 332.
time = 0.65, size = 557, normalized size = 3.09

method	result
derivativedivides	$-\frac{a^3}{2e^3(dx+c)^2}-\frac{b^3\arctan(dx+c)^3}{2e^3}-\frac{3a^2b\arctan(dx+c)}{2e^3(dx+c)^2}-\frac{3a\,b^2\arctan(dx+c)}{e^3(dx+c)}-\frac{3a\,b^2\arctan(dx+c)^2}{2e^3(dx+c)^2}-\frac{3a^2b}{2e^3(dx+c)}-\frac{3a^2b\arctan(dx+c)}{2e^3}$
default	$-\frac{a^3}{2e^3(dx+c)^2}-\frac{b^3\arctan(dx+c)^3}{2e^3}-\frac{3a^2b\arctan(dx+c)}{2e^3(dx+c)^2}-\frac{3a\,b^2\arctan(dx+c)}{e^3(dx+c)}-\frac{3a\,b^2\arctan(dx+c)^2}{2e^3(dx+c)^2}-\frac{3a^2b}{2e^3(dx+c)}-\frac{3a^2b\arctan(dx+c)}{2e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/e^3/(d*x+c)^2-1/2*b^3/e^3*arctan(d*x+c)^3-3/2*I*b^3/e^3*ln(d*
x+c)*ln(1-I*(d*x+c))+3/4*I*b^3/e^3*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-3/2*a^2
*b/e^3/(d*x+c)^2*arctan(d*x+c)-3*a*b^2/e^3*arctan(d*x+c)/(d*x+c)-3/2*a*b^2/
e^3/(d*x+c)^2*arctan(d*x+c)^2-3/4*I*b^3/e^3*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))
+3/4*I*b^3/e^3*ln(d*x+c+I)*ln(1+(d*x+c)^2)+3/2*I*b^3/e^3*ln(d*x+c)*ln(1+I*(
d*x+c))-3/4*I*b^3/e^3*ln(d*x+c-I)*ln(1+(d*x+c)^2)-3/2*a^2*b/e^3/(d*x+c)-3/2
*a^2*b/e^3*arctan(d*x+c)-3/2*a*b^2/e^3*arctan(d*x+c)^2-3/2*a*b^2/e^3*ln(1+(
d*x+c)^2)+3*a*b^2/e^3*ln(d*x+c)-3/2*b^3/e^3*arctan(d*x+c)^2/(d*x+c)-3/2*b^3
/e^3*arctan(d*x+c)*ln(1+(d*x+c)^2)+3*b^3/e^3*ln(d*x+c)*arctan(d*x+c)+3/2*I*
b^3/e^3*dilog(1+I*(d*x+c))+3/4*I*b^3/e^3*dilog(-1/2*I*(d*x+c+I))+3/8*I*b^3/
e^3*ln(d*x+c-I)^2-3/2*I*b^3/e^3*dilog(1-I*(d*x+c))-3/8*I*b^3/e^3*ln(d*x+c+I
)^2-3/4*I*b^3/e^3*dilog(1/2*I*(d*x+c-I))-1/2*b^3/e^3/(d*x+c)^2*arctan(d*x+c
)^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(arctan((d^2*x + c*d)/d)*e^(-3)/d^2 + 1/(d^3*x*e^3 + c*d^2*e^3)) +
arctan(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3))*a^2*b - 3/2*(2*d
*(arctan((d^2*x + c*d)/d)*e^(-3)/d^2 + 1/(d^3*x*e^3 + c*d^2*e^3))*arctan(d*
x + c) - (arctan(d*x + c)^2 - log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*log(d*x
+ c))*e^(-3)/d)*a*b^2 - 3/2*a*b^2*arctan(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*
x*e^3 + c^2*d*e^3) - 1/32*(8*(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^
3 + 12*(d*x + c)*arctan(d*x + c)^2 - 3*(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2 - 32*(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)*integrate(1/32*(16*
(d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c)^3 + 12*(d^3*x^3 + 3*c*d^2*x^2
 + c^3 + (3*c^2 + 1)*d*x + c)*arctan(d*x + c)^2 + 3*(d^3*x^3 + 3*c*d^2*x^2
+ c^3 + (3*c^2 + 1)*d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 24*(d^2*x
^2 + 2*c*d*x + c^2)*arctan(d*x + c) - 12*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
```

```
 + c^3)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 +
(10*c^2*e^3 + e^3)*d^3*x^3 + c^5*e^3 + (10*c^3*e^3 + 3*c*e^3)*d^2*x^2 + c^3
*e^3 + (5*c^4*e^3 + 3*c^2*e^3)*d*x), x))*b^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 +
 c^2*d*e^3) - 1/2*a^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*e^(-3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^3\operatorname{atan}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3a^2b\operatorname{atan}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*atan(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(3*a*b**2*atan(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3), x) + Integral(3*a**2*b*atan(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.20 $\int \frac{(a+b\textbf{ArcTan}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^2(a+b\text{ArcTan}(c+dx))}{de^4(c+dx)}-\frac{b(a+b\text{ArcTan}(c+dx))^2}{2de^4}-\frac{b(a+b\text{ArcTan}(c+dx))^2}{2de^4(c+dx)^2}+\frac{i(a+b\text{ArcTan}(c+dx))^3}{3de^4}$$

```
[Out] -b^2*(a+b*arctan(d*x+c))/d/e^4/(d*x+c)-1/2*b*(a+b*arctan(d*x+c))^2/d/e^4-1/
2*b*(a+b*arctan(d*x+c))^2/d/e^4/(d*x+c)^2+1/3*I*(a+b*arctan(d*x+c))^3/d/e^4
-1/3*(a+b*arctan(d*x+c))^3/d/e^4/(d*x+c)^3+b^3*ln(d*x+c)/d/e^4-1/2*b^3*ln(1
+(d*x+c)^2)/d/e^4-b*(a+b*arctan(d*x+c))^2*ln(2-2/(1-I*(d*x+c)))/d/e^4+I*b^2
*(a+b*arctan(d*x+c))*polylog(2,-1+2/(1-I*(d*x+c)))/d/e^4-1/2*b^3*polylog(3,
-1+2/(1-I*(d*x+c)))/d/e^4
```

Rubi [A]
time = 0.35, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5151, 12, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$\frac{ib^2\text{Li}_2\left(\frac{2}{1-i(c+dx)}-1\right)(a+b\text{ArcTan}(c+dx))}{de^4}-\frac{b^2(a+b\text{ArcTan}(c+dx))}{de^4(c+dx)}-\frac{b(a+b\text{ArcTan}(c+dx))^2}{2de^4(c+dx)^2}-\frac{b(a+b\text{ArcTan}(c+dx))^2}{2de^4}-\frac{(a+b\text{ArcTan}(c+dx))^3}{3de^4(c+dx)^3}+\frac{i(a+b\text{ArcTan}(c+dx))^3}{3de^4}-\frac{b\log\left(2-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{de^4}-\frac{b^3\text{Li}_3\left(\frac{2}{1-i(c+dx)}-1\right)}{2de^4}+\frac{b^3\log(c+dx)}{de^4}-\frac{b^3\log\left((c+dx)^2+1\right)}{2de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcTan[c + d*x]))/(d*e^4*(c + d*x))) - (b*(a + b*ArcTan[c + d
*x])^2)/(2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) + (
(I/3)*(a + b*ArcTan[c + d*x])^3)/(d*e^4) - (a + b*ArcTan[c + d*x])^3/(3*d*e
^4*(c + d*x)^3) + (b^3*Log[c + d*x])/(d*e^4) - (b^3*Log[1 + (c + d*x)^2])/(
2*d*e^4) - (b*(a + b*ArcTan[c + d*x])^2*Log[2 - 2/(1 - I*(c + d*x))])/(d*e^
4) + (I*b^2*(a + b*ArcTan[c + d*x])*PolyLog[2, -1 + 2/(1 - I*(c + d*x))])/(
d*e^4) - (b^3*PolyLog[3, -1 + 2/(1 - I*(c + d*x))])/(2*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^3(1+x^2)}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{x^3}\,dx,x,c+dx\right)}{de^4} - \frac{b\text{Subst}}{}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\tan^{-1}(c+dx))^3}{3de^4} - \frac{(a+b\tan^{-1}(c+dx)}{3de^4(c+dx)^3}$$

$$= -\frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4(c+dx)^2} + \frac{i(a+b\tan^{-1}(c+dx))^3}{3de^4} - \frac{(a+b\tan^{-1}(c+dx)}{3de^4(c+dx)^3}$$

$$= -\frac{b^2(a+b\tan^{-1}(c+dx))}{de^4(c+dx)} - \frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4} - \frac{b(a+b\tan^{-1}(c+dx}{2de^4(c+dx)^2}$$

$$= -\frac{b^2(a+b\tan^{-1}(c+dx))}{de^4(c+dx)} - \frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4} - \frac{b(a+b\tan^{-1}(c+dx}{2de^4(c+dx)^2}$$

$$= -\frac{b^2(a+b\tan^{-1}(c+dx))}{de^4(c+dx)} - \frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4} - \frac{b(a+b\tan^{-1}(c+dx}{2de^4(c+dx)^2}$$

$$= -\frac{b^2(a+b\tan^{-1}(c+dx))}{de^4(c+dx)} - \frac{b(a+b\tan^{-1}(c+dx))^2}{2de^4} - \frac{b(a+b\tan^{-1}(c+dx}{2de^4(c+dx)^2}$$

Mathematica [A]
time = 0.87, size = 360, normalized size = 1.25

$$\frac{-\frac{8a^3}{(c+dx)^3}-\frac{12a^2b}{(c+dx)^2}-\frac{24a^2b\text{ArcTan}(c+dx)}{(c+dx)^3}-24a^2b\log(c+dx)+12a^2b\log\left(1+c^2+2cdx+d^2x^2\right)+24ab^2\left(-\frac{(c+dx)^2+\text{ArcTan}(c+dx)^2}{(c+dx)^3}+\text{ArcTan}(c+dx)\left(-1-\frac{1}{(c+dx)^2}+i\text{ArcTan}(c+dx)-2\log\left(1-e^{2i\text{ArcTan}(c+dx)}\right)\right)+i\text{PolyLog}\left(2,e^{2i\text{ArcTan}(c+dx)}\right)\right)+b^3\left(i\pi^3-\frac{24\text{ArcTan}(c+dx)}{c+dx}-12\text{ArcTan}(c+dx)^2-\frac{12\text{ArcTan}(c+dx)^2}{(c+dx)^2}-8i\text{ArcTan}(c+dx)^3-\frac{8\text{ArcTan}(c+dx)^3}{(c+dx)^3}-24\text{ArcTan}(c+dx)^2\log\left(1-e^{-2i\text{ArcTan}(c+dx)}\right)+24\log\left(\frac{c+dx}{\sqrt{1+(c+dx)^2}}\right)-24i\text{ArcTan}(c+dx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(c+dx)}\right)-12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(c+dx)}\right)\right)}{24de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-8*a^3)/(c + d*x)^3 - (12*a^2*b)/(c + d*x)^2 - (24*a^2*b*ArcTan[c + d*x])
/(c + d*x)^3 - 24*a^2*b*Log[c + d*x] + 12*a^2*b*Log[1 + c^2 + 2*c*d*x + d^2
*x^2] + 24*a*b^2*(-(((c + d*x)^2 + ArcTan[c + d*x]^2)/(c + d*x)^3) + ArcTan
```

```
[c + d*x]*(-1 - (c + d*x)^(-2) + I*ArcTan[c + d*x] - 2*Log[1 - E^((2*I)*Arc
Tan[c + d*x])]) + I*PolyLog[2, E^((2*I)*ArcTan[c + d*x])]) + b^3*(I*Pi^3 -
(24*ArcTan[c + d*x])/(c + d*x) - 12*ArcTan[c + d*x]^2 - (12*ArcTan[c + d*x]
^2)/(c + d*x)^2 - (8*I)*ArcTan[c + d*x]^3 - (8*ArcTan[c + d*x]^3)/(c + d*x)
^3 - 24*ArcTan[c + d*x]^2*Log[1 - E^((-2*I)*ArcTan[c + d*x])] + 24*Log[(c +
 d*x)/Sqrt[1 + (c + d*x)^2]] - (24*I)*ArcTan[c + d*x]*PolyLog[2, E^((-2*I)*
ArcTan[c + d*x])] - 12*PolyLog[3, E^((-2*I)*ArcTan[c + d*x])]))/(24*d*e^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.13, size = 6805, normalized size = 23.71

method	result	size
derivativedivides	Expression too large to display	6805
default	Expression too large to display	6805

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(e^(-4)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2 - 2*e^(-4)*log(d*x + c)
/d^2 - 1/(d^4*x^2*e^4 + 2*c*d^3*x*e^4 + c^2*d^2*e^4)) - 2*arctan(d*x + c)/(
d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4))*a^2*b - 1/3*a
^3/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/96*(4*
b^3*arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)^2 - 96*(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4)*in
tegrate(1/32*(28*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c
)^3 + 4*(24*a*b^2*d^2*x^2 + 24*a*b^2*c^2 + b^3*c + 24*a*b^2 + (48*a*b^2*c +
 b^3)*d*x)*arctan(d*x + c)^2 - 4*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arct
an(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - (b^3*d*x + b^3*c - 3*(b^3*d^
2*x^2 + 2*b^3*c*d*x + b^3*c^2 + b^3)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2)/(d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*e^4 + e^4)*d^4*x^4
+ 4*(5*c^3*e^4 + c*e^4)*d^3*x^3 + c^6*e^4 + 3*(5*c^4*e^4 + 2*c^2*e^4)*d^2*x
^2 + c^4*e^4 + 2*(3*c^5*e^4 + 2*c^3*e^4)*d*x), x))/(d^4*x^3*e^4 + 3*c*d^3*x
^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*e^(-4)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{atan}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{atan}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2b\operatorname{atan}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*atan(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d**
2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*atan(c + d*x)**
2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) +
Integral(3*a**2*b*atan(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.21 $\int \frac{\mathbf{ArcTan}(1+x)}{2+2x} \, dx$

Optimal. Leaf size=31

$$\frac{1}{4}i\text{PolyLog}(2,-i(1+x))-\frac{1}{4}i\text{PolyLog}(2,i(1+x))$$

```
[Out] 1/4*I*polylog(2,-I*(1+x))-1/4*I*polylog(2,I*(1+x))
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5151, 12, 4940, 2438}

$$\frac{1}{4}i\text{Li}_2(-i(x+1))-\frac{1}{4}i\text{Li}_2(i(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(1+x)}{2+2x}\,dx = \text{Subst}\left(\int \frac{\tan^{-1}(x)}{2x}\,dx, x, 1+x\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, 1+x\right)$$
$$= \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, 1+x\right) - \frac{1}{4}i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, 1+x\right)$$
$$= \frac{1}{4}i\text{Li}_2(-i(1+x)) - \frac{1}{4}i\text{Li}_2(i(1+x))$$

Mathematica [**A**]
time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4}i\text{PolyLog}(2,-i(1+x)) - \frac{1}{4}i\text{PolyLog}(2,i(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[1 + x]/(2 + 2*x),x]
```

```
[Out] (I/4)*PolyLog[2, (-I)*(1 + x)] - (I/4)*PolyLog[2, I*(1 + x)]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.
time = 0.06, size = 68, normalized size = 2.19

method	result
risch	$-\frac{i\,\text{dilog}(-ix-i+1)}{4}+\frac{i\,\text{dilog}(ix+i+1)}{4}$
derivativedivides	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i\,\text{dilog}(1+i(1+x))}{4}-\frac{i\,\text{dilog}(1-i(1+$
default	$\frac{\ln(1+x)\arctan(1+x)}{2}+\frac{i\ln(1+x)\ln(1+i(1+x))}{4}-\frac{i\ln(1+x)\ln(1-i(1+x))}{4}+\frac{i\,\text{dilog}(1+i(1+x))}{4}-\frac{i\,\text{dilog}(1-i(1+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1+x)*arctan(1+x)+1/4*I*ln(1+x)*ln(1+I*(1+x))-1/4*I*ln(1+x)*ln(1-I*(1
+x))+1/4*I*dilog(1+I*(1+x))-1/4*I*dilog(1-I*(1+x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.
time = 0.51, size = 44, normalized size = 1.42

$$-\frac{1}{4}\arctan\left(x+1,0\right)\log\left(x^2+2\,x+2\right)+\frac{1}{2}\arctan\left(x+1\right)\log\left(|x+1|\right)-\frac{1}{4}i\,\text{Li}_2(i\,x+i+1)+\frac{1}{4}i\,\text{Li}_2(-i\,x-i+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*arctan2(x + 1, 0)*log(x^2 + 2*x + 2) + 1/2*arctan(x + 1)*log(abs(x + 1
)) - 1/4*I*dilog(I*x + I + 1) + 1/4*I*dilog(-I*x - I + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arctan(x + 1)/(x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{atan}{\left(x+1 \right)}}{x+1}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1+x)/(2+2*x),x)
```

```
[Out] Integral(atan(x + 1)/(x + 1), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.08, size = 25, normalized size = 0.81

$$-\frac{\mathrm{Li_2}\left(1-x\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{4}+\frac{\mathrm{Li_2}\left(x\,1{}\mathrm{i}+1+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x + 1)/(2*x + 2),x)
```

```
[Out] (dilog(x*1i + (1 + 1i))*1i)/4 - (dilog((1 - 1i) - x*1i)*1i)/4
```

3.22 $\int \frac{\text{ArcTan}(a+bx)}{\frac{ad}{b}+dx} dx$

Optimal. Leaf size=41

$$\frac{i\text{PolyLog}(2,-i(a+bx))}{2d}-\frac{i\text{PolyLog}(2,i(a+bx))}{2d}$$

```
[Out] 1/2*I*polylog(2,-I*(b*x+a))/d-1/2*I*polylog(2,I*(b*x+a))/d
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {5151, 12, 4940, 2438}

$$\frac{i\text{Li}_2(-i(a+bx))}{2d}-\frac{i\text{Li}_2(i(a+bx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*(a + b*x)])/d - ((I/2)*PolyLog[2, I*(a + b*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 5151

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcTan[x])^p, x],
x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\tan^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, a+bx\right)}{2d} - \frac{i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, a+bx\right)}{2d}$$
$$= \frac{i\text{Li}_2(-i(a+bx))}{2d} - \frac{i\text{Li}_2(i(a+bx))}{2d}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.83

$$\frac{i(\text{PolyLog}(2,-i(a+bx))-\text{PolyLog}(2,i(a+bx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*(a + b*x)] - PolyLog[2, I*(a + b*x)]))/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(33) = 66.
time = 0.07, size = 98, normalized size = 2.39

method	result	s
risch	$-\frac{i\,\mathrm{dilog}(-ibx-ia+1)}{2d}+\frac{i\,\mathrm{dilog}(ibx+ia+1)}{2d}$	3
derivativedivides	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9
default	$\frac{\frac{b\ln(bx+a)\arctan(bx+a)}{d}-\frac{b\left(-\frac{i\ln(bx+a)\ln(1+i(bx+a))}{2}+\frac{i\ln(bx+a)\ln(1-i(bx+a))}{2}-\frac{i\,\mathrm{dilog}(1+i(bx+a))}{2}+\frac{i\,\mathrm{dilog}(1-i(bx+a))}{2}\right)}{d}}{b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b/d*ln(b*x+a)*arctan(b*x+a)-b/d*(-1/2*I*ln(b*x+a)*ln(1+I*(b*x+a))+1/2*
I*ln(b*x+a)*ln(1-I*(b*x+a))-1/2*I*dilog(1+I*(b*x+a))+1/2*I*dilog(1-I*(b*x+a
))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(29) = 58$.
time = 0.51, size = 123, normalized size = 3.00

$$\frac{\arctan\left(bx+a\right)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{d}-\frac{\arctan\left(bx+a,0\right)\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,\arctan\left(bx+a\right)\log\left(\left|bx+a\right|\right)+i\,\mathrm{Li}_2(i\,bx+i\,a+1)-i\,\mathrm{Li}_2(-i\,bx-i\,a+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + a*d/b)/d - arctan((b^2*x + a*b)/b)*log(d*x + a*d/
b)/d - 1/2*(arctan2(b*x + a, 0)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*arctan
(b*x + a)*log(abs(b*x + a)) + I*dilog(I*b*x + I*a + 1) - I*dilog(-I*b*x - I
*a + 1))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arctan(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{atan}\left(a+bx\right)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(atan(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atan}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(d*x + (a*d)/b),x)
[Out] int(atan(a + b*x)/(d*x + (a*d)/b), x)
```

3.23 $\int (a+bx)^2 \sqrt{\textbf{ArcTan}(a+bx)}\, dx$

Optimal. Leaf size=21

$$\text{Int}\left((a+bx)^2\sqrt{\text{ArcTan}(a+bx)},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+bx)^2\sqrt{\text{ArcTan}(a+bx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

```
[Out] Defer[Int][(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Rubi steps

$$\int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx = \int (a+bx)^2\sqrt{\tan^{-1}(a+bx)}\, dx$$

Mathematica [A]
time = 4.23, size = 0, normalized size = 0.00

$$\int (a+bx)^2\sqrt{\text{ArcTan}(a+bx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]],x]
```

```
[Out] Integrate[(a + b*x)^2*Sqrt[ArcTan[a + b*x]], x]
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int (bx+a)^2\sqrt{\arctan(bx+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^2 \sqrt{\operatorname{atan}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*sqrt(atan(a + b*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sqrt{\operatorname{atan}(a+b\,x)}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(atan(a + b*x)^(1/2)*(a + b*x)^2, x)
```

3.24 $\int (e+fx)^3(a+b\textbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=233

$$-\frac{bf(6d^2e^2-12cdef-(1-6c^2)f^2)x}{4d^3}-\frac{bf^2(de-cf)(c+dx)^2}{2d^4}-\frac{bf^3(c+dx)^3}{12d^4}-\frac{b(d^4e^4-4cd^3e^3f-6(1-c^2)$$

```
[Out] -1/4*b*f*(6*d^2*e^2-12*c*d*e*f-(-6*c^2+1)*f^2)*x/d^3-1/2*b*f^2*(-c*f+d*e)*(
d*x+c)^2/d^4-1/12*b*f^3*(d*x+c)^3/d^4-1/4*b*(d^4*e^4-4*c*d^3*e^3*f-6*(-c^2+
1)*d^2*e^2*f^2+4*c*(-c^2+3)*d*e*f^3+(c^4-6*c^2+1)*f^4)*arctan(d*x+c)/d^4/f+
1/4*(f*x+e)^4*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*(-c*f+d*e+f)*(d*e-(1+c
)*f)*ln(1+(d*x+c)^2)/d^4
```

Rubi [A]
time = 0.28, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\frac{(e+fx)^4(a+b\text{ArcTan}(c+dx))}{4f}-\frac{b\text{ArcTan}(c+dx)\left(-6(1-c^2)\,d^2e^2f^2+4c(3-c^2)\,def^3+(c^4-6c^2+1)\,f^4-4cd^3e^3f+d^4e^4\right)}{4d^4f}-\frac{bfx(-(1-6c^2)\,f^2-12cdef+6d^2e^2)}{4d^3}-\frac{bf^2(c+dx)^2(de-cf)}{2d^4}-\frac{b(de-cf)(-cf+de+f)(de-(c+1)f)\log\left((c+dx)^2+1\right)}{2d^4}-\frac{bf^3(c+dx)^3}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/4*(b*f*(6*d^2*e^2 - 12*c*d*e*f - (1 - 6*c^2)*f^2)*x)/d^3 - (b*f^2*(d*e -
 c*f)*(c + d*x)^2)/(2*d^4) - (b*f^3*(c + d*x)^3)/(12*d^4) - (b*(d^4*e^4 - 4
*c*d^3*e^3*f - 6*(1 - c^2)*d^2*e^2*f^2 + 4*c*(3 - c^2)*d*e*f^3 + (1 - 6*c^2
 + c^4)*f^4)*ArcTan[c + d*x])/(4*d^4*f) + ((e + f*x)^4*(a + b*ArcTan[c + d*
x]))/(4*f) - (b*(d*e - c*f)*(d*e + f - c*f)*(d*e - (1 + c)*f)*Log[1 + (c +
d*x)^2])/(2*d^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3 \left(a+b\tan^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1+x^2}\,dx, x, c+\right.}{4f}$$

$$= \frac{(e+fx)^4\left(a+b\tan^{-1}(c+dx)\right)}{4f} - \frac{b\text{Subst}\left(\int \left(\frac{f^2\left(6d^2e^2-12cdef-(1-6c^2\right.}{d^4}\right.\right.}{}$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} -$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} -$$

$$= -\frac{bf\left(6d^2e^2-12cdef-\left(1-6c^2\right)f^2\right)x}{4d^3} - \frac{bf^2(de-cf)(c+dx)^2}{2d^4} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 157, normalized size = 0.67

$$\frac{(e+fx)^4(a+b\text{ArcTan}(c+dx)) - \frac{b\left(6df^2\left(6d^2e^2-12cdef+\left(-1+6c^2\right)f^2\right)x+12f^3(de-cf)(c+dx)^2+2f^4(c+dx)^3-3i(de-(-i+c)f)^4\log(i-c-dx)+3i(de-(i+c)f)^4\log(i+c+dx)\right)}{6d^4}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^4*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(6*d^2*e^2 - 12*c*d*e*f
+ (-1 + 6*c^2)*f^2)*x + 12*f^3*(d*e - c*f)*(c + d*x)^2 + 2*f^4*(c + d*x)^3
- (3*I)*(d*e - (-I + c)*f)^4*Log[I - c - d*x] + (3*I)*(d*e - (I + c)*f)^4*L
og[I + c + d*x]))/(6*d^4))/(4*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(221) = 442$.
time = 0.53, size = 557, normalized size = 2.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b*ln(1+(d*x+c)^2)*e^3+3*b/d^2*f^2*c*e*(d*x+c)+b/d^2*f^2*arctan(d*
x+c)*e*(d*x+c)^3+3/2*b/d*f*arctan(d*x+c)*e^2*(d*x+c)^2-3*b/d^2*f^2*arctan(d
*x+c)*c*e-3/2*b/d^2*f^2*ln(1+(d*x+c)^2)*c^2*e+3/2*b/d*f*ln(1+(d*x+c)^2)*c*e
^2-b/d^3*f^3*arctan(d*x+c)*c^3*(d*x+c)+3/2*b/d^3*f^3*arctan(d*x+c)*c^2*(d*x
+c)^2-b/d^3*f^3*arctan(d*x+c)*c*(d*x+c)^3+3*b/d^2*f^2*arctan(d*x+c)*c^2*e*(
d*x+c)-3*b/d*f*arctan(d*x+c)*c*e^2*(d*x+c)-3*b/d^2*f^2*arctan(d*x+c)*c*e*(d
*x+c)^2+1/2*b/d^3*f^3*c*(d*x+c)^2-3/2*b/d*f*e^2*(d*x+c)-1/2*b/d^2*f^2*e*(d*
x+c)^2+1/2*b/d^2*f^2*ln(1+(d*x+c)^2)*e+3/2*b/d*f*arctan(d*x+c)*e^2+1/4*b/d^
3*f^3*arctan(d*x+c)*(d*x+c)^4+1/2*b/d^3*f^3*ln(1+(d*x+c)^2)*c^3-1/2*b/d^3*f
^3*ln(1+(d*x+c)^2)*c+3/2*b/d^3*f^3*arctan(d*x+c)*c^2-3/2*b/d^3*f^3*c^2*(d*x
+c)+1/4*(c*f-d*e-f*(d*x+c))^4*a/d^3/f-1/12*b/d^3*f^3*(d*x+c)^3+1/4*b/d^3*f^
3*(d*x+c)+b*arctan(d*x+c)*e^3*(d*x+c)-1/4*b/d^3*f^3*arctan(d*x+c))
```

Maxima [A]
time = 0.48, size = 344, normalized size = 1.48

$$\frac{1}{4}\,af^3x^4+af^2x^3e+\frac{1}{12}\left(3\,x^4\arctan\left(dx+c\right)-d\left(\frac{d^2x^3-3\,cdx^2+3\,(3\,c^2-1)x}{d^4}+\frac{3\,(c^4-6\,c^2+1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^5}-\frac{6\,(c^3-c)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^5}\right)\right)bf^3+\frac{3}{2}\,afx^2e^2+\frac{1}{2}\left(2\,x^3\arctan\left(dx+c\right)-d\left(\frac{dx^2-4\,cx}{d^3}-\frac{2\,(c^3-3\,c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3\,c^2-1)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^4}\right)\right)bf^2e+\frac{3}{2}\left(x^2\arctan\left(dx+c\right)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log\left(d^2x^2+2\,cdx+c^2+1\right)}{d^3}\right)\right)bfe^2+axe^3+\frac{(2\,(dx+c)\arctan\left(dx+c\right)-\log\left((dx+c)^2+1\right))be^3}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*f^2*x^3*e + 1/12*(3*x^4*arctan(d*x + c) - d*((d^2*x^3 - 3
*c*d*x^2 + 3*(3*c^2 - 1)*x)/d^4 + 3*(c^4 - 6*c^2 + 1)*arctan((d^2*x + c*d)/
d)/d^5 - 6*(c^3 - c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^5))*b*f^3 + 3/2*a*f
*x^2*e^2 + 1/2*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3
*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^4))*b*f^2*e + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan(
(d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f*e^2 + a
*x*e^3 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e^3/d
```

Fricas [A]
time = 2.04, size = 317, normalized size = 1.36

$$\frac{3\,ad^4f^3x^4-bd^3f^3x^3+3\,bcd^2f^3x^2+12\,ad^4xe^3-3\,(3\,bc^2-b)df^3x+3\,(bd^4f^3x^4-(bc^4-6\,bc^2+b)f^3+4\,(bd^4x+bcd^3)e^3+6\,(bd^4fx^2-(bc^2-b)d^2f)e^2+4\,(bd^4f^2x^3+(bc^3-3\,bc)df^2)e)\arctan\left(dx+c\right)+18\,(ad^4fx^2-bd^3fx)e^2+6\,(2\,ad^4f^2x^3-bd^3f^2x^2+4\,bcd^2f^2x)e+6\,(3\,bcd^2fe^2-bd^3e^3-(3\,bc^2-b)df^2e+(bc^3-bc)f^3)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*f^3*x^4 - b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 12*a*d^4*x*e^3
- 3*(3*b*c^2 - b)*d*f^3*x + 3*(b*d^4*f^3*x^4 - (b*c^4 - 6*b*c^2 + b)*f^3 +
4*(b*d^4*x + b*c*d^3)*e^3 + 6*(b*d^4*f*x^2 - (b*c^2 - b)*d^2*f)*e^2 + 4*(b*
d^4*f^2*x^3 + (b*c^3 - 3*b*c)*d*f^2)*e)*arctan(d*x + c) + 18*(a*d^4*f*x^2 -
 b*d^3*f*x)*e^2 + 6*(2*a*d^4*f^2*x^3 - b*d^3*f^2*x^2 + 4*b*c*d^2*f^2*x)*e +
 6*(3*b*c*d^2*f*e^2 - b*d^3*e^3 - (3*b*c^2 - b)*d*f^2*e + (b*c^3 - b*c)*f^3
)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/d^4
```

Sympy [C] Result contains complex when optimal does not.
time = 12.06, size = 654, normalized size = 2.81

$$\begin{cases} ae^3x + \frac{3ae^2fx^2}{2} + aef^2x^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3\operatorname{atan}(c+dx)}{4d^4} + \frac{bc^3ef^2\operatorname{atan}(c+dx)}{d^3} + \frac{bc^3f^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^4} - \frac{ibc^3f^3\operatorname{atan}(c+dx)}{d^4} - \frac{3bc^2e^2f\operatorname{atan}(c+dx)}{2d^2} - \frac{3bc^2ef^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{3ibc^2ef^2\operatorname{atan}(c+dx)}{d^3} - \frac{3bc^2f^3x}{4d^3} + \frac{3bc^2f^3\operatorname{atan}(c+dx)}{2d^4} + \frac{bce^3\operatorname{atan}(c+dx)}{d} + \frac{3bce^2f\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{3ibce^2f\operatorname{atan}(c+dx)}{d^2} + \frac{2bcef^2x}{d^2} + \frac{bcf^3x^2}{4d^2} - \frac{3bcef^2\operatorname{atan}(c+dx)}{d^3} - \frac{bcf^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^4} + \frac{ibcf^3\operatorname{atan}(c+dx)}{d^4} + be^3x\operatorname{atan}(c+dx) + \frac{3be^2fx^2\operatorname{atan}(c+dx)}{2} + bef^2x^3\operatorname{atan}(c+dx) + \frac{bf^3x^4\operatorname{atan}(c+dx)}{4} - \frac{be^3\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe^3\operatorname{atan}(c+dx)}{d} - \frac{3be^2fx}{2d} - \frac{bef^2x^2}{2d} - \frac{bf^3x^3}{12d} + \frac{3be^2f\operatorname{atan}(c+dx)}{2d^2} + \frac{bef^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} - \frac{ibef^2\operatorname{atan}(c+dx)}{d^3} + \frac{bf^3x}{4d^3} - \frac{bf^3\operatorname{atan}(c+dx)}{4d^4} & \text{for}\: d \neq 0 \\ (a + b\operatorname{atan}(c))\left(e^3x + \frac{3e^2fx^2}{2} + ef^2x^3 + \frac{f^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*atan(c + d*x)/(4*d**4) + b*c**3*e*f**2*atan(c + d*x)/d**3 + b*c*
*3*f**3*log(c/d + x - I/d)/d**4 - I*b*c**3*f**3*atan(c + d*x)/d**4 - 3*b*c*
*2*e**2*f*atan(c + d*x)/(2*d**2) - 3*b*c**2*e*f**2*log(c/d + x - I/d)/d**3
+ 3*I*b*c**2*e*f**2*atan(c + d*x)/d**3 - 3*b*c**2*f**3*x/(4*d**3) + 3*b*c**
2*f**3*atan(c + d*x)/(2*d**4) + b*c*e**3*atan(c + d*x)/d + 3*b*c*e**2*f*log
(c/d + x - I/d)/d**2 - 3*I*b*c*e**2*f*atan(c + d*x)/d**2 + 2*b*c*e*f**2*x/d
**2 + b*c*f**3*x**2/(4*d**2) - 3*b*c*e*f**2*atan(c + d*x)/d**3 - b*c*f**3*l
og(c/d + x - I/d)/d**4 + I*b*c*f**3*atan(c + d*x)/d**4 + b*e**3*x*atan(c +
d*x) + 3*b*e**2*f*x**2*atan(c + d*x)/2 + b*e*f**2*x**3*atan(c + d*x) + b*f*
*3*x**4*atan(c + d*x)/4 - b*e**3*log(c/d + x - I/d)/d + I*b*e**3*atan(c + d
*x)/d - 3*b*e**2*f*x/(2*d) - b*e*f**2*x**2/(2*d) - b*f**3*x**3/(12*d) + 3*b
*e**2*f*atan(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x - I/d)/d**3 - I*b*e*f
**2*atan(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*atan(c + d*x)/(4*d**4),
 Ne(d, 0)), ((a + b*atan(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3 + f**3
*x**4/4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.02, size = 787, normalized size = 3.38

$$\mathrm{atan}(c+d\,x)\,\left(b\,e^3\,x+\frac{3\,b\,e^2\,f\,x^2}{2}+b\,e\,f^2\,x^3+\frac{b\,f^3\,x^4}{4}\right)+x\,\left(\frac{e\,\left(6\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+2\,a\,d^2\,e^2-3\,b\,d\,e\,f+6\,a\,f^2\right)}{2\,d^2}-\frac{\left(4\,c^2+4\right)\,\left(\frac{f^2\,\left(8\,a\,c\,f-b\,f+12\,a\,d\,e\right)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{4\,d^2}+\frac{2\,c\,\left(\frac{2\,c\,\left(\frac{f^2\,\left(8\,a\,c\,f-b\,f+12\,a\,d\,e\right)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{4\,d^2}+\frac{a\,f^3\,\left(4\,c^2+4\right)}{4\,d^2}\right)}{d}\right)-x^2\,\left(\frac{c\,\left(\frac{f^2\,\left(8\,a\,c\,f-b\,f+12\,a\,d\,e\right)}{4\,d}-\frac{2\,a\,c\,f^3}{d}\right)}{d}-\frac{4\,a\,c^2\,f^3+24\,a\,c\,d\,e\,f^2+12\,a\,d^2\,e^2\,f-4\,b\,d\,e\,f^2+4\,a\,f^3}{8\,d^2}+\frac{a\,f^3\,\left(4\,c^2+4\right)}{8\,d^2}\right)+x^3\,\left(\frac{f^2\,\left(8\,a\,c\,f-b\,f+12\,a\,d\,e\right)}{12\,d}-\frac{2\,a\,c\,f^3}{3\,d}\right)+\frac{a\,f^3\,x^4}{4}-\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\,\left(-64\,b\,c^3\,d^4\,f^3+192\,b\,c^2\,d^5\,e\,f^2-192\,b\,c\,d^6\,e^2\,f+64\,b\,c\,d^4\,f^3+64\,b\,d^7\,e^3-64\,b\,d^5\,e\,f^2\right)}{128\,d^8}-\frac{b\,\mathrm{atan}\left(\frac{4\,d^3\,\left(\frac{c\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^3}+\frac{x\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^2}\right)}{c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3}\right)\,\left(c^4\,f^3-4\,c^3\,d\,e\,f^2+6\,c^2\,d^2\,e^2\,f-6\,c^2\,f^3-4\,c\,d^3\,e^3+12\,c\,d\,e\,f^2-6\,d^2\,e^2\,f+f^3\right)}{4\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*atan(c + d*x)),x)
```

```
[Out] atan(c + d*x)*((b*f^3*x^4)/4 + b*e^3*x + (3*b*e^2*f*x^2)/2 + b*e*f^2*x^3) +
 x*((e*(6*a*f^2 + 6*a*c^2*f^2 + 2*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f))/(2
*d^2) - ((4*c^2 + 4)*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/
d))/(4*d^2) + (2*c*((2*c*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f
^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*
d*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 + 4))/(4*d^2)))/d) - x^2*((c*((f^2*(8*a*c*
f - b*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*f^3 + 4*a*c^2*f^3 - 4
*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f^3*(4*c^2 + 4))
/(8*d^2)) + x^3*((f^2*(8*a*c*f - b*f + 12*a*d*e))/(12*d) - (2*a*c*f^3)/(3*d
)) + (a*f^3*x^4)/4 - (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(64*b*d^7*e^3 - 64*b
*c^3*d^4*f^3 + 64*b*c*d^4*f^3 - 64*b*d^5*e*f^2 - 192*b*c*d^6*e^2*f + 192*b*
c^2*d^5*e*f^2))/(128*d^8) - (b*atan((4*d^3*((c*(f^3 - 6*c^2*f^3 + c^4*f^3 -
 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2
))/(4*d^3) + (x*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*
c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^2)))/(f^3 - 6*c^2*f^3 +
 c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2*d^2*e^2*f + 12*c*d*e*f^2 - 4*c
^3*d*e*f^2))*(f^3 - 6*c^2*f^3 + c^4*f^3 - 4*c*d^3*e^3 - 6*d^2*e^2*f + 6*c^2
*d^2*e^2*f + 12*c*d*e*f^2 - 4*c^3*d*e*f^2))/(4*d^4)
```

3.25 $\int (e+fx)^2(a+b\mathbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=155

$$-\frac{bf(de-cf)x}{d^2}-\frac{bf^2(c+dx)^2}{6d^3}-\frac{b(de-cf)\left(d^2e^2-2cdef-(3-c^2)\,f^2\right)\text{ArcTan}(c+dx)}{3d^3f}+\frac{(e+fx)^3(a+b\text{A}}{3}$$

```
[Out] -b*f*(-c*f+d*e)*x/d^2-1/6*b*f^2*(d*x+c)^2/d^3-1/3*b*(-c*f+d*e)*(d^2*e^2-2*c
*d*e*f-(-c^2+3)*f^2)*arctan(d*x+c)/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))/
f-1/6*b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*ln(1+(d*x+c)^2)/d^3
```

Rubi [A]
time = 0.15, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\frac{(e+fx)^3(a+b\text{ArcTan}(c+dx))}{3f}-\frac{b\text{ArcTan}(c+dx)(de-cf)\left(-(3-c^2)\,f^2-2cdef+d^2e^2\right)}{3d^3f}-\frac{b(-(1-3c^2)\,f^2-6cdef+3d^2e^2)\log\left((c+dx)^2+1\right)}{6d^3}-\frac{bf^2(c+dx)^2}{6d^3}-\frac{bfx(de-cf)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -((b*f*(d*e - c*f)*x)/d^2) - (b*f^2*(c + d*x)^2)/(6*d^3) - (b*(d*e - c*f)*(
d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*ArcTan[c + d*x])/(3*d^3*f) + ((e + f*x
)^3*(a + b*ArcTan[c + d*x]))/(3*f) - (b*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2
)*f^2)*Log[1 + (c + d*x)^2])/(6*d^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tan^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1+x^2}\,dx, x, c+\ldots\right.}{3f}$$

$$= \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)}{d^3}+\frac{f^3x}{d^3}+\frac{(de-\ldots}{}\right.\right.}{}$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \ldots$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3\left(a+b\tan^{-1}(c+dx)\right)}{3f} - \ldots$$

$$= -\frac{bf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2}{6d^3} - \frac{b(de-cf)\left(d^2e^2-2cdef-(3-\ldots\right.}{3d^3f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 118, normalized size = 0.76

$$\frac{(e+fx)^3(a+b\text{ArcTan}(c+dx)) - \frac{b\left(6df^2(de-cf)x+f^3(c+dx)^2-i(de-(-i+c)f)^3\log(i-c-dx)+i(de-(i+c)f)^3\log(i+c+dx)\right)}{2d^3}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^3*(a + b*ArcTan[c + d*x]) - (b*(6*d*f^2*(d*e - c*f)*x + f^3*(c +
 d*x)^2 - I*(d*e - (-I + c)*f)^3*Log[I - c - d*x] + I*(d*e - (I + c)*f)^3*L
og[I + c + d*x]))/(2*d^3))/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(147) = 294$.
time = 0.24, size = 303, normalized size = 1.95

method	result
derivativedivides	$\frac{-\frac{(cf-de-f(dx+c))^3 a}{3d^2 f}+\frac{b f^2 \arctan(dx+c)c^2(dx+c)}{d^2}-\frac{2bf \arctan(dx+c)ce(dx+c)}{d}-\frac{b f^2 \arctan(dx+c)c(dx+c)^2}{d^2}+b\arctan(dx+c)e}{}$
default	$\frac{-\frac{(cf-de-f(dx+c))^3 a}{3d^2 f}+\frac{b f^2 \arctan(dx+c)c^2(dx+c)}{d^2}-\frac{2bf \arctan(dx+c)ce(dx+c)}{d}-\frac{b f^2 \arctan(dx+c)c(dx+c)^2}{d^2}+b\arctan(dx+c)e}{}$
risch	$-\frac{i(fx+e)^3 b\ln(1+i(dx+c))}{6f}+\frac{x^3 f^2 a}{3}-\frac{f^2 b x^2}{6d}-\frac{f^2 b c^2 \ln(d^2x^2+2cdx+c^2+1)}{2d^3}-\frac{b e^3 \arctan(dx+c)}{6f}+\frac{f^2 b c^3 a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*(c*f-d*e-f*(d*x+c))^3*a/d^2/f+b/d^2*f^2*arctan(d*x+c)*c^2*(d*x+c)
-2*b/d*f*arctan(d*x+c)*c*e*(d*x+c)-b/d^2*f^2*arctan(d*x+c)*c*(d*x+c)^2+b*ar
ctan(d*x+c)*e^2*(d*x+c)+b/d*f*arctan(d*x+c)*e*(d*x+c)^2+1/3*b/d^2*f^2*arcta
n(d*x+c)*(d*x+c)^3+b/d^2*f^2*c*(d*x+c)-b/d*f*e*(d*x+c)-1/6*b/d^2*f^2*(d*x+c
)^2-1/2*b/d^2*f^2*ln(1+(d*x+c)^2)*c^2+b/d*f*ln(1+(d*x+c)^2)*c*e-1/2*e^2*b*l
n(1+(d*x+c)^2)+1/6*b/d^2*f^2*ln(1+(d*x+c)^2)-b/d^2*f^2*arctan(d*x+c)*c+b/d*
f*arctan(d*x+c)*e)
```

Maxima [A]
time = 0.48, size = 220, normalized size = 1.42

$$\frac{1}{3}af^2x^3+afx^2e+\frac{1}{6}\left(2x^3\arctan(dx+c)-d\left(\frac{dx^2-4cx}{d^3}-\frac{2(c^3-3c)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^4}+\frac{(3c^2-1)\log(d^2x^2+2cdx+c^2+1)}{d^4}\right)\right)bf^2+\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bfe+axe^2+\frac{(2(dx+c)\arctan(dx+c)-\log((dx+c)^2+1))be^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + 1/6*(2*x^3*arctan(d*x + c) - d*((d*x^2 - 4*c*x)
/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (3*c^2 - 1)*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/d^4))*b*f^2 + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 -
 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*
b*f*e + a*x*e^2 + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*
b*e^2/d
```

Fricas [A]
time = 2.33, size = 199, normalized size = 1.28

$$\frac{2ad^3f^2x^3-bd^2f^2x^2+4bcdf^2x+6ad^3xe^2+2(bd^3f^2x^3+(bc^3-3bc)f^2+3(bd^3x+bcd^2)e^2+3(bd^3fx^2-(bc^2-b)df)e)\arctan(dx+c)+6(ad^3fx^2-bd^2fx)e+(6bcdfe-3bd^2e^2-(3bc^2-b)f^2)\log(d^2x^2+2cdx+c^2+1)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 - b*d^2*f^2*x^2 + 4*b*c*d*f^2*x + 6*a*d^3*x*e^2 + 2*(b
*d^3*f^2*x^3 + (b*c^3 - 3*b*c)*f^2 + 3*(b*d^3*x + b*c*d^2)*e^2 + 3*(b*d^3*f
*x^2 - (b*c^2 - b)*d*f)*e)*arctan(d*x + c) + 6*(a*d^3*f*x^2 - b*d^2*f*x)*e
+ (6*b*c*d*f*e - 3*b*d^2*e^2 - (3*b*c^2 - b)*f^2)*log(d^2*x^2 + 2*c*d*x + c
^2 + 1))/d^3
```

Sympy [C] Result contains complex when optimal does not.
time = 4.60, size = 376, normalized size = 2.43

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{atan}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{atan}(c+dx)}{d^2} - \frac{bc^2f^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^3} + \frac{ibc^2f^2\operatorname{atan}(c+dx)}{d^3} + \frac{bce^2\operatorname{atan}(c+dx)}{d} + \frac{2bcef\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{2ibcef\operatorname{atan}(c+dx)}{d^2} + \frac{2bcf^2x}{3d^2} - \frac{bcf^2\operatorname{atan}(c+dx)}{d^3} + be^2x\operatorname{atan}(c+dx) + befx^2\operatorname{atan}(c+dx) + \frac{bf^2x^3\operatorname{atan}(c+dx)}{3} - \frac{be^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe^2\operatorname{atan}(c+dx)}{d} - \frac{befx}{d} - \frac{bf^2x^2}{6d} + \frac{bef\operatorname{atan}(c+dx)}{d^2} + \frac{bf^2\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{3d^3} - \frac{ibf^2\operatorname{atan}(c+dx)}{3d^3} & \text{for}\ d \neq 0 \\ (a + b\operatorname{atan}(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*atan(c + d*x
)/(3*d**3) - b*c**2*e*f*atan(c + d*x)/d**2 - b*c**2*f**2*log(c/d + x - I/d)
/d**3 + I*b*c**2*f**2*atan(c + d*x)/d**3 + b*c*e**2*atan(c + d*x)/d + 2*b*c
*e*f*log(c/d + x - I/d)/d**2 - 2*I*b*c*e*f*atan(c + d*x)/d**2 + 2*b*c*f**2*
x/(3*d**2) - b*c*f**2*atan(c + d*x)/d**3 + b*e**2*x*atan(c + d*x) + b*e*f*x
**2*atan(c + d*x) + b*f**2*x**3*atan(c + d*x)/3 - b*e**2*log(c/d + x - I/d)
/d + I*b*e**2*atan(c + d*x)/d - b*e*f*x/d - b*f**2*x**2/(6*d) + b*e*f*atan(
c + d*x)/d**2 + b*f**2*log(c/d + x - I/d)/(3*d**3) - I*b*f**2*atan(c + d*x)
/(3*d**3), Ne(d, 0)), ((a + b*atan(c))*(e**2*x + e*f*x**2 + f**2*x**3/3), T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.78, size = 411, normalized size = 2.65

$$x^2\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-x\left(\frac{2\,c\left(\frac{f\,(6\,a\,c\,f-b\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2-3\,b\,d\,e\,f+3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2+3)}{3\,d^2}\right)+\operatorname{atan}(c+d\,x)\left(b\,e^2\,x+b\,e\,f\,x^2+\frac{b\,f^2\,x^3}{3}\right)+\frac{a\,f^2\,x^3}{3}-\frac{\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)\left(36\,b\,c^2\,d^3\,f^2-72\,b\,c\,d^4\,e\,f+36\,b\,d^5\,e^2-12\,b\,d^3\,f^2\right)}{72\,d^6}+\frac{b\operatorname{atan}\left(\frac{3\,d^2\left(\frac{c\,(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f)}{3\,d^2}+\frac{x\,(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f)}{3\,d}\right)}{c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f}\right)\left(c^3\,f^2-3\,c^2\,d\,e\,f+3\,c\,d^2\,e^2-3\,c\,f^2+3\,d\,e\,f\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x)),x)
```

```
[Out] x^2*((f*(6*a*c*f - b*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - x*((2*c*((f*(6*a*
c*f - b*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*f^2 + 3*a*c^2*f^2 +
3*a*d^2*e^2 - 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 + 3))/(3*d^
2)) + atan(c + d*x)*((b*f^2*x^3)/3 + b*e^2*x + b*e*f*x^2) + (a*f^2*x^3)/3 -
 (log(c^2 + d^2*x^2 + 2*c*d*x + 1)*(36*b*d^5*e^2 - 12*b*d^3*f^2 + 36*b*c^2*
d^3*f^2 - 72*b*c*d^4*e*f))/(72*d^6) + (b*atan((3*d^2*((c*(c^3*f^2 - 3*c*f^2
 + 3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d^2) + (x*(c^3*f^2 - 3*c*f^2 +
3*c*d^2*e^2 + 3*d*e*f - 3*c^2*d*e*f))/(3*d)))/(c^3*f^2 - 3*c*f^2 + 3*c*d^2*
e^2 + 3*d*e*f - 3*c^2*d*e*f))*(c^3*f^2 - 3*c*f^2 + 3*c*d^2*e^2 + 3*d*e*f -
3*c^2*d*e*f))/(3*d^3)
```

3.26 $\int (e+fx)(a+b\textbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{bfx}{2d}-\frac{b(de+f-cf)(de-(1+c)f)\text{ArcTan}(c+dx)}{2d^2f}+\frac{(e+fx)^2(a+b\text{ArcTan}(c+dx))}{2f}-\frac{b(de-cf)\log(1}{2d^2}$$

```
[Out] -1/2*b*f*x/d-1/2*b*(-c*f+d*e+f)*(d*e-(1+c)*f)*arctan(d*x+c)/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))/f-1/2*b*(-c*f+d*e)*ln(1+(d*x+c)^2)/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\frac{(e+fx)^2(a+b\text{ArcTan}(c+dx))}{2f}-\frac{b\text{ArcTan}(c+dx)(-cf+de+f)(de-(c+1)f)}{2d^2f}-\frac{b(de-cf)\log\left((c+dx)^2+1\right)}{2d^2}-\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] -1/2*(b*f*x)/d - (b*(d*e + f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*d
^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x]))/(2*f) - (b*(d*e - c*f)*Log[1
+ (c + d*x)^2])/(2*d^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1+x^2}\,dx,x,c+ \right.}{2f}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \left(\frac{f^2}{d^2}+\frac{(de-f-cf)(de+f-c}{d^2(1+x}\right.\right.}{2f}$$

$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \frac{(de-f-cf)(de+f}{1+}\right.}{}$$

$$= -\frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)}{2f} - \frac{(b(de-cf))\text{Subst}\left(\int \frac{}{1+}\right.}{d^2}$$

$$= -\frac{bfx}{2d} - \frac{b(de+f-cf)(de-(1+c)f)\tan^{-1}(c+dx)}{2d^2f} + \frac{(e+fx)^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 163, normalized size = 1.68

$$aex+\frac{1}{2}afx^2+bex\text{ArcTan}(c+dx)+\frac{bf\left(\frac{1}{2}d\left(-\frac{c}{d}+\frac{c+dx}{d}\right)^2\text{ArcTan}(c+dx)-\frac{1}{2}d\left(\frac{x}{d}-\frac{i(i-c)^2\log(i-c-dx)}{2d^2}+\frac{i(i+c)^2\log(i+c+dx)}{2d^2}\right)\right)}{d}-\frac{be\left(-2c\text{ArcTan}(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] a*e*x + (a*f*x^2)/2 + b*e*x*ArcTan[c + d*x] + (b*f*((d*(-(c/d) + (c + d*x)/
d)^2*ArcTan[c + d*x])/2 - (d*(x/d - ((I/2)*(I - c)^2*Log[I - c - d*x])/d^2
```

```
+ ((I/2)*(I + c)^2*Log[I + c + d*x])/d^2))/2))/d - (b*e*(-2*c*ArcTan[c + d*
x] + Log[1 + c^2 + 2*c*d*x + d^2*x^2]))/(2*d)
```

Maple [A]
time = 0.08, size = 150, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\arctan(dx+c)fc(dx+c)}{d}+b\arctan(dx+c)e(dx+c)+\frac{b\arctan(dx+c)f(dx+c)^2}{2d}-\frac{bf(dx+c)}{2d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\arctan(dx+c)fc(dx+c)}{d}+b\arctan(dx+c)e(dx+c)+\frac{b\arctan(dx+c)f(dx+c)^2}{2d}-\frac{bf(dx+c)}{2d}}{d}$
risch	$-\frac{ib(fx^2+2ex)\ln(1+i(dx+c))}{4}+\frac{ibfx^2\ln(1-i(dx+c))}{4}+\frac{ibex\ln(1-i(dx+c))}{2}+\frac{afx^2}{2}-\frac{\arctan(dx+c)bc^2f}{2d^2}+$ b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)-b/d*arctan(d*x+c)*f*c*(
d*x+c)+b*arctan(d*x+c)*e*(d*x+c)+1/2*b/d*arctan(d*x+c)*f*(d*x+c)^2-1/2*b/d*
f*(d*x+c)+1/2*b/d*ln(1+(d*x+c)^2)*f*c-1/2*b*ln(1+(d*x+c)^2)*e+1/2*b/d*f*arc
tan(d*x+c))
```

Maxima [A]
time = 0.47, size = 118, normalized size = 1.22

$$\frac{1}{2}afx^2+\frac{1}{2}\left(x^2\arctan(dx+c)-d\left(\frac{x}{d^2}+\frac{(c^2-1)\arctan\left(\frac{d^2x+cd}{d}\right)}{d^3}-\frac{c\log(d^2x^2+2cdx+c^2+1)}{d^3}\right)\right)bf+axe+\frac{(2(dx+c)\arctan(dx+c)-\log((dx+c)^2+1))be}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x
 + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*b*f + a*x*e + 1/2
*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b*e/d
```

Fricas [A]
time = 2.52, size = 104, normalized size = 1.07

$$\frac{ad^2fx^2+2ad^2xe-bdfx+(bd^2fx^2-(bc^2-b)f+2(bd^2x+bcd)e)\arctan(dx+c)+(bcf-bde)\log(d^2x^2+2cdx+c^2+1)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*f*x^2 + 2*a*d^2*x*e - b*d*f*x + (b*d^2*f*x^2 - (b*c^2 - b)*f + 2
*(b*d^2*x + b*c*d)*e)*arctan(d*x + c) + (b*c*f - b*d*e)*log(d^2*x^2 + 2*c*d
*x + c^2 + 1))/d^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.82, size = 177, normalized size = 1.82

$$\begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2f\operatorname{atan}(c+dx)}{2d^2} + \frac{bce\operatorname{atan}(c+dx)}{d} + \frac{bcf\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d^2} - \frac{ibcf\operatorname{atan}(c+dx)}{d^2} + bex\operatorname{atan}(c+dx) + \frac{bfx^2\operatorname{atan}(c+dx)}{2} - \frac{be\log\left(\frac{c}{d}+x-\frac{i}{d}\right)}{d} + \frac{ibe\operatorname{atan}(c+dx)}{d} - \frac{bfx}{2d} + \frac{bf\operatorname{atan}(c+dx)}{2d^2} & \text{for } d \neq 0 \\ (a + b\operatorname{atan}(c))\left(ex + \frac{fx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*atan(c + d*x)/(2*d**2) + b*c*e*ata
n(c + d*x)/d + b*c*f*log(c/d + x - I/d)/d**2 - I*b*c*f*atan(c + d*x)/d**2 +
 b*e*x*atan(c + d*x) + b*f*x**2*atan(c + d*x)/2 - b*e*log(c/d + x - I/d)/d
+ I*b*e*atan(c + d*x)/d - b*f*x/(2*d) + b*f*atan(c + d*x)/(2*d**2), Ne(d, 0
)), ((a + b*atan(c))*(e*x + f*x**2/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.80, size = 136, normalized size = 1.40

$$a\,e\,x+\frac{a\,f\,x^2}{2}-\frac{b\,e\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2}+\frac{b\,f\,x^2\,\mathrm{atan}(c+d\,x)}{2}-\frac{b\,f\,x}{2\,d}+b\,e\,x\,\mathrm{atan}(c+d\,x)-\frac{b\,c^2\,f\,\mathrm{atan}(c+d\,x)}{2\,d^2}+\frac{b\,c\,f\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d^2}+\frac{b\,c\,e\,\mathrm{atan}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 - (b*e*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*f*a
tan(c + d*x))/(2*d^2) + (b*f*x^2*atan(c + d*x))/2 - (b*f*x)/(2*d) + b*e*x*a
tan(c + d*x) - (b*c^2*f*atan(c + d*x))/(2*d^2) + (b*c*f*log(c^2 + d^2*x^2 +
 2*c*d*x + 1))/(2*d^2) + (b*c*e*atan(c + d*x))/d
```

3.27 $\int (a + b\mathbf{ArcTan}(c + dx))\, dx$

Optimal. Leaf size=38

$$ax + \frac{b(c + dx)\text{ArcTan}(c + dx)}{d} - \frac{b \log\left(1 + (c + dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arctan(d*x+c)/d-1/2*b*ln(1+(d*x+c)^2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5147, 4930, 266}

$$ax + \frac{b(c + dx)\text{ArcTan}(c + dx)}{d} - \frac{b \log\left((c + dx)^2 + 1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcTan[c + d*x])/d - (b*Log[1 + (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tan^{-1}(c+dx)\right)\,dx &= ax+b\int \tan^{-1}(c+dx)\,dx\\
&= ax+\frac{b\text{Subst}\left(\int \tan^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\tan^{-1}(c+dx)}{d}-\frac{b\log\left(1+(c+dx)^2\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.29

$$
ax+bx\text{ArcTan}(c+dx)-\frac{b(-2c\text{ArcTan}(c+dx)+\log\left(1+c^2+2cdx+d^2x^2\right))}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c + d*x],x]
```

```
[Out] a*x + b*x*ArcTan[c + d*x] - (b*(-2*c*ArcTan[c + d*x] + Log[1 + c^2 + 2*c*d*
x + d^2*x^2]))/(2*d)
```

Maple [A]
time = 0.04, size = 42, normalized size = 1.11

method	result	size
derivativedivides	$\frac{(dx+c)a+b(dx+c)\arctan(dx+c)-\frac{b\ln\left(1+(dx+c)^2\right)}{2}}{d}$	39
default	$ax+b\arctan\left(dx+c\right)x+\frac{b\arctan(dx+c)c}{d}-\frac{b\ln\left(1+(dx+c)^2\right)}{2d}$	42
risch	$ax-\frac{ibx\ln(1+i(dx+c))}{2}+\frac{ibx\ln(1-i(dx+c))}{2}+\frac{b\arctan(dx+c)c}{d}-\frac{b\ln(d^2x^2+2cdx+c^2+1)}{2d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*arctan(d*x+c)*x+b/d*arctan(d*x+c)*c-1/2*b*ln(1+(d*x+c)^2)/d
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.95

$$
ax+\frac{\left(2\left(dx+c\right)\arctan\left(dx+c\right)-\log\left(\left(dx+c\right)^2+1\right)\right)b}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

Fricas [A]
time = 3.23, size = 48, normalized size = 1.26

$$\frac{2\,adx+2\,(bdx+bc)\arctan\left(dx+c\right)-b\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + 2*(b*d*x + b*c)*arctan(d*x + c) - b*log(d^2*x^2 + 2*c*d*x +
c^2 + 1))/d
```

Sympy [A]
time = 0.15, size = 51, normalized size = 1.34

$$ax+b\left(\begin{cases}\frac{c\operatorname{atan}\left(c+dx\right)}{d}+x\operatorname{atan}\left(c+dx\right)-\frac{\log\left(c^2+2cdx+d^2x^2+1\right)}{2d} & \text{for}\ d\neq 0\\ x\operatorname{atan}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*atan(c + d*x)/d + x*atan(c + d*x) - log(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/(2*d), Ne(d, 0)), (x*atan(c), True))
```

Giac [A]
time = 0.40, size = 36, normalized size = 0.95

$$ax+\frac{\left(2\,(dx+c)\arctan\left(dx+c\right)-\log\left((dx+c)^2+1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*b/d
```

Mupad [B]
time = 1.10, size = 49, normalized size = 1.29

$$a\,x+b\,x\,\mathrm{atan}(c+d\,x)-\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2+1\right)}{2\,d}+\frac{b\,c\,\mathrm{atan}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c + d*x),x)
```

```
[Out] a*x + b*x*atan(c + d*x) - (b*log(c^2 + d^2*x^2 + 2*c*d*x + 1))/(2*d) + (b*c
*atan(c + d*x))/d
```

3.28 $\int \frac{a+b\text{ArcTan}(c+dx)}{e+fx}\,dx$

Optimal. Leaf size=162

$$-\frac{(a+b\text{ArcTan}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\text{ArcTan}(c+dx))\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}+\frac{ib\text{PolyLog}\Big(2,}{2}$$

[Out] -(a+b*arctan(d*x+c))*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+1/2*I*b*polylog(2,1-2/(1-I*(d*x+c)))/f-1/2*I*b*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f

Rubi [A]
time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5155, 4966, 2449, 2352, 2497}

$$\frac{(a+b\text{ArcTan}(c+dx))\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{f}-\frac{ib\text{Li}_2\Big(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\Big)}{2f}+\frac{ib\text{Li}_2\Big(1-\frac{2}{1-i(c+dx)}\Big)}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x),x]

[Out] -(((a + b*ArcTan[c + d*x])*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[c + d*x])*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - ((I/2)*b*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLog[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2d(e+f\ldots}{(de+if-cf)(1\ldots}\right.}{f} \ldots$$

$$= -\frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2d(e+f\ldots}{(de+if-cf)(1\ldots}\right.}{f} \ldots$$

$$= -\frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\tan^{-1}(c+dx))\log\left(\frac{2d(e+f\ldots}{(de+if-cf)(1\ldots}\right.}{f} \ldots$$

Mathematica [A]
time = 0.18, size = 315, normalized size = 1.94

$$\frac{a\log(e+fx)+b\text{ArcTan}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)+\log\left(\sin\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\right)\right)+\frac{1}{2}b\left(-\frac{1}{4}i(\pi-2\text{ArcTan}(c+dx))^2-i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)^2+(\pi-2\text{ArcTan}(c+dx))\log\left(1+e^{-2i\text{ArcTan}(c+dx)}\right)+2\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)-(\pi-2\text{ArcTan}(c+dx))\log\left(\frac{2}{\sqrt{1+(c+dx)^2}}\right)-2\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\log\left(2\sin\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\right)-i\text{PolyLog}\left(2,-e^{-2i\text{ArcTan}(c+dx)}\right)-i\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] + b*ArcTan[c + d*x]*(-Log[1/Sqrt[1 + (c + d*x)^2]] + Log[Si
n[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]]) + (b*((-1/4*I)*(Pi - 2*ArcTan[
c + d*x])^2 - I*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x])^2 + (Pi - 2*ArcTa
n[c + d*x])*Log[1 + E^((-2*I)*ArcTan[c + d*x])] + 2*(ArcTan[(d*e - c*f)/f]
```

```
+ ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x
]))] - (Pi - 2*ArcTan[c + d*x])*Log[2/Sqrt[1 + (c + d*x)^2]] - 2*(ArcTan[(d
*e - c*f)/f] + ArcTan[c + d*x])*Log[2*Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c
+ d*x]]] - I*PolyLog[2, -E^((-2*I)*ArcTan[c + d*x])] - I*PolyLog[2, E^((2*I
)*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/2)/f
```

Maple [A]
time = 0.10, size = 238, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}-\frac{ibd\ln(cf-de-f(dx+c))\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)}{2f}+\frac{ibd\ln(cf-de-f(dx+c}{2f}}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\arctan(dx+c)}{f}-\frac{ibd\ln(cf-de-f(dx+c))\ln\left(\frac{if+f(dx+c)}{cf-de+if}\right)}{2f}+\frac{ibd\ln(cf-de-f(dx+c}{2f}}{d}$
risch	$\frac{a\ln(icf-ide+(-idx-ic+1)f-f)}{f}+\frac{ib\,\mathrm{dilog}\left(\frac{icf-ide+(-idx-ic+1)f-f}{icf-ide-f}\right)}{2f}+\frac{ib\ln(-idx-ic+1)\ln\left(\frac{icf-ide+(-idx-ic+}{icf-ide-f}\right.}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d*ln(c*f-d*e-f*(d*x+c))/f+b*d*ln(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)-
1/2*I*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))+1/2*I*b
*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b*d/f*di
log((I*f+f*(d*x+c))/(c*f-d*e+I*f))+1/2*I*b*d/f*dilog((I*f-f*(d*x+c))/(d*e+I
*f-c*f)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(d*x + c)/(f*x + e), x) + a*log(f*x + e)/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(d*x + c) + a)/(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{atan}(c+d\,x)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))/(e + f*x), x)
```

3.29 $\int \frac{a+b\text{ArcTan}(c+dx)}{(e+fx)^2} \, dx$

Optimal. Leaf size=151

$$\frac{bd(de-cf)\text{ArcTan}(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)}-\frac{a+b\text{ArcTan}(c+dx)}{f(e+fx)}+\frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2}-\frac{bd\log\left(1+c^2+2cdx\right.}{2\left(d^2e^2-2cdef+(1-\right.}$$

```
[Out] b*d*(-c*f+d*e)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+(-a-b*arctan
(d*x+c))/f/(f*x+e)+b*d*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-1/2*b*d*ln
(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A]
time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5153, 2007, 719, 31, 648, 632, 210, 642}

$$-\frac{a+b\text{ArcTan}(c+dx)}{f(e+fx)}+\frac{bd\text{ArcTan}(c+dx)(de-cf)}{f\left((c^2+1)f^2-2cdef+d^2e^2\right)}-\frac{bd\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left((c^2+1)f^2-2cdef+d^2e^2\right)}+\frac{bd\log(e+fx)}{(c^2+1)f^2-2cdef+d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])/(f*(e + f*x)) + (b*d*Log[e + f*x])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) - (b*d*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e
^2 - 2*c*d*e*f + (1 + c^2)*f^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{1}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{1}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{(bd)\int \frac{d^2e-2cdf-d^2fx}{1+c^2+2cdx+d^2x^2}\,dx}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} + \frac{(bdf)\int \frac{1}{e+fx}\,d}{d^2e^2-2cdef+(1+}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{(bd)\int \frac{2cd+2d^2x}{1+c^2+2cdx+d^2x^2}}{2\left(d^2e^2-2cdef+(1+\right.}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+c^2)f^2} - \frac{bd\log\left(1+c^2+2cdx+\right.}{2\left(d^2e^2-2cdef+(1+\right.}$$

$$= \frac{bd(de-cf)\tan^{-1}(c+dx)}{f\left(d^2e^2-2cdef+(1+c^2)f^2\right)} - \frac{a+b\tan^{-1}(c+dx)}{f(e+fx)} + \frac{bd\log(e+fx)}{d^2e^2-2cdef+(1+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 121, normalized size = 0.80

$$\frac{-\frac{a+b\text{ArcTan}(c+dx)}{e+fx}+\frac{bd(i(-de+(i+c)f)\log(i-c-dx)+i(de+if-cf)\log(i+c+dx)+2f\log(d(e+fx)))}{2(d^2e^2-2cdef+(1+c^2)f^2)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)) + (b*d*(I*(-(d*e) + (I + c)*f)*Log[I
 - c - d*x] + I*(d*e + I*f - c*f)*Log[I + c + d*x] + 2*f*Log[d*(e + f*x)]))/
(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)))/f
```

Maple [A]
time = 0.16, size = 234, normalized size = 1.55

method	result
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{b\,d^2\arctan(dx+c)c}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{b\,d^3\arctan(dx+c)e}{f\left(c^2f^2-2cdef+d^2e^2+f^2\right)}+\cdots}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\arctan(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln\left(1+(dx+c)^2\right)}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)}-\frac{b\,d^2\arctan(dx+c)c}{c^2f^2-2cdef+d^2e^2+f^2}+\frac{b\,d^3\arctan(dx+c)e}{f\left(c^2f^2-2cdef+d^2e^2+f^2\right)}+\cdots}{d}$
risch	$\frac{ib\ln(1+i(dx+c))}{2f(fx+e)}+\frac{i\ln\left((cdf-d^2e-3idf)x-2icf-ide+c^2f-cde+3f\right)b\,d^2efx-i\ln\left((-cdf+d^2e-3idf)x-2icf-ide-c^2\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d^2/(c*f-d*e-f*(d*x+c))/f+b*d^2/(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)-
1/2*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-b*d^2/(c^2*f^2-2*
c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c+b*d^3/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2
)*arctan(d*x+c)*e+b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c
)))
```

Maxima [A]
time = 0.47, size = 186, normalized size = 1.23

$$\frac{1}{2}\left(d\left(\frac{2\,(cdf-d^2e)\arctan\left(\frac{d^2x+cd}{d}\right)}{(2\,cdf^2e-(c^2+1)f^3-d^2fe^2)d}+\frac{\log\left(d^2x^2+2\,cdx+c^2+1\right)}{2\,cdfe-(c^2+1)f^2-d^2e^2}-\frac{2\,\log\left(fx+e\right)}{2\,cdfe-(c^2+1)f^2-d^2e^2}\right)-\frac{2\,\arctan\left(dx+c\right)}{f^2x+fe}\right)b-\frac{a}{f^2x+fe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1)
*f^3 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2 +
 1)*f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2))
- 2*arctan(d*x + c)/(f^2*x + f*e))*b - a/(f^2*x + f*e)
```

Fricas [A]
time = 2.20, size = 195, normalized size = 1.29

$$\frac{4\,acdfe-2\,ad^2e^2-2\,(ac^2+a)f^2-2\,(bcdf^2x+(bc^2+b)f^2-(bd^2fx+bcdf)e)\arctan\left(dx+c\right)-(bdf^2x+bdfe)\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\,(bdf^2x+bdfe)\log\left(fx+e\right)}{2\,((c^2+1)f^4x+d^2fe^3+(d^2f^2x-2\,cdf^2)e^2-(2\,cdf^3x-(c^2+1)f^3)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*c*d*f*e - 2*a*d^2*e^2 - 2*(a*c^2 + a)*f^2 - 2*(b*c*d*f^2*x + (b*c^
2 + b)*f^2 - (b*d^2*f*x + b*c*d*f)*e)*arctan(d*x + c) - (b*d*f^2*x + b*d*f*
e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*d*f^2*x + b*d*f*e)*log(f*x + e))
/((c^2 + 1)*f^4*x + d^2*f*e^3 + (d^2*f^2*x - 2*c*d*f^2)*e^2 - (2*c*d*f^3*x
- (c^2 + 1)*f^3)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.83, size = 127, normalized size = 0.84

$$\frac{b\,d\,\ln\left(e+f\,x\right)}{d^2\,e^2-2\,c\,d\,e\,f+\left(c^2+1\right)\,f^2}-\frac{b\,\mathrm{atan}(c+d\,x)}{f\,\left(e+f\,x\right)}-\frac{a}{x\,f^2+e\,f}-\frac{b\,d\,\ln\left(c+d\,x-\mathrm{i}\right)\,1\mathrm{i}}{2\,f\,\left(d\,e-c\,f+f\,1\mathrm{i}\right)}-\frac{b\,d\,\ln\left(c+d\,x+1\mathrm{i}\right)}{2\,f\,\left(f-c\,f\,1\mathrm{i}+d\,e\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^2,x)
```

```
[Out] (b*d*log(e + f*x))/(f^2*(c^2 + 1) + d^2*e^2 - 2*c*d*e*f) - (b*atan(c + d*x)
)/(f*(e + f*x)) - a/(e*f + f^2*x) - (b*d*log(c + d*x - 1i)*1i)/(2*f*(f*1i -
 c*f + d*e)) - (b*d*log(c + d*x + 1i))/(2*f*(f - c*f*1i + d*e*1i))
```

3.30 $\int \frac{a+b\,\text{ArcTan}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{bd}{2\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)(e+fx)}+\frac{bd^2(de+f-cf)(de-(1+c)f)\text{ArcTan}(c+dx)}{2f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)^2}-\frac{a+b\text{ArcTan}(c}{2f(e+fx}$$

```
[Out] -1/2*b*d/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)/(f*x+e)+1/2*b*d^2*(-c*f+d*e+f)*(d*
e-(1+c)*f)*arctan(d*x+c)/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)^2+1/2*(-a-b*arct
an(d*x+c))/f/(f*x+e)^2+b*d^2*(-c*f+d*e)*ln(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2+1
)*f^2)^2-1/2*b*d^2*(-c*f+d*e)*ln(d^2*x^2+2*c*d*x+c^2+1)/(d^2*e^2-2*c*d*e*f+
(c^2+1)*f^2)^2
```

Rubi [A]
time = 0.22, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5153, 2007, 723, 814, 648, 632, 210, 642}

$$-\frac{a+b\text{ArcTan}(c+dx)}{2f(e+fx)^2}+\frac{bd^2\text{ArcTan}(c+dx)(-cf+de+f)(de-(c+1)f)}{2f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd^2(de-cf)\log\left(c^2+2cdx+d^2x^2+1\right)}{2\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}-\frac{bd}{2(e+fx)\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}+\frac{bd^2(de-cf)\log(e+fx)}{\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] -1/2*(b*d)/((d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)*(e + f*x)) + (b*d^2*(d*e
+ f - c*f)*(d*e - (1 + c)*f)*ArcTan[c + d*x])/(2*f*(d^2*e^2 - 2*c*d*e*f + (
1 + c^2)*f^2)^2) - (a + b*ArcTan[c + d*x])/(2*f*(e + f*x)^2) + (b*d^2*(d*e
- c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2 - (b*d^2*(d*e
- c*f)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)^3}\,dx = -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{1}{(e+fx)^2(1+(c+dx)^2)}\,dx}{2f}$$

$$= -\frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{1}{(e+fx)^2(1+c^2+2cdx+d^2x^2)}\,dx}{2f}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \frac{d(}{(e+fx}}{2f\left(d^2e^2-2\right.}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{(bd)\int \Big(\frac{}{(d^2e^2}}{}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-c}{(d^2e^2-2cde}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-c}{(d^2e^2-2cde}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} - \frac{a+b\tan^{-1}(c+dx)}{2f(e+fx)^2} + \frac{bd^2(de-c}{(d^2e^2-2cde}$$

$$= -\frac{bd}{2\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)} + \frac{bd^2(de-f-cf)(de+f-cf)\tan^{}}{2f\left(d^2e^2-2cdef+f^2+c^2f\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 175, normalized size = 0.77

$$\frac{-\frac{a+b\text{ArcTan}(c+dx)}{(e+fx)^2}+\frac{1}{2}bd^2\left(-\frac{2f}{d\left(d^2e^2-2cdef+(1+c^2)f^2\right)(e+fx)}-\frac{i\log(i-c-dx)}{(de-(-i+c)f)^2}+\frac{i\log(i+c+dx)}{(de-(i+c)f)^2}-\frac{4f(-de+cf)\log(d(e+fx))}{\left(d^2e^2-2cdef+(1+c^2)f^2\right)^2}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (-((a + b*ArcTan[c + d*x])/(e + f*x)^2) + (b*d^2*((-2*f)/(d*(d^2*e^2 - 2*c*
d*e*f + (1 + c^2)*f^2)*(e + f*x)) - (I*Log[I - c - d*x])/(d*e - (-I + c)*f)
^2 + (I*Log[I + c + d*x])/(d*e - (I + c)*f)^2 - (4*f*(-(d*e) + c*f)*Log[d*(
e + f*x)])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)^2))/2)/(2*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(222) = 444.
time = 0.33, size = 468, normalized size = 2.06

method	result

derivativedivides	$-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{b\,d^3 f\arctan(dx+c)c^2}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}-\frac{b\,d^4\arctan(dx+c)ce}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}+\frac{b\,d^5\arctan(dx}{2f\left(c^2f^2-2cdef+d\right.}$
default	$-\frac{a\,d^3}{2(cf-de-f(dx+c))^2 f}-\frac{b\,d^3\arctan(dx+c)}{2(cf-de-f(dx+c))^2 f}+\frac{b\,d^3 f\arctan(dx+c)c^2}{2\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}-\frac{b\,d^4\arctan(dx+c)ce}{\left(c^2f^2-2cdef+d^2e^2+f^2\right)^2}+\frac{b\,d^5\arctan(dx}{2f\left(c^2f^2-2cdef+d\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a*d^3/(c*f-d*e-f*(d*x+c))^2/f-1/2*b*d^3/(c*f-d*e-f*(d*x+c))^2/f*a
rctan(d*x+c)+1/2*b*d^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c^
2-b*d^4/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*c*e+1/2*b*d^5/f/(c^
2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arctan(d*x+c)*e^2+1/2*b*d^3*f/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)^2*ln(1+(d*x+c)^2)*c-1/2*b*d^4/(c^2*f^2-2*c*d*e*f+d^2*e^2+
f^2)^2*ln(1+(d*x+c)^2)*e-1/2*b*d^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*arct
an(d*x+c)+1/2*b*d^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(c*f-d*e-f*(d*x+c))-b*d
^3*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(c*f-d*e-f*(d*x+c))*c+b*d^4/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)^2*ln(c*f-d*e-f*(d*x+c))*e)
```

Maxima [A]
time = 0.50, size = 436, normalized size = 1.92

$$-\frac{1}{2}\left(d\left(\frac{(cdf-d^2e)\log\left(d^2x^2+2\,cdx+c^2+1\right)}{4\,cd^3fe^3-2\,(3\,c^2e^2+e^2)d^2f^2+4\,(c^3e+ce)df^3-(c^4+2\,c^2+1)f^4-d^4e^4}-\frac{2\,(cdf-d^2e)\log\left(fx+e\right)}{4\,cd^3fe^3-2\,(3\,c^2e^2+e^2)d^2f^2+4\,(c^3e+ce)df^3-(c^4+2\,c^2+1)f^4-d^4e^4}-\frac{\left(2\,cd^3fe-(c^2-1)d^2f^2-d^4e^2\right)\arctan\left(\frac{d^2x+cd}{d}\right)}{\left(4\,cd^3f^2e^3-2\,(3\,c^2e^2+e^2)d^2f^3+4\,(c^3e+ce)df^4-(c^4+2\,c^2+1)f^5-d^4fe^4\right)d}-\frac{1}{2\,cdfe^2-(c^2e+e)f^2-d^2e^3+\left(2\,cdf^2e-(c^2+1)f^3-d^2fe^2\right)x}\right)+\frac{\arctan\left(dx+c\right)}{f^3x^2+2\,f^2xe+fe^2}\right)b-\frac{a}{2\,(f^3x^2+2\,f^2xe+fe^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(d*((c*d*f - d^2*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(4*c*d^3*f*e^3 -
2*(3*c^2*e^2 + e^2)*d^2*f^2 + 4*(c^3*e + c*e)*d*f^3 - (c^4 + 2*c^2 + 1)*f^4
 - d^4*e^4) - 2*(c*d*f - d^2*e)*log(f*x + e)/(4*c*d^3*f*e^3 - 2*(3*c^2*e^2
+ e^2)*d^2*f^2 + 4*(c^3*e + c*e)*d*f^3 - (c^4 + 2*c^2 + 1)*f^4 - d^4*e^4) -
 (2*c*d^3*f*e - (c^2 - 1)*d^2*f^2 - d^4*e^2)*arctan((d^2*x + c*d)/d)/((4*c*
d^3*f^2*e^3 - 2*(3*c^2*e^2 + e^2)*d^2*f^3 + 4*(c^3*e + c*e)*d*f^4 - (c^4 +
2*c^2 + 1)*f^5 - d^4*f*e^4)*d) - 1/(2*c*d*f*e^2 - (c^2*e + e)*f^2 - d^2*e^3
 + (2*c*d*f^2*e - (c^2 + 1)*f^3 - d^2*f*e^2)*x)) + arctan(d*x + c)/(f^3*x^2
 + 2*f^2*x*e + f*e^2))*b - 1/2*a/(f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(237) = 474$.
time = 6.84, size = 679, normalized size = 2.99

$$-\frac{(bc^2+b)df^4x+ad^4e^4-(4\,ac-b)d^3fe^3+(ac^4+2\,ac^2+a)f^4-\left((bc^2-b)d^2f^4x^2-(bc^4+2\,bc^2+b)f^4+2\,(bd^4fx+bcd^3f)e^3+(bd^4f^2x^2-4\,bcd^3f^2x-(5\,bc^2+3\,b)d^2f^2)e^2-2\,(bcd^3f^3x^2-(bc^2-b)d^2f^3x-2\,(bc^3+bc)df^3)e\right)\arctan\left(dx+c\right)+(bd^3f^2x+2\,(3\,ac^2-bc+a)d^2f^2)e^2-(2\,bcd^2f^3x+(4\,ac^3-bc^2+4\,ac-b)df^3)e-(bcd^2f^4x^2-bd^3fe^3-(2\,bd^3f^2x-bcd^2f^2)e^2-(bd^3f^3x^2-2\,bcd^2f^3x)e)\log\left(d^2x^2+2\,cdx+c^2+1\right)+2\,(bcd^2f^4x^2-bd^3fe^3-(2\,bd^3f^2x-bcd^2f^2)e^2-(bd^3f^3x^2-2\,bcd^2f^3x)e)\log\left(fx+e\right)}{2\,\left((c^4+2\,c^2+1)f^7x^2+d^4fe^6+2\,(d^4f^2x-2\,cd^3f^2)e^5+(d^4f^3x^2-8\,cd^3f^3x+2\,(3\,c^2+1)d^2f^3)e^4-4\,(cd^3f^4x^2-(3\,c^2+1)d^2f^4x+(c^3+c)df^4)e^3+(2\,(3\,c^2+1)d^2f^5x^2-8\,(c^3+c)df^5x+(c^4+2\,c^2+1)f^5)e^2-2\,(2\,(c^3+c)df^6x^2-(c^4+2\,c^2+1)f^6x)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b*c^2 + b)*d*f^4*x + a*d^4*e^4 - (4*a*c - b)*d^3*f*e^3 + (a*c^4 + 2*
a*c^2 + a)*f^4 - ((b*c^2 - b)*d^2*f^4*x^2 - (b*c^4 + 2*b*c^2 + b)*f^4 + 2*(
b*d^4*f*x + b*c*d^3*f)*e^3 + (b*d^4*f^2*x^2 - 4*b*c*d^3*f^2*x - (5*b*c^2 +
3*b)*d^2*f^2)*e^2 - 2*(b*c*d^3*f^3*x^2 - (b*c^2 - b)*d^2*f^3*x - 2*(b*c^3 +
 b*c)*d*f^3)*e)*arctan(d*x + c) + (b*d^3*f^2*x + 2*(3*a*c^2 - b*c + a)*d^2*
f^2)*e^2 - (2*b*c*d^2*f^3*x + (4*a*c^3 - b*c^2 + 4*a*c - b)*d*f^3)*e - (b*c
*d^2*f^4*x^2 - b*d^3*f*e^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*e^2 - (b*d^3*f^3
*x^2 - 2*b*c*d^2*f^3*x)*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b*c*d^2*f^
4*x^2 - b*d^3*f*e^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*e^2 - (b*d^3*f^3*x^2 -
2*b*c*d^2*f^3*x)*e)*log(f*x + e))/((c^4 + 2*c^2 + 1)*f^7*x^2 + d^4*f*e^6 +
2*(d^4*f^2*x - 2*c*d^3*f^2)*e^5 + (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 +
 1)*d^2*f^3)*e^4 - 4*(c*d^3*f^4*x^2 - (3*c^2 + 1)*d^2*f^4*x + (c^3 + c)*d*f
^4)*e^3 + (2*(3*c^2 + 1)*d^2*f^5*x^2 - 8*(c^3 + c)*d*f^5*x + (c^4 + 2*c^2 +
 1)*f^5)*e^2 - 2*(2*(c^3 + c)*d*f^6*x^2 - (c^4 + 2*c^2 + 1)*f^6*x)*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 7.53, size = 399, normalized size = 1.76

$$\frac{b\,d^3\,e\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}-\frac{a\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,d\,e}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{a\,c^2\,f}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,\mathrm{atan}\left(c+d\,x\right)}{2\,f\,{\left(e+f\,x\right)}^2}-\frac{b\,c\,d^2\,f\,\ln\left(e+f\,x\right)}{{\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}^2}+\frac{a\,c\,d\,e}{{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,d\,f\,x}{2\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{a\,d^2\,e^2}{2\,f\,{\left(e+f\,x\right)}^2\,\left(c^2\,f^2-2\,c\,d\,e\,f+d^2\,e^2+f^2\right)}-\frac{b\,d^2\,\ln\left(c+d\,x-\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(d\,e-c\,f+f\,1{}\mathrm{i}\right)}^2}+\frac{b\,d^2\,\ln\left(c+d\,x+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,f\,{\left(c\,f-d\,e+f\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*d^2*log(c + d*x + 1i)*1i)/(4*f*(f*1i + c*f - d*e)^2) - (a*f)/(2*(e + f*x
)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d*e)/(2*(e + f*x)^2*(f^2 +
```

```
c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) - (b*d^2*log(c + d*x - 1i)*1i)/(4*f*(f*1i -
 c*f + d*e)^2) - (b*atan(c + d*x))/(2*f*(e + f*x)^2) - (a*c^2*f)/(2*(e + f*
x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f)) + (b*d^3*e*log(e + f*x))/(f^2 +
 c^2*f^2 + d^2*e^2 - 2*c*d*e*f)^2 - (b*c*d^2*f*log(e + f*x))/(f^2 + c^2*f^2
 + d^2*e^2 - 2*c*d*e*f)^2 + (a*c*d*e)/((e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2
 - 2*c*d*e*f)) - (b*d*f*x)/(2*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*
e*f)) - (a*d^2*e^2)/(2*f*(e + f*x)^2*(f^2 + c^2*f^2 + d^2*e^2 - 2*c*d*e*f))
```

3.31 $\int (e+fx)^2(a+b\mathbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=382

$$\frac{b^2f^2x}{3d^2}-\frac{2abf(de-cf)x}{d^2}-\frac{b^2f^2\text{ArcTan}(c+dx)}{3d^3}-\frac{2b^2f(de-cf)(c+dx)\text{ArcTan}(c+dx)}{d^3}-\frac{bf^2(c+dx)^2(a+b}{3d^3}$$

```
[Out] 1/3*b^2*f^2*x/d^2-2*a*b*f*(-c*f+d*e)*x/d^2-1/3*b^2*f^2*arctan(d*x+c)/d^3-2*
b^2*f*(-c*f+d*e)*(d*x+c)*arctan(d*x+c)/d^3-1/3*b*f^2*(d*x+c)^2*(a+b*arctan(
d*x+c))/d^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^
2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^2
/d^3/f+1/3*(f*x+e)^3*(a+b*arctan(d*x+c))^2/f+2/3*b*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^3+b^2*f*(-c*f+d*e)*l
n(1+(d*x+c)^2)/d^3+1/3*I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(2
,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A]

time = 0.43, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5155, 4974, 4930, 266, 4946, 327, 209, 5104, 5004, 5040, 4964, 2449, 2352}

$$\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\text{ArcTan}(c+dx))^2}{3d^3}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\text{ArcTan}(c+dx))^2}{3d^3f}+\frac{2b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{3d^3}-\frac{bf^2(c+dx)^2(a+b\text{ArcTan}(c+dx))}{3d^3}+\frac{(e+fx)^3(a+b\text{ArcTan}(c+dx))^2}{3f}-\frac{2abfx(de-cf)}{d^2}-\frac{2b^2f(c+dx)\text{ArcTan}(c+dx)(de-cf)}{d^3}-\frac{b^2f^2\text{ArcTan}(c+dx)}{3d^3}+\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{3d^3}+\frac{b^2f(de-cf)\log\left((c+dx)^2+1\right)}{d^3}+\frac{b^2f^2x}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) - (2*a*b*f*(d*e - c*f)*x)/d^2 - (b^2*f^2*ArcTan[c + d*x
])/(3*d^3) - (2*b^2*f*(d*e - c*f)*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(
c + d*x)^2*(a + b*ArcTan[c + d*x]))/(3*d^3) + ((I/3)*(3*d^2*e^2 - 6*c*d*e*f
 - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/d^3 - ((d*e - c*f)*(d^2*e^2
- 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^2)/(3*d^3*f) + ((e + f
*x)^3*(a + b*ArcTan[c + d*x])^2)/(3*f) + (2*b*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/(3*d^3) + (b
^2*f*(d*e - c*f)*Log[1 + (c + d*x)^2])/d^3 + ((I/3)*b^2*(3*d^2*e^2 - 6*c*d*
e*f - (1 - 3*c^2)*f^2)*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 (a+b\tan^{-1}(x))^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 (a+b\tan^{-1}(c+dx))^2}{3f} - \frac{(2b)\text{Subst}\left(\int \left(\frac{3f^2(de-cf)(a+b\tan\right.\right.}{d^3}$$

$$= \frac{(e+fx)^3 (a+b\tan^{-1}(c+dx))^2}{3f} - \frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef\right.}{}$$

$$= -\frac{2abf(de-cf)x}{d^2} - \frac{bf^2(c+dx)^2 (a+b\tan^{-1}(c+dx))}{3d^3} + \frac{(e+f}{}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{2b^2f(de-cf)(c+dx)\tan^{-1}(c+dx)}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(}{}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(}{}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(}{}$$

$$= \frac{b^2f^2x}{3d^2} - \frac{2abf(de-cf)x}{d^2} - \frac{b^2f^2\tan^{-1}(c+dx)}{3d^3} - \frac{2b^2f(de-cf)(}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 801 vs. $2(382) = 764$.
time = 2.66, size = 801, normalized size = 2.10

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(-(d*f*x*(6*d*e - 4*c*f +
d*f*x)) + 2*(3*d*e*f - 3*c^2*d*e*f + c^3*f^2 + 3*c*(d^2*e^2 - f^2) + d^3*x*
(3*e^2 + 3*e*f*x + f^2*x^2))*ArcTan[c + d*x] + (-3*d^2*e^2 + 6*c*d*e*f + (1
 - 3*c^2)*f^2)*Log[1 + (c + d*x)^2]))/(3*d^3) + (b^2*e^2*(ArcTan[c + d*x]*(
(-I + c + d*x)*ArcTan[c + d*x] + 2*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*
PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d + (b^2*e*f*((1 + (2*I)*c - c^2 +
```

```
 d^2*x^2)*ArcTan[c + d*x]^2 - 2*ArcTan[c + d*x]*(c + d*x + 2*c*Log[1 + E^((
2*I)*ArcTan[c + d*x])]) + Log[1 + (c + d*x)^2] + (2*I)*c*PolyLog[2, -E^((2*
I)*ArcTan[c + d*x])]))/d^2 + (b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sq
rt[1 + (c + d*x)^2] + (6*c*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2]
 + (3*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)
*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTa
n[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan
[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2
*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])]
+ 6*c*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 + (c + d*x)^2]] + ((3*I - 12*c -
(9*I)*c^2)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d*x]*(-2 + (-3 + 9*c^2)*Log[1 +
 E^((2*I)*ArcTan[c + d*x])]) + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 +
(c + d*x)^2] - ((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/
(1 + (c + d*x)^2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[
3*ArcTan[c + d*x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTa
n[c + d*x]^2*Sin[3*ArcTan[c + d*x]]))/(12*d^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1607 vs. 2(362) = 724.
time = 0.43, size = 1608, normalized size = 4.21

method	result	size
derivativedivides	Expression too large to display	1608
default	Expression too large to display	1608
risch	Expression too large to display	2653

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*I*b^2/d^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*f^2-I*b^2/d*ln(d*x+c+I)*
ln(1+(d*x+c)^2)*c*e*f-4*a*b/d*f*arctan(d*x+c)*c*e*(d*x+c)+I*b^2/d*ln(d*x+c-
I)*ln(1+(d*x+c)^2)*c*e*f+I*b^2/d*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*c*e*f-I*b^
2/d*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*c*e*f+1/12*I*b^2/d^2*ln(d*x+c+I)^2*f^2
+1/6*I*b^2/d^2*dilog(1/2*I*(d*x+c-I))*f^2-1/6*I*b^2/d^2*dilog(-1/2*I*(d*x+c
+I))*f^2-1/12*I*b^2/d^2*ln(d*x+c-I)^2*f^2+1/2*I*b^2*ln(d*x+c-I)*ln(-1/2*I*(
d*x+c+I))*e^2+1/2*I*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*e^2-1/2*I*b^2*ln(d*x+c+
I)*ln(1/2*I*(d*x+c-I))*e^2-1/3*a*b/d^2*f^2*(d*x+c)^2+2*a*b*arctan(d*x+c)*e^
2*(d*x+c)+1/3*a*b/d^2*f^2*ln(1+(d*x+c)^2)+b^2/d*f*ln(1+(d*x+c)^2)*e+b^2/d*f
*arctan(d*x+c)^2*e+1/3*b^2/d^2*f^2*arctan(d*x+c)^2*(d*x+c)^3-b^2/d^2*f^2*ln
(1+(d*x+c)^2)*c-1/3*b^2/d^2*f^2*arctan(d*x+c)*(d*x+c)^2-b^2/d^2*f^2*arctan(
d*x+c)^2*c+1/3*b^2/d^2*f^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-1/2*I*b^2*ln(d*x+c
-I)*ln(1+(d*x+c)^2)*e^2+1/4*I*b^2*ln(d*x+c-I)^2*e^2-1/2*I*b^2*dilog(1/2*I*(
d*x+c-I))*e^2-1/3*(c*f-d*e-f*(d*x+c))^3*a^2/d^2/f-1/4*I*b^2*ln(d*x+c+I)^2*e
^2+1/3*b^2/d^2*f^2*(d*x+c)+b^2*arctan(d*x+c)^2*e^2*(d*x+c)-1/3*b^2/d^2*f^2*
arctan(d*x+c)+1/2*I*b^2*dilog(-1/2*I*(d*x+c+I))*e^2-2*a*b/d^2*f^2*arctan(d*
```

```
x+c)*c+2*a*b/d*f*arctan(d*x+c)*e+2*a*b/d^2*f^2*c*(d*x+c)-2*a*b/d*f*e*(d*x+c
)-1/6*I*b^2/d^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*f^2-1/2*I*b^2/d^2*dilog(1/2*I*(
d*x+c-I))*c^2*f^2+b^2/d*f*arctan(d*x+c)^2*e*(d*x+c)^2+b^2/d^2*f^2*arctan(d*
x+c)^2*c^2*(d*x+c)-2*b^2/d*f*arctan(d*x+c)*e*(d*x+c)+2*b^2/d^2*f^2*arctan(d
*x+c)*c*(d*x+c)-b^2/d^2*f^2*arctan(d*x+c)*ln(1+(d*x+c)^2)*c^2+2/3*a*b/d^2*f
^2*arctan(d*x+c)*(d*x+c)^3-a*b/d^2*f^2*ln(1+(d*x+c)^2)*c^2-1/4*I*b^2/d^2*ln
(d*x+c+I)^2*c^2*f^2-b^2/d^2*f^2*arctan(d*x+c)^2*c*(d*x+c)^2+1/6*I*b^2/d^2*l
n(d*x+c-I)*ln(1+(d*x+c)^2)*f^2+1/2*I*b^2/d^2*dilog(-1/2*I*(d*x+c+I))*c^2*f^
2-1/6*I*b^2/d^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*f^2+1/4*I*b^2/d^2*ln(d*x+c
-I)^2*c^2*f^2-b^2*e^2*arctan(d*x+c)*ln(1+(d*x+c)^2)-e^2*a*b*ln(1+(d*x+c)^2)
-2*b^2/d*f*arctan(d*x+c)^2*c*e*(d*x+c)+I*b^2/d*dilog(1/2*I*(d*x+c-I))*c*e*f
+2*a*b/d^2*f^2*arctan(d*x+c)*c^2*(d*x+c)-2*a*b/d^2*f^2*arctan(d*x+c)*c*(d*x
+c)^2+2*a*b/d*f*arctan(d*x+c)*e*(d*x+c)^2+2*a*b/d*f*ln(1+(d*x+c)^2)*c*e+2*b
^2/d*f*arctan(d*x+c)*ln(1+(d*x+c)^2)*c*e-I*b^2/d*dilog(-1/2*I*(d*x+c+I))*c*
e*f-1/2*I*b^2/d*ln(d*x+c-I)^2*c*e*f+1/2*I*b^2/d*ln(d*x+c+I)^2*c*e*f-1/2*I*b
^2/d^2*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c^2*f^2+1/2*I*b^2/d^2*ln(d*x+c-I)*ln(-1/
2*I*(d*x+c+I))*c^2*f^2+1/2*I*b^2/d^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*c^2*f^2-1/
2*I*b^2/d^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*c^2*f^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*f^2*x^3 + 3/4*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2
/d + 36*b^2*d^2*f^2*integrate(1/48*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
 + c^2 + 1), x) + 3*b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*c*d*f^2*integrate(1/4
8*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*d^2*f*e*
integrate(1/48*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*
b^2*d^2*f^2*integrate(1/48*x^4*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*d^2*f*e*integra
te(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1
), x) + 36*b^2*c^2*f^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*
d*x + c^2 + 1), x) + 144*b^2*c*d*f*e*integrate(1/48*x^2*arctan(d*x + c)^2/(
d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*f^2*integrate(1/48*x^3*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*f*
e*integrate(1/48*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 3*b^2*c^2*f^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c*d*f*e*integrate(1/48*x
^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 7
```

```
2*b^2*c^2*f*e*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
 1), x) + 12*b^2*c*d*f*e*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c^2*f*e*integrate(1/48*x*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 1/4*(3*arc
tan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2
*c^2*e^2 + a^2*f*x^2*e - 8*b^2*d*f^2*integrate(1/48*x^3*arctan(d*x + c)/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 36*b^2*d^2*e^2*integrate(1/48*x^2*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*d^2*e^2*integrate(1/48*
x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) -
24*b^2*d*f*e*integrate(1/48*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + 72*b^2*c*d*e^2*integrate(1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d
*x + c^2 + 1), x) + 12*b^2*d^2*e^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*c*d*e^2*integrate(1/4
8*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) +
12*b^2*c*d*e^2*integrate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 +
 2*c*d*x + c^2 + 1), x) + 3*b^2*c^2*e^2*integrate(1/48*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/3*(2*x^3*arctan(d*x +
c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/d^4 + (
3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a*b*f^2 + 3/4*b^2*arctan(
d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d + 2*(x^2*arctan(d*x + c) - d*(x/d^
2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 +
 1)/d^3))*a*b*f*e + 36*b^2*f^2*integrate(1/48*x^2*arctan(d*x + c)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 3*b^2*f^2*integrate(1/48*x^2*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 72*b^2*f*e*integrate(
1/48*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^2*f*e*inte
grate(1/48*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) - 24*b^2*d*e^2*integrate(1/48*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) - 1/4*(3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan
((d^2*x + c*d)/d)^3/d)*b^2*e^2 + a^2*x*e^2 + 3*b^2*e^2*integrate(1/48*log(d
^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/12*(b^2
*f^2*x^3 + 3*b^2*f*x^2*e + 3*b^2*x*e^2)*arctan(d*x + c)^2 + (2*(d*x + c)*ar
ctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*e^2/d - 1/48*(b^2*f^2*x^3 + 3*b^2
*f*x^2*e + 3*b^2*x*e^2)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*f*x*e + (b^2*f^2*x^2 + 2*b^2*f*x*e + b^2*e^2)*
arctan(d*x + c)^2 + a^2*e^2 + 2*(a*b*f^2*x^2 + 2*a*b*f*x*e + a*b*e^2)*arcta
n(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}\,(c + dx))^2\,(e + fx)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2\,(a + b\,\mathrm{atan}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^2, x)
```

3.32 $\int (e+fx)(a+b\mathbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=222

$$-\frac{abfx}{d}-\frac{b^2f(c+dx)\mathrm{ArcTan}(c+dx)}{d^2}+\frac{i(de-cf)(a+b\mathrm{ArcTan}(c+dx))^2}{d^2}-\frac{(de+f-cf)(de-(1+c)f)(a-}{2d^2f}$$

```
[Out] -a*b*f*x/d-b^2*f*(d*x+c)*arctan(d*x+c)/d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))
^2/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(a+b*arctan(d*x+c))^2/d^2/f+1/2*(f*x+
e)^2*(a+b*arctan(d*x+c))^2/f+2*b*(-c*f+d*e)*(a+b*arctan(d*x+c))*ln(2/(1+I*(
d*x+c)))/d^2+1/2*b^2*f*ln(1+(d*x+c)^2)/d^2+I*b^2*(-c*f+d*e)*polylog(2,1-2/(
1+I*(d*x+c)))/d^2
```

Rubi [A]
time = 0.28, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = $\{5155, 4974, 4930, 266, 5104, 5004, 5040, 4964, 2449, 2352\}$

$$\frac{i(de-cf)(a+b\mathrm{ArcTan}(c+dx))^2}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)(a+b\mathrm{ArcTan}(c+dx))^2}{2d^2f}+\frac{2b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\mathrm{ArcTan}(c+dx))}{d^2}+\frac{(e+fx)^2(a+b\mathrm{ArcTan}(c+dx))^2}{2f}-\frac{abfx}{d}-\frac{b^2f(c+dx)\mathrm{ArcTan}(c+dx)}{d^2}+\frac{ib^2(de-cf)\mathrm{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{d^2}+\frac{b^2f\log\left((c+dx)^2+1\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] -((a*b*f*x)/d) - (b^2*f*(c + d*x)*ArcTan[c + d*x])/d^2 + (I*(d*e - c*f)*(a
+ b*ArcTan[c + d*x])^2)/d^2 - ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*Arc
Tan[c + d*x])^2)/(2*d^2*f) + ((e + f*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*f)
+ (2*b*(d*e - c*f)*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^2 +
(b^2*f*Log[1 + (c + d*x)^2])/(2*d^2) + (I*b^2*(d*e - c*f)*PolyLog[2, 1 - 2/
(1 + I*(c + d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\left(\frac{f^2(a+b\tan^{-1}(x))}{d^2}+\frac{((de-$$

$$=\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\frac{((de-f-cf)(de+f-cf)+2f(c}{1+x^2}\right.}{d^2}$$

$$=-\frac{abfx}{d}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\left(\frac{(de+f-cf)(d}{}\right.\right.}{}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{(e+fx)^2\left(a+b\tan^{-1}(c+d\right.}{2f}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+\right.}{d^2}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+\right.}{d^2}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+\right.}{d^2}$$

$$=-\frac{abfx}{d}-\frac{b^2f(c+dx)\tan^{-1}(c+dx)}{d^2}+\frac{i(de-cf)\left(a+b\tan^{-1}(c+\right.}{d^2}$$

Mathematica [A]
time = 0.27, size = 264, normalized size = 1.19

$$\frac{2a^2cde-2abcf-a^2c^2f+2a^2d^2ex-2abdfx+a^2d^2fx^2+b^2(-i+c+dx)(2de+if-cf+dfx)\text{ArcTan}(c+dx)^2-2b\text{ArcTan}(c+dx)\left(bf(c+dx)+a(-2cde+c^2f-2d^2ex-f(1+d^2x^2))-2b(de-cf)\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)\right)+4abde\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-2b^2f\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-4abcf\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)-2ib^2(de-cf)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e - 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x - 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-I + c + d*x)*(2*d*e + I*f - c*f + d*f*x)*ArcTan[c + d*x]^2
- 2*b*ArcTan[c + d*x]*(b*f*(c + d*x) + a*(-2*c*d*e + c^2*f - 2*d^2*e*x - f*
```

```
(1 + d^2*x^2)) - 2*b*(d*e - c*f)*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 4*a*
b*d*e*Log[1/Sqrt[1 + (c + d*x)^2]] - 2*b^2*f*Log[1/Sqrt[1 + (c + d*x)^2]] -
 4*a*b*c*f*Log[1/Sqrt[1 + (c + d*x)^2]] - (2*I)*b^2*(d*e - c*f)*PolyLog[2,
-E^((2*I)*ArcTan[c + d*x])])/(2*d^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(212) = 424$.
time = 0.20, size = 701, normalized size = 3.16

method	result
derivativedivides	$\frac{\frac{ab\arctan(dx+c)f(dx+c)^2}{d} - \frac{2ab\arctan(dx+c)fc(dx+c)}{d} + \frac{ib^2 \operatorname{dilog}\left(-\frac{i(dx+c+i)}{2}\right)e}{2} - \frac{ib^2 \operatorname{dilog}\left(\frac{i(dx+c-i)}{2}\right)e}{2} - \frac{ib^2 \ln(dx+c+i)^2 e}{4} +}{}$
default	$\frac{\frac{ab\arctan(dx+c)f(dx+c)^2}{d} - \frac{2ab\arctan(dx+c)fc(dx+c)}{d} + \frac{ib^2 \operatorname{dilog}\left(-\frac{i(dx+c+i)}{2}\right)e}{2} - \frac{ib^2 \operatorname{dilog}\left(\frac{i(dx+c-i)}{2}\right)e}{2} - \frac{ib^2 \ln(dx+c+i)^2 e}{4} +}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*b/d*arctan(d*x+c)*f*(d*x+c)^2-2*a*b/d*arctan(d*x+c)*f*c*(d*x+c)+a*b/
d*ln(1+(d*x+c)^2)*c*f-1/2*I*b^2/d*dilog(-1/2*I*(d*x+c+I))*c*f-1/4*I*b^2/d*l
n(d*x+c-I)^2*c*f-b^2/d*arctan(d*x+c)^2*f*c*(d*x+c)+b^2/d*ln(1+(d*x+c)^2)*ar
ctan(d*x+c)*c*f+1/2*I*b^2/d*dilog(1/2*I*(d*x+c-I))*c*f+1/4*I*b^2/d*ln(d*x+c
+I)^2*c*f-a^2/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)+1/2*I*b^2/d*ln(d*
x+c+I)*ln(1/2*I*(d*x+c-I))*c*f+1/2*I*b^2/d*ln(d*x+c-I)*ln(1+(d*x+c)^2)*c*f-
1/2*I*b^2/d*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))*c*f-1/2*I*b^2/d*ln(d*x+c+I)*ln
(1+(d*x+c)^2)*c*f-1/4*I*b^2*ln(d*x+c+I)^2*e+1/2*b^2/d*arctan(d*x+c)^2*f*(d*
x+c)^2-b^2/d*arctan(d*x+c)*f*(d*x+c)+1/2*I*b^2*ln(d*x+c-I)*ln(-1/2*I*(d*x+c
+I))*e+1/2*I*b^2*ln(d*x+c+I)*ln(1+(d*x+c)^2)*e-1/2*I*b^2*ln(d*x+c-I)*ln(1+(
d*x+c)^2)*e-1/2*I*b^2*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))*e+2*e*(d*x+c)*a*b*arc
tan(d*x+c)+a*b/d*f*arctan(d*x+c)-a*b/d*f*(d*x+c)-a*b*ln(1+(d*x+c)^2)*e+b^2*
arctan(d*x+c)^2*e*(d*x+c)-b^2*ln(1+(d*x+c)^2)*arctan(d*x+c)*e+1/2*b^2/d*arc
tan(d*x+c)^2*f+1/2*b^2/d*f*ln(1+(d*x+c)^2)+1/2*I*b^2*dilog(-1/2*I*(d*x+c+I)
)*e+1/4*I*b^2*ln(d*x+c-I)^2*e-1/2*I*b^2*dilog(1/2*I*(d*x+c-I))*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d - 1/4*(3*arctan(d
*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*c^2*
```

```
e + 12*b^2*d^2*f*integrate(1/16*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + b^2*d^2*f*integrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^2*c*d*f*integrate(1/16*x^2*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*d^2*e*integrate(1
/16*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 2*b^2*d^2*f*i
ntegrate(1/16*x^3*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
 + 1), x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*d^2*e*integrate(1/16*x^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^2*c^2*
f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24
*b^2*c*d*e*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1)
, x) + 2*b^2*c*d*f*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2
*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*d^2*e*integrate(1/16*x^2*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*f*integrat
e(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 2*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x
^2 + 2*c*d*x + c^2 + 1), x) + 4*b^2*c*d*e*integrate(1/16*x*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^2*c^2*e*integrate(1/
16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
/2*a^2*f*x^2 + 3/4*b^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d - 4*b^
2*d*f*integrate(1/16*x^2*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
- 8*b^2*d*e*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + (x^2*arctan(d*x + c) - d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d
^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^3))*a*b*f - 1/4*(3*arctan(d*x + c
)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*b^2*e + a^2*x*
e + 12*b^2*f*integrate(1/16*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 +
1), x) + b^2*f*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2
 + 2*c*d*x + c^2 + 1), x) + b^2*e*integrate(1/16*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/8*(b^2*f*x^2 + 2*b^2*x*e)*ar
ctan(d*x + c)^2 + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a*b*
e/d - 1/32*(b^2*f*x^2 + 2*b^2*x*e)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + (b^2*f*x + b^2*e)*arctan(d*x + c)^2 + a^2*e + 2*(a*b*f*x
 + a*b*e)*arctan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^2 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**2*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)\,\left(a+b\,\mathrm{atan}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^2,x)
```

```
[Out] int((e + f*x)*(a + b*atan(c + d*x))^2, x)
```

3.33 $\int (a+b\mathbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=102

$$\frac{i(a+b\text{ArcTan}(c+dx))^2}{d}+\frac{(c+dx)(a+b\text{ArcTan}(c+dx))^2}{d}+\frac{2b(a+b\text{ArcTan}(c+dx))\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{ib^2\text{P}}{}$$

```
[Out] I*(a+b*arctan(d*x+c))^2/d+(d*x+c)*(a+b*arctan(d*x+c))^2/d+2*b*(a+b*arctan(d
*x+c))*ln(2/(1+I*(d*x+c)))/d+I*b^2*polylog(2,1-2/(1+I*(d*x+c)))/d
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5147, 4930, 5040, 4964, 2449, 2352}

$$\frac{(c+dx)(a+b\text{ArcTan}(c+dx))^2}{d}+\frac{i(a+b\text{ArcTan}(c+dx))^2}{d}+\frac{2b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{d}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^2)/d +
 (2*b*(a + b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d + (I*b^2*PolyLog[
2, 1 - 2/(1 + I*(c + d*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(2b)\text{Subst}\left(\int\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}\right.}{}$$

$$= \frac{i\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d} + \frac{2b\left(a+b\tan^{-1}\right.}{}$$

Mathematica [A]
time = 0.13, size = 104, normalized size = 1.02

$$\frac{a^2(c+dx)}{d} + \frac{b\left(b(-i+c+dx)\text{ArcTan}(c+dx)^2 + 2\text{ArcTan}(c+dx)\left(a(c+dx)+b\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)\right) - a\log\left(1+(c+dx)^2\right) - ib\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2,x]
```

```
[Out] (a^2*(c + d*x))/d + (b*(b*(-I + c + d*x)*ArcTan[c + d*x]^2 + 2*ArcTan[c + d
*x]*(a*(c + d*x) + b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) - a*Log[1 + (c + d
*x)^2] - I*b*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]))/d
```

Maple [A]
time = 0.14, size = 146, normalized size = 1.43

method	result
derivativedivides	$\frac{(dx+c)a^2-i\arctan(dx+c)^2b^2+\arctan(dx+c)^2b^2(dx+c)-i\,\mathrm{polylog}\left(2,-\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)b^2+2\arctan(dx+c)\ln\left(1+\frac{(1+i(dx}{1+(dx+}\right.}{d}$
default	$\frac{(dx+c)a^2-i\arctan(dx+c)^2b^2+\arctan(dx+c)^2b^2(dx+c)-i\,\mathrm{polylog}\left(2,-\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)b^2+2\arctan(dx+c)\ln\left(1+\frac{(1+i(dx}{1+(dx+}\right.}{d}$
risch	$-\frac{\ln(-idx-ic+1)^2x\,b^2}{4}+\frac{\ln(-idx-ic+1)x\,b^2}{2}-\frac{ib^2\ln(-idx-ic+1)(-idx-ic+1)}{2d}+\frac{ib^2\ln(d^2x^2+2cdx+c^2+1)}{4d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2-I*arctan(d*x+c)^2*b^2+arctan(d*x+c)^2*b^2*(d*x+c)-I*polylo
g(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))*b^2+2*arctan(d*x+c)*ln(1+(1+I*(d*x+c))^
2/(1+(d*x+c)^2))*b^2+2*a*b*(d*x+c)*arctan(d*x+c)-a*b*ln(1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(12*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 4*(3*arctan(d*x
+ c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*c^2 + 4*x*a
rctan(d*x + c)^2 + 192*d^2*integrate(1/16*x^2*arctan(d*x + c)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 16*d^2*integrate(1/16*x^2*log(d^2*x^2 + 2*c*d*x +
c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*c*d*integrate(1/16*x*arc
tan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 64*d^2*integrate(1/16*x^
2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 32*c
*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) + 64*c*d*integrate(1/16*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*c^2*integrate(1/16*log(d^2*x^2 + 2*c*d
*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - x*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2 + 12*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 12*arctan(
d*x + c)*arctan((d^2*x + c*d)/d)^2/d + 4*arctan((d^2*x + c*d)/d)^3/d - 128*
d*integrate(1/16*x*arctan(d*x + c)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 16*i
ntegrate(1/16*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 +
```

```
 1), x))*b^2 + a^2*x + (2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))
*a*b/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2,x)
[Out] Integral((a + b*atan(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{atan}(c + d\,x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2,x)
[Out] int((a + b*atan(c + d*x))^2, x)
```

3.34 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{e+fx}\,dx$

Optimal. Leaf size=261

$$-\frac{(a+b\text{ArcTan}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\text{ArcTan}(c+dx))^2\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}+\frac{ib(a+b\text{ArcTa}}{}$$

```
[Out] -(a+b*arctan(d*x+c))^2*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^2*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+I*b*(a+b*arctan(d*x+c))*polylog(2,1-2
/(1-I*(d*x+c)))/f-I*b*(a+b*arctan(d*x+c))*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-
c*f)/(1-I*(d*x+c)))/f-1/2*b^2*polylog(3,1-2/(1-I*(d*x+c)))/f+1/2*b^2*polylo
g(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A]
time = 0.13, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5155, 4968}

$$-\frac{ib(a+b\text{ArcTan}(c+dx))\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{f}+\frac{(a+b\text{ArcTan}(c+dx))^2\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{ib\text{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{f}+\frac{b^2\text{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{b^2\text{Li}_3\left(1-\frac{2}{1-i(c+dx)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^2*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^2*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (I*b*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/f - (I*b
*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*
(1 - I*(c + d*x)))])/f - (b^2*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) +
(b^2*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])
/(2*f)
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
```

```
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{(a+b\tan^{-1}(c+dx))^2\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\tan^{-1}(c+dx))^2\log\left(\frac{\cdots}{(de+i\cdots}\right.}{f}$$

Mathematica [F]
time = 5.13, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(c+dx))^2}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.12, size = 2022, normalized size = 7.75

method	result	size
derivativedivides	Expression too large to display	2022
default	Expression too large to display	2022

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d*ln(c*f-d*e-f*(d*x+c))/f+b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arctan(d*x
+c)^2-b^2*d/f*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)+1/
2*I*b^2*d/f*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*
f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d
*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3+1/2*I*b^2*d/f*arctan(d*x+c)^2*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(
1+(d*x+c)^2)))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^
```

```
2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x
+c))^2/(1+(d*x+c)^2)))+I*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c))/(
d*e+I*f-c*f))-I*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*ln((I*f+f*(d*x+c))/(c*f-d*e+I
*f))+1/2*I*b^2*d/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(
d*x+c)^2)/(d*e+I*f-c*f))-1/2*b^2*d/f*polylog(3,-(1+I*(d*x+c))^2/(1+(d*x+c)^
2))-1/2*I*b^2*d/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*
f-d*e))*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))^2+b^2*d/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I
*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+I*b^2*d/(c*f-d*e+I*f)*arct
an(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))
+b^2*d*c/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(
1+(d*x+c)^2)/(d*e+I*f-c*f))-I*a*b*d/f*dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))+
1/2*b^2*d*c/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)/(d*e+I*f-c*f))-b^2*d^2/f*e/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e
+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+2*I*b^2*d^2/f*e*arctan(d
*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/
(2*I*f+2*c*f-2*d*e)-1/2*b^2*d^2/f*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+2*a*b*d*ln(c*f-d*e-f*(d*x+c))/f
*arctan(d*x+c)+I*a*b*d/f*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/2*I*b^2*d/f
*arctan(d*x+c)^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(I*f*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*
x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+I*b
^2*d/f*arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-I*b^2*d*c/(c
*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c
)^2)/(d*e+I*f-c*f)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/16*(12*b^2*arctan(d*x + c)^2 + b^2*log(d^2
*x^2 + 2*c*d*x + c^2 + 1)^2 + 32*a*b*arctan(d*x + c))/(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f*x + e), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(e + f*x), x)
```

3.35 $\int \frac{(a+b\text{ArcTan}(c+dx))^2}{(e+fx)^2} \, dx$

Optimal. Leaf size=568

$$\frac{2abd(de-cf)\text{ArcTan}(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{ib^2d\text{ArcTan}(c+dx)^2}{d^2e^2-2cdef+\left(1+c^2\right)f^2}+\frac{b^2d(de-cf)\text{ArcTan}(c+dx)^2}{f\left(d^2e^2-2cdef+\left(1+c^2\right)f^2\right)}-\frac{(a+b\text{ArcTan}(}{f(e+f}$$

```
[Out] 2*a*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)+I*b^2*d*arctan(d*x+c)
^2/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*
e^2-2*c*d*e*f+(c^2+1)*f^2)-(a+b*arctan(d*x+c))^2/f/(f*x+e)+2*a*b*d*ln(f*x+e
)/(f^2+(-c*f+d*e)^2)-2*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+2*b^2*d*arctan(d*x+c)*ln(2/(1+I*
(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-a*b*d*ln(1+(d*x+c)^2)/(f^2+(-c*f+
d*e)^2)+I*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)-I*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c
*d*e*f+(c^2+1)*f^2)+I*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f
+(c^2+1)*f^2)
```

Rubi [A]

time = 1.03, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5153, 2007, 719, 31, 648, 632, 210, 642, 6873, 5165, 720, 649, 209, 266, 6820, 12, 6857, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964}

$$\frac{2abd\text{ArcTan}(c+dx)(de-cf)}{f\left((de-cf)^2+f^2\right)}-\frac{(a+b\text{ArcTan}(c+dx))^2}{f(e+fx)}+\frac{2abd\log(e+fx)}{(de-cf)^2+f^2}-\frac{abd\log\left((c+dx)^2+1\right)}{(de-cf)^2+f^2}+\frac{ib^2d\text{ArcTan}(c+dx)^2}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{b^2d\text{ArcTan}(c+dx)^2(de-cf)}{f\left(\left(c^2+1\right)f^2-2cdef+d^2e^2\right)}-\frac{2b^2d\text{ArcTan}(c+dx)\log\left(\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2b^2d\text{ArcTan}(c+dx)\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{2b^2d\text{ArcTan}(c+dx)\log\left(\frac{2}{1+i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{ib^2d\text{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}-\frac{ib^2d\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}+\frac{ib^2d\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{\left(c^2+1\right)f^2-2cdef+d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (2*a*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + (I*b^2*d*
ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(d*e - c*
f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)) - (a + b*Ar
cTan[c + d*x])^2/(f*(e + f*x)) + (2*a*b*d*Log[e + f*x])/(f^2 + (d*e - c*f)^
2) - (2*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2) + (2*b^2*d*ArcTan[c + d*x]*Log[(2*d*(e + f*x))/((d*e + I
*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (2*b
^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 +
c^2)*f^2) - (a*b*d*Log[1 + (c + d*x)^2])/(f^2 + (d*e - c*f)^2) + (I*b^2*d*P
olyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) -
 (I*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x
)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
```

```
EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^2}{(e+fx)^2}\,dx = -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\tan^{-1}(c+dx)}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{d(a+b\tan^{-1}(x))}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \frac{a+b\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \left(\frac{a}{(de-cf+fx)(1+x^2)} + \frac{b\tan^{-1}(x)}{(de-cf+fx)(1+x^2}\right.\right.}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2abd)\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} +$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{(2b^2d)\,\text{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} +\right.\right.}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2} + \frac{(2b^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\tan}{1+x^2}\right.}{f\,(d^2e^2-2cdef+(}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} - \frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} - \frac{(a+b\tan^{-1}(c+dx))^2}{f(e+fx)} + \frac{2abd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf}{f\,(d^2e^2-2cd}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf}{f\,(d^2e^2-2cd}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf}{f\,(d^2e^2-2cd}$$

$$= \frac{2abd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{ib^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{b^2d(de-cf}{f\,(d^2e^2-2cd}$$

Mathematica [**A**]
time = 5.00, size = 419, normalized size = 0.74

$$\frac{-\frac{a^2}{f}+\frac{2ab\left(-\left(\left(-cde+f+c^2f-d^2ex+cdfx\right)\text{ArcTan}(c+dx)\right)+d(e+fx)\log\left(\frac{d(e+fx)}{\sqrt{1+(c+dx)^2}}\right)\right)}{d^2e^2-2cdef+(1+c^2)f^2}+\frac{b^2d(e+fx)\left(-\frac{e^{i\text{ArcTan}\left(\frac{de-cf}{f}\right)}\text{ArcTan}(c+dx)^2}{f\sqrt{1+\frac{(de-cf)^2}{f^2}}}+\frac{(c+dx)\text{ArcTan}(c+dx)^2}{d(e+fx)}-\frac{(de-cf)\left(-i\left(\pi-2\text{ArcTan}\left(\frac{de-cf}{f}\right)\right)\text{ArcTan}(c+dx)-\pi\log\left(1+e^{-2i\text{ArcTan}(c+dx)}\right)-2\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\log\left(1-e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)+\pi\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)+2\text{ArcTan}\left(\frac{de-cf}{f}\right)\log\left(\sin\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)\right)+i\text{PolyLog}\left(2,e^{2i\left(\text{ArcTan}\left(\frac{de-cf}{f}\right)+\text{ArcTan}(c+dx)\right)}\right)\right)}{f^2\left(1+\frac{(de-cf)^2}{f^2}\right)}\right)}{de-cf}}{e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*(-((-(c*d*e) + f + c^2*f - d^2*e*x + c*d*f*x)*ArcTan[c +
 d*x]) + d*(e + f*x)*Log[(d*(e + f*x))/Sqrt[1 + (c + d*x)^2]]))/(d^2*e^2 -
2*c*d*e*f + (1 + c^2)*f^2) + (b^2*d*(e + f*x)*(-((E^(I*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x]^2)/(f*Sqrt[1 + (d*e - c*f)^2/f^2])) + ((c + d*x)*ArcTan[
c + d*x]^2)/(d*(e + f*x)) - ((d*e - c*f)*((-I)*(Pi - 2*ArcTan[(d*e - c*f)/f
])*ArcTan[c + d*x] - Pi*Log[1 + E^((-2*I)*ArcTan[c + d*x])] - 2*(ArcTan[(d*
e - c*f)/f] + ArcTan[c + d*x])*Log[1 - E^((2*I)*(ArcTan[(d*e - c*f)/f] + Ar
cTan[c + d*x]))] + Pi*Log[1/Sqrt[1 + (c + d*x)^2]] + 2*ArcTan[(d*e - c*f)/f
]*Log[Sin[ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]]] + I*PolyLog[2, E^((2*I)
*(ArcTan[(d*e - c*f)/f] + ArcTan[c + d*x]))]))/(f^2*(1 + (d*e - c*f)^2/f^2)
)))/(d*e - c*f))/(e + f*x)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(556) = 1112$.
time = 1.58, size = 1159, normalized size = 2.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d^2/(c*f-d*e-f*(d*x+c))/f+b^2*d^2/(c*f-d*e-f*(d*x+c))/f*arctan(d*x
+c)^2-b^2*d^2*arctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)
-b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c+b^2*d^3/f/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*e+2*b^2*d^2*arctan(d*x+c)/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))-I*b^2*d^2/(c^2*f^2-2*c*d*e*f
+d^2*e^2+f^2)*dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))+I*b^2*d^2/(c^2*f^2-2*c*d
*e*f+d^2*e^2+f^2)*dilog((I*f-f*(d*x+c))/(d*e+I*f-c*f))-1/4*I*b^2*d^2/(c^2*f
^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)^2+1/2*I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^
2*e^2+f^2)*ln(d*x+c+I)*ln(1+(d*x+c)^2)-1/2*I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2
*e^2+f^2)*dilog(1/2*I*(d*x+c-I))+1/2*I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-I*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f
^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))-1/2*I*b^2*d^2/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)*ln(1+(d*x+c)^2)-1/2*I*b^2*d^2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1/2*I*(d*x+c-I))+1/4*I*b^2*d^2
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c-I)^2+I*b^2*d^2/(c^2*f^2-2*c*d*e*f
+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))+1/2*I
*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*dilog(-1/2*I*(d*x+c+I))+2*a*b*d^2/
(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)-a*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)
```

```
*ln(1+(d*x+c)^2)-2*a*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*c+
2*a*b*d^3/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)*e+2*a*b*d^2/(c^2*
f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] (d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1)*f^3
 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2 + 1)*
f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2)) - 2*
arctan(d*x + c)/(f^2*x + f*e))*a*b - 1/16*(4*arctan(d*x + c)^2 - 16*(f^2*x
+ f*e)*integrate(1/16*(12*(d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*arctan(d*x
+ c)^2 + (d^2*f*x^2 + 2*c*d*f*x + (c^2 + 1)*f)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2 + 8*(d*f*x + d*e)*arctan(d*x + c) - 4*(d^2*f*x^2 + c*d*e + (c*d*f +
d^2*e)*x)*log(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^2*f^3*x^4 + 2*(c*d*f^3 + d^2
*f^2*e)*x^3 + (4*c*d*f^2*e + (c^2 + 1)*f^3 + d^2*f*e^2)*x^2 + (c^2*e^2 + e^
2)*f + 2*(c*d*f*e^2 + (c^2*e + e)*f^2)*x), x) - log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2)*b^2/(f^2*x + f*e) - a^2/(f^2*x + f*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)/(f^2*x^2 + 2
*f*x*e + e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**2/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c+d\,x))^2}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^2/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^2/(e + f*x)^2, x)
```

3.36 $\int (e+fx)^2(a+b\mathbf{ArcTan}(c+dx))^3\,dx$

Optimal. Leaf size=564

$$\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\mathrm{ArcTan}(c+dx)}{d^3}-\frac{bf^2(a+b\mathrm{ArcTan}(c+dx))^2}{2d^3}-\frac{3ibf(de-cf)(a+b\mathrm{ArcTan}(c+dx))^2}{d^3}$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arctan(d*x+c)/d^3-1/2*b*f^2*(a+b*arctan(d*x
+c))^2/d^3-3*I*b*f*(-c*f+d*e)*(a+b*arctan(d*x+c))^2/d^3-3*b*f*(-c*f+d*e)*(d
*x+c)*(a+b*arctan(d*x+c))^2/d^3-1/2*b*f^2*(d*x+c)^2*(a+b*arctan(d*x+c))^2/d
^3+1/3*I*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(d*x+c))^3/d^3-1/3
*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f-(-c^2+3)*f^2)*(a+b*arctan(d*x+c))^3/d^3/f+1/
3*(f*x+e)^3*(a+b*arctan(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arctan(d*x+c))*
ln(2/(1+I*(d*x+c)))/d^3+b*(3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*(a+b*arctan(
d*x+c))^2*ln(2/(1+I*(d*x+c)))/d^3-1/2*b^3*f^2*ln(1+(d*x+c)^2)/d^3-3*I*b^3*f
*(-c*f+d*e)*polylog(2,1-2/(1+I*(d*x+c)))/d^3+I*b^2*(3*d^2*e^2-6*c*d*e*f-(-3
*c^2+1)*f^2)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^3+1/2*b^3*(
3*d^2*e^2-6*c*d*e*f-(-3*c^2+1)*f^2)*polylog(3,1-2/(1+I*(d*x+c)))/d^3
```

Rubi [A]
time = 0.70, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5155, 4974, 4930, 5040, 4964, 2449, 2352, 4946, 5036, 266, 5004, 5104, 5114, 6745}

$$\frac{ib^2\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\mathrm{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\mathrm{ArcTan}(c+dx))}{d^3}-\frac{6b^2f(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\mathrm{ArcTan}(c+dx))}{d^3}+\frac{i\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)(a+b\mathrm{ArcTan}(c+dx))^3}{3d^3}-\frac{(de-cf)\left(-\left(3-c^2\right)f^2-2cdef+d^2e^2\right)(a+b\mathrm{ArcTan}(c+dx))^3}{3d^3f}+\frac{b\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\mathrm{ArcTan}(c+dx))^2}{d^3}-\frac{3ibf(de-cf)(a+b\mathrm{ArcTan}(c+dx))^2}{d^3}-\frac{3bf(c+dx)(de-cf)(a+b\mathrm{ArcTan}(c+dx))^2}{d^3}-\frac{bf^2(a+b\mathrm{ArcTan}(c+dx))^2}{2d^3}-\frac{bf^2(c+dx)^2(a+b\mathrm{ArcTan}(c+dx))^2}{2d^3}+\frac{(e+fx)^3(a+b\mathrm{ArcTan}(c+dx))^3}{3f}+\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\mathrm{ArcTan}(c+dx)}{d^3}+\frac{b^3\left(-\left(1-3c^2\right)f^2-6cdef+3d^2e^2\right)\mathrm{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d^3}-\frac{3ib^3f(de-cf)\mathrm{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{d^3}-\frac{b^3f^2\log\left((c+dx)^2+1\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcTan[c + d*x])/d^3 - (b*f^2*(a + b
*ArcTan[c + d*x])^2)/(2*d^3) - ((3*I)*b*f*(d*e - c*f)*(a + b*ArcTan[c + d*x
])^2)/d^3 - (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcTan[c + d*x])^2)/d^3 - (
b*f^2*(c + d*x)^2*(a + b*ArcTan[c + d*x])^2)/(2*d^3) + ((I/3)*(3*d^2*e^2 -
6*c*d*e*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/d^3 - ((d*e - c*f)*
(d^2*e^2 - 2*c*d*e*f - (3 - c^2)*f^2)*(a + b*ArcTan[c + d*x])^3)/(3*d^3*f)
+ ((e + f*x)^3*(a + b*ArcTan[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a +
 b*ArcTan[c + d*x])*Log[2/(1 + I*(c + d*x))])/d^3 + (b*(3*d^2*e^2 - 6*c*d*e
*f - (1 - 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d
^3 - (b^3*f^2*Log[1 + (c + d*x)^2])/(2*d^3) - ((3*I)*b^3*f*(d*e - c*f)*Poly
Log[2, 1 - 2/(1 + I*(c + d*x))])/d^3 + (I*b^2*(3*d^2*e^2 - 6*c*d*e*f - (1 -
 3*c^2)*f^2)*(a + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d
^3 + (b^3*(3*d^2*e^2 - 6*c*d*e*f - (1 - 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 + I
*(c + d*x))])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\tan^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\tan^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \left(\frac{3f^2(de-cf)(a+b\tan^{-1}(x)}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\tan^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef-3f^2}{}\right.}{}$$

$$= -\frac{3bf(de-cf)(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{bf^2(c+dx)^2 (a+}{2}$$

$$= -\frac{3ibf(de-cf)\left(a+b\tan^{-1}(c+dx)\right)^2}{d^3} - \frac{3bf(de-cf)(c+dx)(a}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3} - \frac{3ibf(de-cf)\left(a+b\tan^{-1}(\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\tan^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^3}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. 2(564) = 1128.
time = 8.57, size = 1844, normalized size = 3.27

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 - 3*b*d*e*f + 2*b*c*f^2)*x)/d^2 - (a^2*f*(-2*a*d*e + b*f)*x
^2)/(2*d) + (a^3*f^2*x^3)/3 + ((3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 3*a^2*b
*c^2*d*e*f - 3*a^2*b*c*f^2 + a^2*b*c^3*f^2)*ArcTan[c + d*x])/d^3 + a^2*b*x*
```

(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[c + d*x] + ((-3*a^2*b*d^2*e^2 + 6*a^2*b*
c*d*e*f + a^2*b*f^2 - 3*a^2*b*c^2*f^2)*Log[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d^3) + (3*a*b^2*e^2*((-I)*ArcTan[c + d*x]^2 + (c + d*x)*ArcTan[c + d*x]^2
+ 2*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcTan[c + d*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[c + d*x])]))/d + 6*a*b^2*e*f*(-(((c + d*x)*ArcTan[c + d*x])/d^2
) + (I*c*ArcTan[c + d*x]^2)/d^2 - (c*(c + d*x)*ArcTan[c + d*x]^2)/d^2 + ((1
 + (c + d*x)^2)*ArcTan[c + d*x]^2)/(2*d^2) - (2*c*ArcTan[c + d*x]*Log[1 + E
^((2*I)*ArcTan[c + d*x])])/d^2 - Log[1/Sqrt[1 + (c + d*x)^2]]/d^2 + (I*c*Po
lyLog[2, -E^((2*I)*ArcTan[c + d*x])])/d^2) + (b^3*e^2*((-I)*ArcTan[c + d*x]
^3 + (c + d*x)*ArcTan[c + d*x]^3 + 3*ArcTan[c + d*x]^2*Log[1 + E^((2*I)*Arc
Tan[c + d*x])] - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x]
)] + (3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2))/d + (b^3*e*f*(ArcTan[c
+ d*x]*((3*I)*ArcTan[c + d*x] + (2*I)*c*ArcTan[c + d*x]^2 + (1 + (c + d*x)^
2)*ArcTan[c + d*x]^2 - (c + d*x)*ArcTan[c + d*x]*(3 + 2*c*ArcTan[c + d*x])
- 6*Log[1 + E^((2*I)*ArcTan[c + d*x])] - 6*c*ArcTan[c + d*x]*Log[1 + E^((2*
I)*ArcTan[c + d*x])]) + (3*I)*(1 + 2*c*ArcTan[c + d*x])*PolyLog[2, -E^((2*I
)*ArcTan[c + d*x])] - 3*c*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])]))/d^2 + (a
*b^2*f^2*(1 + (c + d*x)^2)^(3/2)*((c + d*x)/Sqrt[1 + (c + d*x)^2] + (6*c*(c
 + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d*
x]^2)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^2)/Sqrt[1 +
(c + d*x)^2] + I*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTa
n[c + d*x]^2*Cos[3*ArcTan[c + d*x]] - 2*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*
x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c^2*ArcTan[c + d*x]*Cos[3*ArcTan
[c + d*x]]*Log[1 + E^((2*I)*ArcTan[c + d*x])] + 6*c*Cos[3*ArcTan[c + d*x]]*
Log[1/Sqrt[1 + (c + d*x)^2]] + (ArcTan[c + d*x]*(-4 + (3*I - 12*c - (9*I)*c
^2)*ArcTan[c + d*x]) + 6*(-1 + 3*c^2)*ArcTan[c + d*x]*Log[1 + E^((2*I)*ArcT
an[c + d*x])] + 18*c*Log[1/Sqrt[1 + (c + d*x)^2]])/Sqrt[1 + (c + d*x)^2] -
((4*I)*(-1 + 3*c^2)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^
2)^(3/2) + Sin[3*ArcTan[c + d*x]] + 6*c*ArcTan[c + d*x]*Sin[3*ArcTan[c + d*
x]] - ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^2*Si
n[3*ArcTan[c + d*x]]))/(4*d^3) + (b^3*f^2*((-I)*(3*c - ArcTan[c + d*x] + 3*
c^2*ArcTan[c + d*x])*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + ((1 + (c + d*
x)^2)^(3/2)*((3*(c + d*x)*ArcTan[c + d*x])/Sqrt[1 + (c + d*x)^2] + (9*c*(c
+ d*x)*ArcTan[c + d*x]^2)/Sqrt[1 + (c + d*x)^2] + (3*(c + d*x)*ArcTan[c + d
*x]^3)/Sqrt[1 + (c + d*x)^2] + (3*c^2*(c + d*x)*ArcTan[c + d*x]^3)/Sqrt[1 +
 (c + d*x)^2] - (9*I)*c*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]] + I*ArcTan
[c + d*x]^3*Cos[3*ArcTan[c + d*x]] - (3*I)*c^2*ArcTan[c + d*x]^3*Cos[3*ArcT
an[c + d*x]] + 18*c*ArcTan[c + d*x]*Cos[3*ArcTan[c + d*x]]*Log[1 + E^((2*I)
*ArcTan[c + d*x])] - 3*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Log[1 + E^(
(2*I)*ArcTan[c + d*x])] + 9*c^2*ArcTan[c + d*x]^2*Cos[3*ArcTan[c + d*x]]*Lo
g[1 + E^((2*I)*ArcTan[c + d*x])] + 3*Cos[3*ArcTan[c + d*x]]*Log[1/Sqrt[1 +
(c + d*x)^2]] + (3*(ArcTan[c + d*x]^2*(-2 - (9*I)*c + I*ArcTan[c + d*x] - 4
*c*ArcTan[c + d*x] - (3*I)*c^2*ArcTan[c + d*x]) + 3*ArcTan[c + d*x]*(6*c -
ArcTan[c + d*x] + 3*c^2*ArcTan[c + d*x])*Log[1 + E^((2*I)*ArcTan[c + d*x])]
 + 3*Log[1/Sqrt[1 + (c + d*x)^2]]))/Sqrt[1 + (c + d*x)^2] + (6*(-1 + 3*c^2)

```
*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/(1 + (c + d*x)^2)^(3/2) + 3*ArcTan
[c + d*x]*Sin[3*ArcTan[c + d*x]] + 9*c*ArcTan[c + d*x]^2*Sin[3*ArcTan[c + d
*x]] - ArcTan[c + d*x]^3*Sin[3*ArcTan[c + d*x]] + 3*c^2*ArcTan[c + d*x]^3*S
in[3*ArcTan[c + d*x]]))/12))/d^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.24, size = 6607, normalized size = 11.71

method	result	size
derivativedivides	Expression too large to display	6607
default	Expression too large to display	6607

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e^2/d + 1/3*a^3*f^2*x
^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d + 28*b^3*d
^2*f^2*integrate(1/32*x^4*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 3*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*f^2*integrat
e(1/32*x^4*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d
*f^2*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x)
 + 56*b^3*d^2*f*e*integrate(1/32*x^3*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x +
 c^2 + 1), x) + 4*b^3*d^2*f^2*integrate(1/32*x^4*arctan(d*x + c)*log(d^2*x^
2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c*d*f^2*in
tegrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^
2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*e*integrate(1/32*x^3*arctan(d*x +
c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1
92*a*b^2*c*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x +
c^2 + 1), x) + 192*a*b^2*d^2*f*e*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 28*b^3*c^2*f^2*integrate(1/32*x^2*arctan(d*x
 + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*e*integrate(1/32*
x^2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 4*b^3*c*d*f^2*int
egrate(1/32*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 +
 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*e*integrate(1/32*x^3*arctan(d*x + c)
```

*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3
*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1
)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*e*integrate(1/32*x^2*a
rctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 96*a*b^2*c^2*f^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 +
 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*e*integrate(1/32*x^2*arctan(d*x +
 c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*f*e*integrate(1/32*x*a
rctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*f*e*integra
te(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 6*b^3*c^2*f*e*integrate(1/32*x*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*
c^2*f*e*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d
)^3/d)*a*b^2*c^2*e^2 - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/
d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)
^4/d)*b^3*c^2*e^2 + 7/8*b^3*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e^2/d
 + a^3*f*x^2*e - 4*b^3*d*f^2*integrate(1/32*x^3*arctan(d*x + c)^2/(d^2*x^2
+ 2*c*d*x + c^2 + 1), x) + 28*b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + b^3*d*f^2*integrate(1/32*x^3*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*
e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(
d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 96*a*b^2*d^2*e^2*integrate(1/32*x^2*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*f*e*integrate(1/
32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*e^2
*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*
b^3*d^2*e^2*integrate(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*e*integrate(1/32*x^2*log
(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*c
*d*e^2*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*e^2*integrate(1/32*x*arct
an(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e^2*integrate(
1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 +
 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*(2*x^3*arctan
(d*x + c) - d*((d*x^2 - 4*c*x)/d^3 - 2*(c^3 - 3*c)*arctan((d^2*x + c*d)/d)/
d^4 + (3*c^2 - 1)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^4))*a^2*b*f^2 + 3*a*b^
2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e^2/d + 3*(x^2*arctan(d*x + c)
- d*(x/d^2 + (c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*
x + c^2 + 1)/d^3))*a^2*b*f*e + 28*b^3*f^2*integrate(1/32*x^2*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*f^2*integrate(1/32*x^2*arctan
(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) + 96*a*b^2*f^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 56*b^3*f*e*integrate(1/32*x*arctan(d*x + c)^3/(d^2*x^2 + 2
*c*d*x + c^2 + 1), x) + 6*b^3*f*e*integrate(1/32*x*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - 12*b^3*d*e^2

```
*integrate(1/32*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192
*a*b^2*f*e*integrate(1/32*x*arctan(d*x + c)^2/(...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*f*x*e + (b^3*f^2*x^2 + 2*b^3*f*x*e + b^3*e^2)*
arctan(d*x + c)^3 + a^3*e^2 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*f*x*e + a*b^2*e^2)
*arctan(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e + a^2*b*e^2)*arctan(d
*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{atan}(c + dx))^3 (e + fx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2\,(a + b\,\mathrm{atan}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^2*(a + b*atan(c + d*x))^3, x)
```

3.37 $\int (e+fx)(a+b\mathbf{ArcTan}(c+dx))^3\,dx$

Optimal. Leaf size=337

$$-\frac{3ibf(a+b\text{ArcTan}(c+dx))^2}{2d^2}-\frac{3bf(c+dx)(a+b\text{ArcTan}(c+dx))^2}{2d^2}+\frac{i(de-cf)(a+b\text{ArcTan}(c+dx))^3}{d^2}-$$

```
[Out] -3/2*I*b*f*(a+b*arctan(d*x+c))^2/d^2-3/2*b*f*(d*x+c)*(a+b*arctan(d*x+c))^2/
d^2+I*(-c*f+d*e)*(a+b*arctan(d*x+c))^3/d^2-1/2*(-c*f+d*e+f)*(d*e-(1+c)*f)*(
a+b*arctan(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arctan(d*x+c))^3/f-3*b^2*f*(a
+b*arctan(d*x+c))*ln(2/(1+I*(d*x+c)))/d^2+3*b*(-c*f+d*e)*(a+b*arctan(d*x+c)
)^2*ln(2/(1+I*(d*x+c)))/d^2-3/2*I*b^3*f*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3*
I*b^2*(-c*f+d*e)*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d^2+3/2*b
^3*(-c*f+d*e)*polylog(3,1-2/(1+I*(d*x+c)))/d^2
```

Rubi [A]
time = 0.46, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5155, 4974, 4930, 5040, 4964, 2449, 2352, 5104, 5004, 5114, 6745}

$$\frac{3ib^2(de-cf)\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))}{d^2}-\frac{3b^2f\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{d^2}+\frac{i(de-cf)(a+b\text{ArcTan}(c+dx))^3}{d^2}-\frac{(-cf+de+f)(de-(c+1)f)(a+b\text{ArcTan}(c+dx))^3}{2d^2f}+\frac{3b(de-cf)\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{d^2}-\frac{3ibf(a+b\text{ArcTan}(c+dx))^2}{2d^2}-\frac{3bf(c+dx)(a+b\text{ArcTan}(c+dx))^2}{2d^2}+\frac{(e+fx)^2(a+b\text{ArcTan}(c+dx))^3}{2f}+\frac{3b^3(de-cf)\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d^2}-\frac{3ib^3f\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*f*(a + b*ArcTan[c + d*x])^2)/d^2 - (3*b*f*(c + d*x)*(a + b*Ar
cTan[c + d*x])^2)/(2*d^2) + (I*(d*e - c*f)*(a + b*ArcTan[c + d*x])^3)/d^2 -
 ((d*e + f - c*f)*(d*e - (1 + c)*f)*(a + b*ArcTan[c + d*x])^3)/(2*d^2*f) +
((e + f*x)^2*(a + b*ArcTan[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcTan[c +
d*x])*Log[2/(1 + I*(c + d*x))])/d^2 + (3*b*(d*e - c*f)*(a + b*ArcTan[c + d*
x])^2*Log[2/(1 + I*(c + d*x))])/d^2 - (((3*I)/2)*b^3*f*PolyLog[2, 1 - 2/(1
+ I*(c + d*x))])/d^2 + ((3*I)*b^2*(d*e - c*f)*(a + b*ArcTan[c + d*x])*PolyL
og[2, 1 - 2/(1 + I*(c + d*x))])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(
1 + I*(c + d*x))])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4974

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - D
ist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\tan^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \left(\frac{f^2\left(a+b\tan^{-1}(x)\right)^2}{d^2}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\tan^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \frac{((de-f-cf)(de+f-cf)}{}\right.}{}$$

$$= -\frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\tan^{-1}(c+\right.}{2f}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2}$$

$$= -\frac{3ibf\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2} - \frac{3bf(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^2}{2d^2}$$

Mathematica [A]
time = 0.43, size = 592, normalized size = 1.76

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (a^2*(2*a*d*e - 3*b*f - 2*a*c*f)*(c + d*x) + a^3*f*(c + d*x)^2 + 3*a^2*b*f*
ArcTan[c + d*x] - 3*a^2*b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcTan[c + d*x] +
 6*a*b^2*f*(-((c + d*x)*ArcTan[c + d*x]) + ((1 + (c + d*x)^2)*ArcTan[c + d*
x]^2)/2 - Log[1/Sqrt[1 + (c + d*x)^2]]) - 3*a^2*b*(d*e - c*f)*Log[1 + (c +
d*x)^2] + 6*a*b^2*d*e*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*
Log[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x
])]) - 6*a*b^2*c*f*(ArcTan[c + d*x]*((-I + c + d*x)*ArcTan[c + d*x] + 2*Log
[1 + E^((2*I)*ArcTan[c + d*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]
) + b^3*f*(ArcTan[c + d*x]*((3*I)*ArcTan[c + d*x] - 3*(c + d*x)*ArcTan[c +
d*x] + (1 + (c + d*x)^2)*ArcTan[c + d*x]^2 - 6*Log[1 + E^((2*I)*ArcTan[c +
d*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])]) + 2*b^3*d*e*(ArcTan
[c + d*x]^2*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c +
 d*x])]) - (3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (
3*PolyLog[3, -E^((2*I)*ArcTan[c + d*x])])/2) - 2*b^3*c*f*(ArcTan[c + d*x]^2
*((-I + c + d*x)*ArcTan[c + d*x] + 3*Log[1 + E^((2*I)*ArcTan[c + d*x])]) -
(3*I)*ArcTan[c + d*x]*PolyLog[2, -E^((2*I)*ArcTan[c + d*x])] + (3*PolyLog[3
, -E^((2*I)*ArcTan[c + d*x])])/2))/(2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.69, size = 16045, normalized size = 47.61

method	result	size
derivativedivides	Expression too large to display	16045
default	Expression too large to display	16045

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)*e/d + 3*a*b^2*c^2*arc
tan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d - (3*arctan(d*x + c)*arctan((d^2
*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2*e - 7/32*(6*arcta
n(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x
+ c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2*e + 7/8*b^3*arctan(d*x
 + c)^3*arctan((d^2*x + c*d)/d)*e/d + 56*b^3*d^2*f*integrate(1/64*x^3*arcta
n(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*f*integrate(1/64
*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 192*a*b^2*d^2*f*integrate(1/64*x^3*arctan(d*x + c)^2/(d^2*
x^2 + 2*c*d*x + c^2 + 1), x) + 112*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x
+ c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*d^2*e*integrate(1/64*x^2*
arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*f*integrat
e(1/64*x^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*
d*x + c^2 + 1), x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^
2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*d^2*
e*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*f*integrate(1/64*x^2*arctan(
d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*d^2*e*integrate(1/
64*x^2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c^2*f*i
ntegrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 112*b
^3*c*d*e*integrate(1/64*x*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 12*b^3*c*d*f*integrate(1/64*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
+ c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*d^2*e*integrate(1/64*
x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c
^2 + 1), x) + 6*b^3*c^2*f*integrate(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*
c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*e*integra
te(1/64*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 192*a*b^2*c^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 384*a*b^2*c*d*e*integrate(1/64*x*arctan(d
*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 24*b^3*c*d*e*integrate(1/64*x
*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2
+ 1), x) + 6*b^3*c^2*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/2*a^3*f*x^2 + 3*a*b^2*a
rctan(d*x + c)^2*arctan((d^2*x + c*d)/d)*e/d - 12*b^3*d*f*integrate(1/64*x^
2*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*f*integrate
(1/64*x^2*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1),
 x) - 24*b^3*d*e*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^
2 + 1), x) + 6*b^3*d*e*integrate(1/64*x*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/
(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3/2*(x^2*arctan(d*x + c) - d*(x/d^2 + (
c^2 - 1)*arctan((d^2*x + c*d)/d)/d^3 - c*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/d
^3))*a^2*b*f - (3*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2
*x + c*d)/d)^3/d)*a*b^2*e - 7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/
d)^2/d - 4*arctan(d*x + c)*arctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*
d)/d)^4/d)*b^3*e + a^3*x*e + 56*b^3*f*integrate(1/64*x*arctan(d*x + c)^3/(d
^2*x^2 + 2*c*d*x + c^2 + 1), x) + 6*b^3*f*integrate(1/64*x*arctan(d*x + c)*

```
log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*
a*b^2*f*integrate(1/64*x*arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x
) + 6*b^3*e*integrate(1/64*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)
^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 1/16*(b^3*f*x^2 + 2*b^3*x*e)*arctan(
d*x + c)^3 + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2 + 1))*a^2*b
*e/d - 3/64*(b^3*f*x^2 + 2*b^3*x*e)*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x +
 c^2 + 1)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
[Out] integral(a^3*f*x + (b^3*f*x + b^3*e)*arctan(d*x + c)^3 + a^3*e + 3*(a*b^2*f
*x + a*b^2*e)*arctan(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arctan(d*x + c),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^3 (e + fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*atan(d*x+c))**3,x)
[Out] Integral((a + b*atan(c + d*x))**3*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\,\left(a + b\,\mathrm{atan}(c + d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*atan(c + d*x))^3,x)
[Out] int((e + f*x)*(a + b*atan(c + d*x))^3, x)
```

3.38 $\int (a+b\mathbf{ArcTan}(c+dx))^3\,dx$

Optimal. Leaf size=143

$$\frac{i(a+b\text{ArcTan}(c+dx))^3}{d}+\frac{(c+dx)(a+b\text{ArcTan}(c+dx))^3}{d}+\frac{3b(a+b\text{ArcTan}(c+dx))^2\log\left(\frac{2}{1+i(c+dx)}\right)}{d}+\frac{3i}{}$$

[Out] I*(a+b*arctan(d*x+c))^3/d+(d*x+c)*(a+b*arctan(d*x+c))^3/d+3*b*(a+b*arctan(d*x+c))^2*ln(2/(1+I*(d*x+c)))/d+3*I*b^2*(a+b*arctan(d*x+c))*polylog(2,1-2/(1+I*(d*x+c)))/d+3/2*b^3*polylog(3,1-2/(1+I*(d*x+c)))/d

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5147, 4930, 5040, 4964, 5004, 5114, 6745}

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{i(c+dx)+1}\right)(a+b\text{ArcTan}(c+dx))}{d}+\frac{(c+dx)(a+b\text{ArcTan}(c+dx))^3}{d}+\frac{i(a+b\text{ArcTan}(c+dx))^3}{d}+\frac{3b\log\left(\frac{2}{1+i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{d}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{i(c+dx)+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcTan[c + d*x])^3)/d + 
 (3*b*(a + b*ArcTan[c + d*x])^2*Log[2/(1 + I*(c + d*x))])/d + ((3*I)*b^2*(a
 + b*ArcTan[c + d*x])*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/d + (3*b^3*PolyL
og[3, 1 - 2/(1 + I*(c + d*x))])/(2*d)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\text{Subst}\left(\int \frac{x\left(a+b\tan^{-1}(x)\right)^2}{1+x^2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(3b)\text{Subst}\left(\int \frac{(a\cdots}{\cdots}\right.}{\cdots}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(\cdots\right.}{\cdots}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(\cdots\right.}{\cdots}$$

$$=\frac{i\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\tan^{-1}(c+dx)\right)^3}{d}+\frac{3b\left(a+b\tan^{-1}(\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.12, size = 190, normalized size = 1.33

$$\frac{2b^3(-i+c+dx)\text{ArcTan}(c+dx)^3+6b^2\text{ArcTan}(c+dx)^2\left(a(-i+c+dx)+b\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)\right)+6ab\text{ArcTan}(c+dx)\left(a(c+dx)+2b\log\left(1+e^{2i\text{ArcTan}(c+dx)}\right)\right)+2a^2\left(a(c+dx)+3b\log\left(\frac{1}{\sqrt{1+(c+dx)^2}}\right)\right)-6ib^2(a+b\text{ArcTan}(c+dx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(c+dx)}\right)+3b^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] (2*b^3*(-I + c + d*x)*ArcTan[c + d*x]^3 + 6*b^2*ArcTan[c + d*x]^2*(a*(-I +
c + d*x) + b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 6*a*b*ArcTan[c + d*x]*(a
*(c + d*x) + 2*b*Log[1 + E^((2*I)*ArcTan[c + d*x])]) + 2*a^2*(a*(c + d*x) +
 3*b*Log[1/Sqrt[1 + (c + d*x)^2]]) - (6*I)*b^2*(a + b*ArcTan[c + d*x])*Poly
Log[2, -E^((2*I)*ArcTan[c + d*x])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c +
d*x])])/(2*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(136) = 272.
time = 0.46, size = 297, normalized size = 2.08

method	result
derivativedivides	$\frac{(dx+c)a^3-ib^3\arctan(dx+c)^3+b^3\arctan(dx+c)^3(dx+c)+3b^3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3ib^3\arctan(dx+c)}{}$
default	$\frac{(dx+c)a^3-ib^3\arctan(dx+c)^3+b^3\arctan(dx+c)^3(dx+c)+3b^3\arctan(dx+c)^2\ln\left(1+\frac{(1+i(dx+c))^2}{1+(dx+c)^2}\right)-3ib^3\arctan(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3-I*b^3*arctan(d*x+c)^3+b^3*arctan(d*x+c)^3*(d*x+c)+3*b^3*ar
ctan(d*x+c)^2*ln(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3*I*b^3*arctan(d*x+c)*pol
ylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*b^3*polylog(3,-(1+I*(d*x+c))^2/(
1+(d*x+c)^2))-3*I*arctan(d*x+c)^2*a*b^2+3*arctan(d*x+c)^2*a*b^2*(d*x+c)-3*I
*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))*a*b^2+6*arctan(d*x+c)*ln(1+(1+I*
(d*x+c))^2/(1+(d*x+c)^2))*a*b^2+3*a^2*b*(d*x+c)*arctan(d*x+c)-3/2*a^2*b*ln(
1+(d*x+c)^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c^2*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 1/8*b^3*x*arctan(
d*x + c)^3 + 3*a*b^2*c^2*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)/d - 3/32
*b^3*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - (3*arctan(d*x +
 c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2*c^2 -
7/32*(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*a
rctan((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3*c^2 + 7/8*b^3
*arctan(d*x + c)^3*arctan((d^2*x + c*d)/d)/d + 28*b^3*d^2*integrate(1/32*x^
2*arctan(d*x + c)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d^2*integrate
(1/32*x^2*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c
*d*x + c^2 + 1), x) + 96*a*b^2*d^2*integrate(1/32*x^2*arctan(d*x + c)^2/(d^
2*x^2 + 2*c*d*x + c^2 + 1), x) + 56*b^3*c*d*integrate(1/32*x*arctan(d*x + c
)^3/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*d^2*integrate(1/32*x^2*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x + c^2 + 1),
x) + 6*b^3*c*d*integrate(1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2
 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 192*a*b^2*c*d*integrate(1/32*x*
arctan(d*x + c)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 12*b^3*c*d*integrate(
1/32*x*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(d^2*x^2 + 2*c*d*x
+ c^2 + 1), x) + 3*b^3*c^2*integrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c
*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*a*b^2*arctan(d*x +
c)^2*arctan((d^2*x + c*d)/d)/d - 12*b^3*d*integrate(1/32*x*arctan(d*x + c)^
2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) + 3*b^3*d*integrate(1/32*x*log(d^2*x^2
+ 2*c*d*x + c^2 + 1)^2/(d^2*x^2 + 2*c*d*x + c^2 + 1), x) - (3*arctan(d*x +
c)*arctan((d^2*x + c*d)/d)^2/d - arctan((d^2*x + c*d)/d)^3/d)*a*b^2 - 7/32*
(6*arctan(d*x + c)^2*arctan((d^2*x + c*d)/d)^2/d - 4*arctan(d*x + c)*arctan
((d^2*x + c*d)/d)^3/d + arctan((d^2*x + c*d)/d)^4/d)*b^3 + a^3*x + 3*b^3*in
tegrate(1/32*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2/(d^2*x^2 +
2*c*d*x + c^2 + 1), x) + 3/2*(2*(d*x + c)*arctan(d*x + c) - log((d*x + c)^2
 + 1))*a^2*b/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arctan
(d*x + c) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3,x)
[Out] Integral((a + b*atan(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3,x)
[Out] int((a + b*atan(c + d*x))^3, x)
```

3.39 $\int \frac{(a+b\textbf{ArcTan}(c+dx))^3}{e+fx}\,dx$

Optimal. Leaf size=372

$$-\frac{(a+b\text{ArcTan}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f}+\frac{(a+b\text{ArcTan}(c+dx))^3\log\left(\frac{2d(e+fx)}{(de+if-cf)(1-i(c+dx))}\right)}{f}+\frac{3ib(a+b\text{ArcT}}{}$$

```
[Out] -(a+b*arctan(d*x+c))^3*ln(2/(1-I*(d*x+c)))/f+(a+b*arctan(d*x+c))^3*ln(2*d*(
f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f+3/2*I*b*(a+b*arctan(d*x+c))^2*polylog
(2,1-2/(1-I*(d*x+c)))/f-3/2*I*b*(a+b*arctan(d*x+c))^2*polylog(2,1-2*d*(f*x+
e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f-3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2
/(1-I*(d*x+c)))/f+3/2*b^2*(a+b*arctan(d*x+c))*polylog(3,1-2*d*(f*x+e)/(d*e+
I*f-c*f)/(1-I*(d*x+c)))/f-3/4*I*b^3*polylog(4,1-2/(1-I*(d*x+c)))/f+3/4*I*b^
3*polylog(4,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/f
```

Rubi [A]
time = 0.16, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5155, 4970}

$$\frac{3b^2(a+b\text{ArcTan}(c+dx))\text{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}-\frac{3b^2\text{Li}_3\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))}{2f}-\frac{3ib(a+b\text{ArcTan}(c+dx))^2\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{2f}+\frac{(a+b\text{ArcTan}(c+dx))^3\log\left(\frac{2d(e+fx)}{(1-i(c+dx))(-cf+de+if)}\right)}{f}+\frac{3ib\text{Li}_2\left(1-\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^2}{2f}-\frac{\log\left(\frac{2}{1-i(c+dx)}\right)(a+b\text{ArcTan}(c+dx))^3}{f}+\frac{3ib^3\text{Li}_4\left(1-\frac{2d(e+fx)}{(de-cf+if)(1-i(c+dx))}\right)}{4f}-\frac{3ib^3\text{Li}_4\left(1-\frac{2}{1-i(c+dx)}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcTan[c + d*x])^3*Log[2/(1 - I*(c + d*x))])/f) + ((a + b*ArcTan[
c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f +
 (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - 2/(1 - I*(c + d*x))]
)/f - (((3*I)/2)*b*(a + b*ArcTan[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))
/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f - (3*b^2*(a + b*ArcTan[c + d*x])
*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*f) + (3*b^2*(a + b*ArcTan[c + d*x]
)*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2
*f) - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*(c + d*x))])/f + (((3*I)/4)*b^
3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/f
```

Rule 4970

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^3)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^3*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[3
*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2/(1 - I*c*x)]/(2*e)), x] - Simp
[3*I*b*(a + b*ArcTan[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1
 - I*c*x)))]/(2*e)), x] - Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2/
(1 - I*c*x)]/(2*e)), x] + Simp[3*b^2*(a + b*ArcTan[c*x])*(PolyLog[3, 1 - 2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x] - Simp[3*I*b^3*(PolyLog
```

```
[4, 1 - 2/(1 - I*c*x)]/(4*e)), x] + Simp[3*I*b^3*(PolyLog[4, 1 - 2*c*((d +
e*x)/((c*d + I*e)*(1 - I*c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{(a+b\tan^{-1}(c+dx))^3\log\left(\frac{2}{1-i(c+dx)}\right)}{f} + \frac{(a+b\tan^{-1}(c+dx))^3\log\left(\frac{}{(de+i}\right.}{f}$$

Mathematica [F]
time = 56.24, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(c+dx))^3}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.81, size = 4124, normalized size = 11.09

method	result	size
derivativedivides	Expression too large to display	4124
default	Expression too large to display	4124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*I*b^3*d/f*arctan(d*x+c)^3*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c
```

```
*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*Pi+3*a*b^2*d*c/(c*f-d*e+I*f)*a
rctan(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*
f))-3/2*a*b^2*d^2/f*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*a*b^2*d/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(
1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3*I*a*b^2*d/f*
arctan(d*x+c)*polylog(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))-3/2*I*a^2*b*d*ln(c*
f-d*e-f*(d*x+c))/f*ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))+3/2*I*a^2*b*d*ln(c*f-d
*e-f*(d*x+c))/f*ln((I*f-f*(d*x+c))/(d*e+I*f-c*f))-3/2*I*b^3*d*c/(c*f-d*e+I*
f)*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d
*e+I*f-c*f))-b^3*d^2/f*e/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c*f-d*e+I*f)*(
1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+1/2*I*b^3*d/f*arctan(d*x+c)^3*c
sgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-
d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c
)^2)))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I/(1+(1+I*(d*x
+c))^2/(1+(d*x+c)^2)))*Pi+3/2*I*a*b^2*d/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c)
)^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(I
*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c
)^2)))*Pi-3/2*I*a*b^2*d/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2
)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(I*f*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)-I*f+c*f-d*e))*Pi+6*I*a*b^2*d^2/f*e*arctan(d*x+c)*polylog(2,(c*f
-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)-
3/2*I*a*b^2*d/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c
*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-
d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x
+c)^2)))*Pi-3*a*b^2*d^2/f*e/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1-(c*f-d*e+I*f
)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*I*a*b^2*d*c/(c*f-d*e+I*f)*
arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*
f-c*f))+3/2*I*a*b^2*d/f*arctan(d*x+c)^2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x
+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-
I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^3*Pi-1/2*I*b^3*d/f*arctan(d
*x+c)^3*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/
(1+(d*x+c)^2)))^2*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c
))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*Pi+3*I*b
^3*d^2/f*e*arctan(d*x+c)^2*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+
c)^2)/(d*e+I*f-c*f))/(2*I*f+2*c*f-2*d*e)-1/2*I*b^3*d/f*arctan(d*x+c)^3*csgn
(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e
*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2
)))^2*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))*Pi+3*a*b^2*d*ln(c*f-d*e-f*(
d*x+c))/f*arctan(d*x+c)^2-3*a*b^2*d/f*arctan(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^
```

```
2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d
*x+c)^2)-I*f+c*f-d*e)+3*a*b^2*d/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-
d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*a*b^2*d*c/(c*f-d*
e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))
+3/2*I*a*b^2*d/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)/(d*e+I*f-c*f))+3*a^2*b*d*ln(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)-3/2*
I*a^2*b*d/f*dilog((I*f+f*(d*x+c))/(c*f-d*e+I*f))+3/2*I*a^2*b*d/f*dilog((I*f
-f*(d*x+c))/(d*e+I*f-c*f))+b^3*d*c/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c*f-
d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*b^3*d*c/(c*f-d*e+
I*f)*arctan(d*x+c)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d
*e+I*f-c*f))+I*b^3*d/(c*f-d*e+I*f)*arctan(d*x+c)^3*ln(1-(c*f-d*e+I*f)*(1+I*
(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3*d/f*arctan(d*x+c)^2*polyl
og(2,-(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/2*I*b^3*d/(c*f-d*e+I*f)*arctan(d*x+c
)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/4*
I*b^3*d*c/(c*f-d*e+I*f)*polylog(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^
2)/(d*e+I*f-c*f))+a^3*d*ln(c*f-d*e-f*(d*x+c))/f-3/4*b^3*d/(c*f-d*e+I*f)*pol
ylog(4,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/2*b^3*d
^2/f*e/(c*f-d*e+I*f)*arctan(d*x+c)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/
(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*I*b^3*d^2/f*e*polylog(4,(c*f-d*e+I*f)*(1+I*(
d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))/(4*I*f+4...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/32*(28*b^3*arctan(d*x + c)^3 + 3*b^3*arcta
n(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + 96*a*b^2*arctan(d*x + c)^2
+ 96*a^2*b*arctan(d*x + c))/(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)^3}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x),x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(e + f*x), x)
```

3.40 $\int \frac{(a+b\textbf{ArcTan}(c+dx))^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1233

$$\frac{3a^2bd(de-cf)\text{ArcTan}(c+dx)}{f\left(f^2+(de-cf)^2\right)}+\frac{3iab^2d\text{ArcTan}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2}+\frac{3ab^2d(de-cf)\text{ArcTan}(c+dx)^2}{f\left(d^2e^2-2cdef+(1+c^2)\,f^2\right)}+\frac{ib^3d\text{Arc}}{d^2e^2-2cd}$$

```
[Out] 3*a^2*b*d*(-c*f+d*e)*arctan(d*x+c)/f/(f^2+(-c*f+d*e)^2)-3*I*b^3*d*arctan(d*
x+c)*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*
f+(c^2+1)*f^2)+3*a*b^2*d*(-c*f+d*e)*arctan(d*x+c)^2/f/(d^2*e^2-2*c*d*e*f+(c
^2+1)*f^2)+3*I*a*b^2*d*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2
+1)*f^2)+b^3*d*(-c*f+d*e)*arctan(d*x+c)^3/f/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
-(a+b*arctan(d*x+c))^3/f/(f*x+e)+3*a^2*b*d*ln(f*x+e)/(f^2+(-c*f+d*e)^2)-6*a
*b^2*d*arctan(d*x+c)*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3*
b^3*d*arctan(d*x+c)^2*ln(2/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6
*a*b^2*d*arctan(d*x+c)*ln(2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I*(d*x+c)))/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*ln(2*d*(f*x+e)/(d*e+I*f-c*f
)/(1-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+6*a*b^2*d*arctan(d*x+c)*ln
(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3*b^3*d*arctan(d*x+c)^2*l
n(2/(1+I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*a^2*b*d*ln(1+(d*x+c)
^2)/(f^2+(-c*f+d*e)^2)+3*I*a*b^2*d*polylog(2,1-2/(1-I*(d*x+c)))/(d^2*e^2-2*
c*d*e*f+(c^2+1)*f^2)-3*I*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1-I
*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+I*b^3*d*arctan(d*x+c)^3/(d^2*e^2
-2*c*d*e*f+(c^2+1)*f^2)+3*I*a*b^2*d*arctan(d*x+c)^2/(d^2*e^2-2*c*d*e*f+(c^2
+1)*f^2)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1+I*(d*x+c)))/(d^2*e^2-2*c*
d*e*f+(c^2+1)*f^2)+3*I*b^3*d*arctan(d*x+c)*polylog(2,1-2/(1-I*(d*x+c)))/(d^
2*e^2-2*c*d*e*f+(c^2+1)*f^2)-3/2*b^3*d*polylog(3,1-2/(1-I*(d*x+c)))/(d^2*e^
2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2*d*(f*x+e)/(d*e+I*f-c*f)/(1
-I*(d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)+3/2*b^3*d*polylog(3,1-2/(1+I*(
d*x+c)))/(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)
```

Rubi [A]

time = 1.66, antiderivative size = 1233, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5153, 6873, 5165, 6820, 12, 6857, 720, 31, 649, 209, 266, 4966, 2449, 2352, 2497, 5104, 5004, 5040, 4964, 4968, 5114, 6745}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] (3*a^2*b*d*(d*e - c*f)*ArcTan[c + d*x])/(f*(f^2 + (d*e - c*f)^2)) + ((3*I)*
a*b^2*d*ArcTan[c + d*x]^2)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*a*b^2
```

```
*d*(d*e - c*f)*ArcTan[c + d*x]^2)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 + (I*b^3*d*ArcTan[c + d*x]^3)/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (b^3
*d*(d*e - c*f)*ArcTan[c + d*x]^3)/(f*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2))
 - (a + b*ArcTan[c + d*x])^3/(f*(e + f*x)) + (3*a^2*b*d*Log[e + f*x])/(f^2
+ (d*e - c*f)^2) - (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 - I*(c + d*x))])/(d^
2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 -
I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + (6*a*b^2*d*ArcTan[c
+ d*x]*Log[(2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[(2*d*(e + f*
x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2
)*f^2) + (6*a*b^2*d*ArcTan[c + d*x]*Log[2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*
c*d*e*f + (1 + c^2)*f^2) + (3*b^3*d*ArcTan[c + d*x]^2*Log[2/(1 + I*(c + d*x
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - (3*a^2*b*d*Log[1 + (c + d*x)^2
])/(2*(f^2 + (d*e - c*f)^2)) + ((3*I)*a*b^2*d*PolyLog[2, 1 - 2/(1 - I*(c +
d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*ArcTan[c + d*x
]*PolyLog[2, 1 - 2/(1 - I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2
) - ((3*I)*a*b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I
*(c + d*x)))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) - ((3*I)*b^3*d*ArcTan[
c + d*x]*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + I*f - c*f)*(1 - I*(c + d*x)
))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*a*b^2*d*PolyLog[2, 1 -
2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f^2) + ((3*I)*b^3*d*
ArcTan[c + d*x]*PolyLog[2, 1 - 2/(1 + I*(c + d*x))])/(d^2*e^2 - 2*c*d*e*f +
 (1 + c^2)*f^2) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - I*(c + d*x))])/(2*(d^2*e^2
 - 2*c*d*e*f + (1 + c^2)*f^2)) + (3*b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((
d*e + I*f - c*f)*(1 - I*(c + d*x)))])/(2*(d^2*e^2 - 2*c*d*e*f + (1 + c^2)*f
^2)) + (3*b^3*d*PolyLog[3, 1 - 2/(1 + I*(c + d*x))])/(2*(d^2*e^2 - 2*c*d*e*
f + (1 + c^2)*f^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
```

```
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4968

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5104

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
```

```
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(c+dx))^3}{(e+fx)^2}\,dx = -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3bd)\int \frac{(a+b\tan^{-1}(c+dx))^2}{(e+fx)(1+(c+dx)^2)}\,dx}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3bd)\int \frac{(a+b\tan^{-1}(c+dx))^2}{(e+fx)(1+c^2+2cdx+d^2x^2)}\,dx}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{d(a+b\tan^{-1}(x))^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(x))^2}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \left(\frac{a^2}{(de-cf+fx)(1+x^2)} + \frac{2ab\tan^{-1}(x)}{(de-cf+fx)(1+x^2)}\right.\right.}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(de-cf+fx)(1+x^2)}\,dx, x, c+dx\right)}{f} +$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{(6ab^2d)\,\text{Subst}\left(\int \left(\frac{f^2\tan^{-1}(x)}{(d^2e^2-2cdef+(1+c^2)f^2)(de-cf+fx)} + \right.\right.}{f}$$

$$= -\frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2} + \frac{(6ab^2d)\,\text{Subst}\left(\int \frac{(de-cf-fx)\,\text{ta}}{1+x^2}\right.}{f\,(d^2e^2-2cdef+(}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} - \frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} - \frac{(a+b\tan^{-1}(c+dx))^3}{f(e+fx)} + \frac{3a^2bd\log(e+fx)}{f^2+(de-cf)^2}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{3ab^2d(de-cf}{f\,(d^2e^2-2cde}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{3ab^2d(de-cf}{f\,(d^2e^2-2cde}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{3ab^2d(de-cf}{f\,(d^2e^2-2cde}$$

$$= \frac{3a^2bd(de-cf)\tan^{-1}(c+dx)}{f\,(f^2+(de-cf)^2)} + \frac{3iab^2d\tan^{-1}(c+dx)^2}{d^2e^2-2cdef+(1+c^2)\,f^2} + \frac{3ab^2d(de-cf}{f\,(d^2e^2-2cde}$$

Mathematica [F]
time = 52.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\text{ArcTan}(c+dx))^3}{(e+fx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c + d*x])^3/(e + f*x)^2, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.92, size = 4691, normalized size = 3.80

method	result	size
derivativedivides	Expression too large to display	4691
default	Expression too large to display	4691

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/2*a^2*b*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)+3*a^2*b
*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))+b^3*d^2/(c*f-d*e
-f*(d*x+c))/f*arctan(d*x+c)^3-b^3*d^2*arctan(d*x+c)^3/(c^2*f^2-2*c*d*e*f+d^
2*e^2+f^2)*c+3*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*ln(2
)+3*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*ln((1+I*(d*x+c)
)/(1+(d*x+c)^2)^(1/2))-3/2*b^3*d^2*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e
^2+f^2)*ln(1+(d*x+c)^2)+3*b^3*d^2*arctan(d*x+c)^2/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)*ln(c*f-d*e-f*(d*x+c))-3*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arct
an(d*x+c)^2*ln(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*
x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)-I*b^3*d^2/(c^2*f^2-2
*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^3-3/2*I*a*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^
2*e^2+f^2)*ln(d*x+c-I)*ln(1+(d*x+c)^2)+3/2*I*a*b^2*d^2/(c^2*f^2-2*c*d*e*f+d
^2*e^2+f^2)*ln(d*x+c-I)*ln(-1/2*I*(d*x+c+I))-3*I*a*b^2*d^2/(c^2*f^2-2*c*d*e
*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f+f*(d*x+c))/(c*f-d*e+I*f))+3/2
*I*a*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1+(d*x+c)^2)+3*
I*a*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))*ln((I*f-f
*(d*x+c))/(d*e+I*f-c*f))+3*b^3*d^2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(c*f-d
*e+I*f)*arctan(d*x+c)*polylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)
/(d*e+I*f-c*f))+3/2*b^3*d^2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(c*f-d*e+I*
f)*polylog(3,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3*b
^3*d^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(c*f-d*e+I*f)*arctan(d*x+c)^2*ln(1
-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+3/2*I*b^3*d^2*f
/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*(1+I
*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/4*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d
```

```
^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^3+3/4*
I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I
*(d*x+c))^2/(1+(d*x+c)^2))^2)^3+3/2*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^
2)*arctan(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1
+I*(d*x+c))^2/(1+(d*x+c)^2)))^3-3/4*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^
2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2))^2)^3+3*a^2*b*d^3/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan
(d*x+c)*e+3*b^3*d^2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*c/(c*f-d*e+I*f)*arcta
n(d*x+c)^2*ln(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+
3*I*b^3*d^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e/(c*f-d*e+I*f)*arctan(d*x+c)*p
olylog(2,(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))-3/4*I*b
^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+
c))/(1+(d*x+c)^2)^(1/2))^2*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))+3/4*I*b^3*
d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*(d*x+c))
^2/(1+(d*x+c)^2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(
1+(d*x+c)^2))^2)^2+3/4*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x
+c)^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)*csgn(I*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2-3/2*I*b^3*d^2/(c^2*f^
2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I/(1+(1+I*(d*x+c))^2/(1+(d
*x+c)^2)))*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1
+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))
^2/(1+(d*x+c)^2)))^2+3/4*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d
*x+c)^2*Pi*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2*csgn(I*(1+(1+I*(d*x+
c))^2/(1+(d*x+c)^2))^2)-3/2*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arcta
n(d*x+c)^2*Pi*csgn(I*(I*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2
/(1+(d*x+c)^2)-d*e*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e))*csgn(I*(I*f*
(1+I*(d*x+c))^2/(1+(d*x+c)^2)+c*f*(1+I*(d*x+c))^2/(1+(d*x+c)^2)-d*e*(1+I*(d
*x+c))^2/(1+(d*x+c)^2)-I*f+c*f-d*e)/(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2)))^2+3/
2*I*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+I*
(d*x+c))/(1+(d*x+c)^2)^(1/2))*csgn(I*(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2-3/2*I
*b^3*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*Pi*csgn(I*(1+(1+I*
(d*x+c))^2/(1+(d*x+c)^2)))*csgn(I*(1+(1+I*(d*x+c))^2/(1+(d*x+c)^2))^2)^2+3*
I*b^3*d^2*f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)/(c*f-d*e+I*f)*arctan(d*x+c)^2*l
n(1-(c*f-d*e+I*f)*(1+I*(d*x+c))^2/(1+(d*x+c)^2)/(d*e+I*f-c*f))+a^3*d^2/(c*f
-d*e-f*(d*x+c))/f+3*a*b^2*d^3/f/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+
c)^2*e-3/2*I*a*b^2*d^2/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(d*x+c+I)*ln(1/2*I
*(d*x+c-I))+3*a*b^2*d^2/(c*f-d*e-f*(d*x+c))/f*arctan(d*x+c)^2-3*a*b^2*d^2*a
rctan(d*x+c)/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(1+(d*x+c)^2)-3*a*b^2*d^2/(c
^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*arctan(d*x+c)^2*c+6*a*b^2*d^2*arctan(d*x+c)/(
c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*ln(c*f-d*e-f*(d*x+c))+b^3*d^3/f*arctan(d*x+c
)^3/(c^2*f^2-2*c*d*e*f+d^2*e^2+f^2)*e-3/2*b^3*d^3/(c^2*f^2-2*c*d*e*f+d^2*e^
2+f^2)*e/(c*f-d*e+I*f)*polylog(3,(c*f-d*e+I*f)*...
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 3/2*(d*(2*(c*d*f - d^2*e)*arctan((d^2*x + c*d)/d)/((2*c*d*f^2*e - (c^2 + 1)
*f^3 - d^2*f*e^2)*d) + log(d^2*x^2 + 2*c*d*x + c^2 + 1)/(2*c*d*f*e - (c^2 +
 1)*f^2 - d^2*e^2) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 + 1)*f^2 - d^2*e^2))
- 2*arctan(d*x + c)/(f^2*x + f*e))*a^2*b - a^3/(f^2*x + f*e) - 1/32*(4*b^3*
arctan(d*x + c)^3 - 3*b^3*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^
2 - 32*(f^2*x + f*e)*integrate(1/32*(28*(b^3*d^2*f*x^2 + 2*b^3*c*d*f*x + (b
^3*c^2 + b^3)*f)*arctan(d*x + c)^3 + 12*(8*a*b^2*d^2*f*x^2 + b^3*d*e + (16*
a*b^2*c + b^3)*d*f*x + 8*(a*b^2*c^2 + a*b^2)*f)*arctan(d*x + c)^2 - 12*(b^3
*d^2*f*x^2 + b^3*c*d*e + (b^3*c*d*f + b^3*d^2*e)*x)*arctan(d*x + c)*log(d^2
*x^2 + 2*c*d*x + c^2 + 1) - 3*(b^3*d*f*x + b^3*d*e - (b^3*d^2*f*x^2 + 2*b^3
*c*d*f*x + (b^3*c^2 + b^3)*f)*arctan(d*x + c))*log(d^2*x^2 + 2*c*d*x + c^2
+ 1)^2)/(d^2*f^3*x^4 + 2*(c*d*f^3 + d^2*f^2*e)*x^3 + (4*c*d*f^2*e + (c^2 +
1)*f^3 + d^2*f*e^2)*x^2 + (c^2*e^2 + e^2)*f + 2*(c*d*f*e^2 + (c^2*e + e)*f^
2)*x), x))/(f^2*x + f*e)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c+d\,x))^3}{(e+f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*atan(c + d*x))^3/(e + f*x)^2, x)
```

3.41 $\int (e+fx)^m(a+b\textbf{ArcTan}(c+dx))\,dx$

Optimal. Leaf size=177

$$\frac{(e+fx)^{1+m}(a+b\text{ArcTan}(c+dx))}{f(1+m)}-\frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de+if-cf}\right)}{2f(de+(i-c)f)(1+m)(2+m)}+\frac{ibd(e+fx)^{2+m}\,{}_2F_1\Big(}{2f(de-(i+}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arctan(d*x+c))/f/(1+m)-1/2*I*b*d*(f*x+e)^(2+m)*hypergeom
([1, 2+m],[3+m],d*(f*x+e)/(d*e+I*f-c*f))/f/(d*e+(I-c)*f)/(1+m)/(2+m)+1/2*I*
b*d*(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(d*e-(I+c)*f))/f/(d*e-
(I+c)*f)/(1+m)/(2+m)
```

Rubi [A]
time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5155, 4972, 726, 70}

$$\frac{(e+fx)^{m+1}(a+b\text{ArcTan}(c+dx))}{f(m+1)}-\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf+if}\right)}{2f(m+1)(m+2)(de+(-c+i)f)}+\frac{ibd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-(c+i)f}\right)}{2f(m+1)(m+2)(de-(c+i)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcTan[c + d*x]))/(f*(1 + m)) - ((I/2)*b*d*(e + f
*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + I*f - c
*f)])/(f*(d*e + (I - c)*f)*(1 + m)*(2 + m)) + ((I/2)*b*d*(e + f*x)^(2 + m)*
Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (I + c)*f)])/(f*(d*
e - (I + c)*f)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
```

```
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\tan^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\tan^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1+x^2} dx, x \ldots\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\text{Subst}\left(\int \left(\frac{i\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(i-x)} + \ldots\right.\right.}{f(1 \ldots}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{(ib)\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{i-x} dx \ldots\right.}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\tan^{-1}(c+dx)\right)}{f(1+m)} - \frac{ibd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m; \ldots\right.}{2f(de+(i-c)f)(1+ \ldots}$$

Mathematica [A]
time = 0.28, size = 162, normalized size = 0.92

$$\frac{(e+fx)^{1+m}\left(2(a+b\text{ArcTan}(c+dx))+\frac{bd(e+fx)\left((de-(i+c)f)\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-(-i+c)f}\right)+(-de+(-i+c)f)\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-(i+c)f}\right)\right)}{(ide+f-icf)(de-(-i+c)f)(2+m)}\right)}{2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(2*(a + b*ArcTan[c + d*x]) + (b*d*(e + f*x)*((d*e - (I +
 c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - (-I + c)*f)]
 + (-(d*e) + (-I + c)*f)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(
```

```
d*e - (I + c)*f)]))/((I*d*e + f - I*c*f)*(d*e - (-I + c)*f)*(2 + m))))/(2*f
*(1 + m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (fx+e)^m (a+b\arctan(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((3*m^2*e + (3*f*m^2 + 2*f*m + f)*x + 2*m*e + e)*(f*x + e)^m*arctan(d*x
 + c) + ((f*m + f)*x + m*e + e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1
) + 2*(f*m^3 + f*m^2 + f*m + f)*integrate(-1/2*(2*((c^2 + 1)*f*m^3 + 2*(c^2
 + 1)*f*m^2 + (c^2 + 1)*f*m + (d^2*f*m^3 + 2*d^2*f*m^2 + d^2*f*m)*x^2 + 2*(
c*d*f*m^3 + 2*c*d*f*m^2 + c*d*f*m)*x)*(f*x + e)^m*arctan(d*x + c) - ((c^2 +
 1)*f*m^3 - (c^2 + 1)*f*m + (d^2*f*m^3 - d^2*f*m)*x^2 + 2*(c*d*f*m^3 - c*d*
f*m)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 + 1) - 2*((c*e - e)*d*m^2 -
 (c*e + e)*d*m + (d^2*f*m^2 - d^2*f*m)*x^2 + (((c - 1)*d*f + d^2*e)*m^2 - (
(c + 1)*d*f + d^2*e)*m)*x)*(f*x + e)^m)/((c^2 + 1)*f*m^3 + (c^2 + 1)*f*m^2
+ (c^2 + 1)*f*m + (d^2*f*m^3 + d^2*f*m^2 + d^2*f*m + d^2*f)*x^2 + (c^2 + 1)
*f + 2*(c*d*f*m^3 + c*d*f*m^2 + c*d*f*m + c*d*f)*x), x))*b/(f*m^3 + f*m^2 +
 f*m + f) + (f*x + e)^(m + 1)*a/(f*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="fricas")
[Out] integral((b*arctan(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{atan}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x)),x)
```

```
[Out] int((e + f*x)^m*(a + b*atan(c + d*x)), x)
```

3.42 $\int (e+fx)^m (a+b\mathbf{ArcTan}(c+dx))^2\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left((e+fx)^m(a+b\mathrm{ArcTan}(c+dx))^2,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m(a+b\mathrm{ArcTan}(c+dx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^2, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m\left(a+b\tan^{-1}(c+dx)\right)^2\,dx=\frac{\mathrm{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m\left(a+b\tan^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

Mathematica [A]
time = 3.35, size = 0, normalized size = 0.00

$$\int (e+fx)^m(a+b\mathrm{ArcTan}(c+dx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2,x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^2, x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int (fx+e)^m\left(a+b\arctan(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^2/(f*(m + 1)) + 1/16*(4*(b^2*f*x + b^2*e)*(f*x + e)^m*a
rctan(d*x + c)^2 - (b^2*f*x + b^2*e)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^
2 + 1)^2 + 16*(f*m + f)*integrate(1/16*(12*((b^2*c^2 + b^2)*f*m + (b^2*d^2*
f*m + b^2*d^2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(
f*x + e)^m*arctan(d*x + c)^2 + ((b^2*c^2 + b^2)*f*m + (b^2*d^2*f*m + b^2*d^
2*f)*x^2 + (b^2*c^2 + b^2)*f + 2*(b^2*c*d*f*m + b^2*c*d*f)*x)*(f*x + e)^m*l
og(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 - 8*(b^2*d*e - 4*(a*b*c^2 + a*b)*f*m - 4*
(a*b*d^2*f*m + a*b*d^2*f)*x^2 - 4*(a*b*c^2 + a*b)*f - (8*a*b*c*d*f*m + (8*a
*b*c - b^2)*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 4*(b^2*d^2*f*x^2 + b^2*c*
d*e + (b^2*c*d*f + b^2*d^2*e)*x)*(f*x + e)^m*log(d^2*x^2 + 2*c*d*x + c^2 +
1))/((c^2 + 1)*f*m + (d^2*f*m + d^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d
*f)*x), x))/(f*m + f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(d*x + c)^2 + 2*a*b*arctan(d*x + c) + a^2)*(f*x + e)^m,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e + f\,x)^m\,(a + b\,\mathrm{atan}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^2,x)
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^2, x)
```

3.43 $\int (e+fx)^m(a+b\text{ArcTan}(c+dx))^3\,dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m(a+b\text{ArcTan}(c+dx))^3,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m(a+b\text{ArcTan}(c+dx))^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcTan[x])^3, x]
, x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m\left(a+b\tan^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m\left(a+b\tan^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int (e+fx)^m(a+b\text{ArcTan}(c+dx))^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcTan[c + d*x])^3, x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int (fx+e)^m\left(a+b\arctan\left(dx+c\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arctan(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (f*x + e)^(m + 1)*a^3/(f*(m + 1)) + 1/32*(4*(b^3*f*x + b^3*e)*(f*x + e)^m*a
rctan(d*x + c)^3 - 3*(b^3*f*x + b^3*e)*(f*x + e)^m*arctan(d*x + c)*log(d^2*
x^2 + 2*c*d*x + c^2 + 1)^2 + 32*(f*m + f)*integrate(1/32*(28*((b^3*c^2 + b^
3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*f*m
 + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c)^3 - 12*(b^3*d*e - 8*(a*b^2*c^2
 + a*b^2)*f*m - 8*(a*b^2*d^2*f*m + a*b^2*d^2*f)*x^2 - 8*(a*b^2*c^2 + a*b^2)
*f - (16*a*b^2*c*d*f*m + (16*a*b^2*c - b^3)*d*f)*x)*(f*x + e)^m*arctan(d*x
+ c)^2 + 12*(b^3*d^2*f*x^2 + b^3*c*d*e + (b^3*c*d*f + b^3*d^2*e)*x)*(f*x +
e)^m*arctan(d*x + c)*log(d^2*x^2 + 2*c*d*x + c^2 + 1) + 96*((a^2*b*c^2 + a^
2*b)*f*m + (a^2*b*d^2*f*m + a^2*b*d^2*f)*x^2 + (a^2*b*c^2 + a^2*b)*f + 2*(a
^2*b*c*d*f*m + a^2*b*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + 3*(((b^3*c^2 +
 b^3)*f*m + (b^3*d^2*f*m + b^3*d^2*f)*x^2 + (b^3*c^2 + b^3)*f + 2*(b^3*c*d*
f*m + b^3*c*d*f)*x)*(f*x + e)^m*arctan(d*x + c) + (b^3*d*f*x + b^3*d*e)*(f*
x + e)^m)*log(d^2*x^2 + 2*c*d*x + c^2 + 1)^2)/((c^2 + 1)*f*m + (d^2*f*m + d
^2*f)*x^2 + (c^2 + 1)*f + 2*(c*d*f*m + c*d*f)*x), x))/(f*m + f)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(d*x + c)^3 + 3*a*b^2*arctan(d*x + c)^2 + 3*a^2*b*arcta
n(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*atan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arctan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e + f\,x)^m\,(a + b\,\mathrm{atan}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*atan(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^m*(a + b*atan(c + d*x))^3, x)
```

3.44 $\int x^3 \text{ArcTan}(a+bx)\,dx$

Optimal. Leaf size=106

$$\frac{(1-6a^2)\,x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}-\frac{(1-6a^2+a^4)\,\text{ArcTan}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{ArcTan}(a+bx)-\frac{a(1-a^2)\log(1\ldots}{2b^4}$$

```
[Out] 1/4*(-6*a^2+1)*x/b^3+1/2*a*(b*x+a)^2/b^4-1/12*(b*x+a)^3/b^4-1/4*(a^4-6*a^2+
1)*arctan(b*x+a)/b^4+1/4*x^4*arctan(b*x+a)-1/2*a*(-a^2+1)*ln(1+(b*x+a)^2)/b
^4
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$-\frac{a(1-a^2)\log\left((a+bx)^2+1\right)}{2b^4}+\frac{(1-6a^2)\,x}{4b^3}-\frac{(a^4-6a^2+1)\,\text{ArcTan}(a+bx)}{4b^4}+\frac{1}{4}x^4\text{ArcTan}(a+bx)-\frac{(a+bx)^3}{12b^4}+\frac{a(a+bx)^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[a + b*x],x]
```

```
[Out] ((1 - 6*a^2)*x)/(4*b^3) + (a*(a + b*x)^2)/(2*b^4) - (a + b*x)^3/(12*b^4) -
((1 - 6*a^2 + a^4)*ArcTan[a + b*x])/(4*b^4) + (x^4*ArcTan[a + b*x])/4 - (a*
(1 - a^2)*Log[1 + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int x^3\tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^3\tan^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1+x^2}\,dx,x,a+bx\right)$$
$$= \frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\left(-\frac{1-6a^2}{b^4}-\frac{4ax}{b^4}+\frac{x^2}{b^4}+\frac{1-6a^2+a^4+4a(1-}{b^4(1+x^2)}\right.\right.$$
$$= \frac{(1-6a^2)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{1-6a^2+a^4+4a}{1+x^2}\right.}{4}$$
$$= \frac{(1-6a^2)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\tan^{-1}(a+bx)-\frac{(a(1-a^2))\,\text{Subst}\left(\int\right.}{b^4}$$
$$= \frac{(1-6a^2)x}{4b^3}+\frac{a(a+bx)^2}{2b^4}-\frac{(a+bx)^3}{12b^4}-\frac{(1-6a^2+a^4)\tan^{-1}(a+bx)}{4b^4}+\frac{1}{4}x^4\tan^{-1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 95, normalized size = 0.90

$$\frac{6(1-6a^2)bx+12a(a+bx)^2-2(a+bx)^3+6b^4x^4\text{ArcTan}(a+bx)+3i(-i+a)^4\log(i-a-bx)-3i(i+a)^4\log(i+a+bx)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[a + b*x],x]
```

```
[Out] (6*(1 - 6*a^2)*b*x + 12*a*(a + b*x)^2 - 2*(a + b*x)^3 + 6*b^4*x^4*ArcTan[a
+ b*x] + (3*I)*(-I + a)^4*Log[I - a - b*x] - (3*I)*(I + a)^4*Log[I + a + b*
x])/(24*b^4)
```

Maple [A]
time = 0.06, size = 157, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)a^4}{4} - \arctan(bx+a)a^3(bx+a) + \frac{3\arctan(bx+a)a^2(bx+a)^2}{2} - \arctan(bx+a)a(bx+a)^3 + \frac{\arctan(bx+a)(bx+a)^4}{4} - 3a^2}{b^4}$
default	$\frac{\frac{\arctan(bx+a)a^4}{4} - \arctan(bx+a)a^3(bx+a) + \frac{3\arctan(bx+a)a^2(bx+a)^2}{2} - \arctan(bx+a)a(bx+a)^3 + \frac{\arctan(bx+a)(bx+a)^4}{4} - 3a^2}{b^4}$
risch	$-\frac{ix^4\ln(1+i(bx+a))}{8} + \frac{ix^4\ln(1-i(bx+a))}{8} - \frac{x^3}{12b} - \frac{a^4\arctan(bx+a)}{4b^4} + \frac{a\,x^2}{4b^2} + \frac{a^3\ln(b^2x^2+2abx+a^2+1)}{2b^4} - 3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arctan(b*x+a)*a^4-arctan(b*x+a)*a^3*(b*x+a)+3/2*arctan(b*x+a)*a^
2*(b*x+a)^2-arctan(b*x+a)*a*(b*x+a)^3+1/4*arctan(b*x+a)*(b*x+a)^4-3/2*a^2*(
b*x+a)+1/2*(b*x+a)^2*a-1/12*(b*x+a)^3+1/4*b*x+1/4*a-1/8*(-4*a^3+4*a)*ln(1+(
b*x+a)^2)-1/4*(a^4-6*a^2+1)*arctan(b*x+a))
```

Maxima [A]
time = 0.50, size = 104, normalized size = 0.98

$$\frac{1}{4}x^4\arctan(bx+a) - \frac{1}{12}b\left(\frac{b^2x^3 - 3\,abx^2 + 3\,(3\,a^2-1)x}{b^4} + \frac{3\,(a^4-6\,a^2+1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} - \frac{6\,(a^3-a)\log(b^2x^2+2\,abx+a^2+1)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(b*x + a) - 1/12*b*((b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 - 1)*x)/b
^4 + 3*(a^4 - 6*a^2 + 1)*arctan((b^2*x + a*b)/b)/b^5 - 6*(a^3 - a)*log(b^2*
x^2 + 2*a*b*x + a^2 + 1)/b^5)
```

Fricas [A]
time = 3.19, size = 87, normalized size = 0.82

$$-\frac{b^3x^3 - 3\,ab^2x^2 + 3\,(3\,a^2-1)bx - 3\,(b^4x^4 - a^4 + 6\,a^2 - 1)\arctan(bx+a) - 6\,(a^3-a)\log(b^2x^2+2\,abx+a^2+1)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(b^3*x^3 - 3*a*b^2*x^2 + 3*(3*a^2 - 1)*b*x - 3*(b^4*x^4 - a^4 + 6*a^2
 - 1)*arctan(b*x + a) - 6*(a^3 - a)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

Sympy [A]
time = 0.42, size = 155, normalized size = 1.46

$$\begin{cases} -\frac{a^4 \operatorname{atan}(a+bx)}{4b^4} + \frac{a^3 \log(a^2+2abx+b^2x^2+1)}{2b^4} - \frac{3a^2x}{4b^3} + \frac{3a^2 \operatorname{atan}(a+bx)}{2b^4} + \frac{ax^2}{4b^2} - \frac{a \log(a^2+2abx+b^2x^2+1)}{2b^4} + \frac{x^4 \operatorname{atan}(a+bx)}{4} - \frac{x^3}{12b} + \frac{x}{4b^3} - \frac{\operatorname{atan}(a+bx)}{4b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atan}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**4*atan(a + b*x)/(4*b**4) + a**3*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(2*b**4) - 3*a**2*x/(4*b**3) + 3*a**2*atan(a + b*x)/(2*b**4) + a*x*
*2/(4*b**2) - a*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*b**4) + x**4*atan(a
+ b*x)/4 - x**3/(12*b) + x/(4*b**3) - atan(a + b*x)/(4*b**4), Ne(b, 0)), (x
**4*atan(a)/4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.59, size = 133, normalized size = 1.25

$$\frac{x^4 \operatorname{atan}(a+b\,x)}{4} - \frac{\operatorname{atan}(a+b\,x)}{4\,b^4} + \frac{x}{4\,b^3} - \frac{x^3}{12\,b} + \frac{a^3 \ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2\,b^4} + \frac{3\,a^2 \operatorname{atan}(a+b\,x)}{2\,b^4} - \frac{a^4 \operatorname{atan}(a+b\,x)}{4\,b^4} + \frac{a\,x^2}{4\,b^2} - \frac{3\,a^2\,x}{4\,b^3} - \frac{a \ln(a^2+2\,a\,b\,x+b^2\,x^2+1)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atan(a + b*x),x)
```

```
[Out] (x^4*atan(a + b*x))/4 - atan(a + b*x)/(4*b^4) + x/(4*b^3) - x^3/(12*b) + (a
^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4) + (3*a^2*atan(a + b*x))/(2*b^4
) - (a^4*atan(a + b*x))/(4*b^4) + (a*x^2)/(4*b^2) - (3*a^2*x)/(4*b^3) - (a*
log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^4)
```

3.45 $\int x^2 \mathbf{ArcTan}(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}-\frac{a(3-a^2)\,\text{ArcTan}(a+bx)}{3b^3}+\frac{1}{3}x^3\text{ArcTan}(a+bx)+\frac{(1-3a^2)\log\left(1+(a+bx)^2\right)}{6b^3}$$

```
[Out] a*x/b^2-1/6*(b*x+a)^2/b^3-1/3*a*(-a^2+3)*arctan(b*x+a)/b^3+1/3*x^3*arctan(b
*x+a)+1/6*(-3*a^2+1)*ln(1+(b*x+a)^2)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$-\frac{a(3-a^2)\,\text{ArcTan}(a+bx)}{3b^3}+\frac{(1-3a^2)\log\left((a+bx)^2+1\right)}{6b^3}+\frac{1}{3}x^3\text{ArcTan}(a+bx)-\frac{(a+bx)^2}{6b^3}+\frac{ax}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[a + b*x],x]
```

```
[Out] (a*x)/b^2 - (a + b*x)^2/(6*b^3) - (a*(3 - a^2)*ArcTan[a + b*x])/(3*b^3) + (
x^3*ArcTan[a + b*x])/3 + ((1 - 3*a^2)*Log[1 + (a + b*x)^2])/(6*b^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\tan^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1+x^2}\,dx,x,a+bx\right)\\ &= \frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\left(-\frac{3a}{b^3}+\frac{x}{b^3}+\frac{a(3-a^2)-(1-3a^2)\,x}{b^3(1+x^2)}\right)\,dx,x,a-\right.\\ &= \frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{a(3-a^2)-(1-3a^2)x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}\\ &= \frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)+\frac{(1-3a^2)\,\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{3b^3}-\frac{(a}{}\\ &= \frac{ax}{b^2}-\frac{(a+bx)^2}{6b^3}-\frac{a(3-a^2)\tan^{-1}(a+bx)}{3b^3}+\frac{1}{3}x^3\tan^{-1}(a+bx)+\frac{(1-3a^2)\log(1}{6b^3}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 114, normalized size = 1.44

$$\frac{\frac{1}{3}b\left(-\frac{a}{b}+\frac{a+bx}{b}\right)^3\text{ArcTan}(a+bx)-\frac{1}{3}b\left(-\frac{3ax}{b^2}+\frac{(a+bx)^2}{2b^3}-\frac{(1+ia)^3\log(i-a-bx)}{2b^3}-\frac{(1-ia)^3\log(i+a+bx)}{2b^3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[a + b*x],x]
```

```
[Out] ((b*(-(a/b) + (a + b*x)/b)^3*ArcTan[a + b*x])/3 - (b*((-3*a*x)/b^2 + (a + b
*x)^2/(2*b^3) - ((1 + I*a)^3*Log[I - a - b*x])/(2*b^3) - ((1 - I*a)^3*Log[I
 + a + b*x])/(2*b^3)))/3)/b
```

Maple [A]
time = 0.05, size = 113, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+}{}}{b^3}$
default	$\frac{-\frac{\arctan(bx+a)a^3}{3}+\arctan(bx+a)a^2(bx+a)-\arctan(bx+a)a(bx+a)^2+\frac{\arctan(bx+a)(bx+a)^3}{3}+(bx+a)a-\frac{(bx+a)^2}{6}+\frac{(-3a^2+}{}}{b^3}$
risch	$-\frac{ix^3\ln(1+i(bx+a))}{6}+\frac{ix^3\ln(1-i(bx+a))}{6}+\frac{a^3\arctan(bx+a)}{3b^3}-\frac{x^2}{6b}-\frac{a^2\ln(b^2x^2+2abx+a^2+1)}{2b^3}+\frac{2ax}{3b^2}-\frac{a\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arctan(b*x+a)*a^3+arctan(b*x+a)*a^2*(b*x+a)-arctan(b*x+a)*a*(b*
x+a)^2+1/3*arctan(b*x+a)*(b*x+a)^3+(b*x+a)*a-1/6*(b*x+a)^2+1/6*(-3*a^2+1)*l
n(1+(b*x+a)^2)+1/3*(a^3-3*a)*arctan(b*x+a))
```

Maxima [A]
time = 0.51, size = 85, normalized size = 1.08

$$\frac{1}{3}x^3\arctan(bx+a)-\frac{1}{6}b\left(\frac{bx^2-4\,ax}{b^3}-\frac{2\,(a^3-3\,a)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4}+\frac{(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(b*x + a) - 1/6*b*((b*x^2 - 4*a*x)/b^3 - 2*(a^3 - 3*a)*arctan
((b^2*x + a*b)/b)/b^4 + (3*a^2 - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4)
```

Fricas [A]
time = 1.97, size = 66, normalized size = 0.84

$$-\frac{b^2x^2-4\,abx-2\,(b^3x^3+a^3-3\,a)\arctan(bx+a)+(3\,a^2-1)\log(b^2x^2+2\,abx+a^2+1)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*x^2 - 4*a*b*x - 2*(b^3*x^3 + a^3 - 3*a)*arctan(b*x + a) + (3*a^2
- 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A]
time = 0.30, size = 117, normalized size = 1.48

$$\begin{cases}\frac{a^3\operatorname{atan}(a+bx)}{3b^3}-\frac{a^2\log(a^2+2abx+b^2x^2+1)}{2b^3}+\frac{2ax}{3b^2}-\frac{a\operatorname{atan}(a+bx)}{b^3}+\frac{x^3\operatorname{atan}(a+bx)}{3}-\frac{x^2}{6b}+\frac{\log(a^2+2abx+b^2x^2+1)}{6b^3} & \text{for } b\neq 0\\ \frac{x^3\operatorname{atan}(a)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(b*x+a),x)
```

```
[Out] Piecewise((a**3*atan(a + b*x)/(3*b**3) - a**2*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)/(2*b**3) + 2*a*x/(3*b**2) - a*atan(a + b*x)/b**3 + x**3*atan(a + b*x
)/3 - x**2/(6*b) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*b**3), Ne(b, 0)),
 (x**3*atan(a)/3, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 0.86, size = 102, normalized size = 1.29

$$\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{6\,b^3}+\frac{x^3\,\mathrm{atan}(a+b\,x)}{3}-\frac{x^2}{6\,b}-\frac{a^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,b^3}+\frac{a^3\,\mathrm{atan}(a+b\,x)}{3\,b^3}-\frac{a\,\mathrm{atan}(a+b\,x)}{b^3}+\frac{2\,a\,x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(a + b*x),x)
```

```
[Out] log(a^2 + b^2*x^2 + 2*a*b*x + 1)/(6*b^3) + (x^3*atan(a + b*x))/3 - x^2/(6*b
) - (a^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*b^3) + (a^3*atan(a + b*x))/(3
*b^3) - (a*atan(a + b*x))/b^3 + (2*a*x)/(3*b^2)
```

3.46 $\int x\mathbf{ArcTan}(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{x}{2b}+\frac{(1-a^2)\,\text{ArcTan}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{ArcTan}(a+bx)+\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}$$

```
[Out] -1/2*x/b+1/2*(-a^2+1)*arctan(b*x+a)/b^2+1/2*x^2*arctan(b*x+a)+1/2*a*ln(1+(b
*x+a)^2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.750$, Rules used = {5155, 4972, 716, 649, 209, 266}

$$\frac{(1-a^2)\,\text{ArcTan}(a+bx)}{2b^2}+\frac{1}{2}x^2\text{ArcTan}(a+bx)+\frac{a\log\left((a+bx)^2+1\right)}{2b^2}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[a + b*x],x]
```

```
[Out] -1/2*x/b + ((1 - a^2)*ArcTan[a + b*x])/(2*b^2) + (x^2*ArcTan[a + b*x])/2 +
(a*Log[1 + (a + b*x)^2])/(2*b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 4972

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Dist[b*(
c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\tan^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1+x^2}\,dx,x,a+bx\right)\\
&= \frac{1}{2}x^2\tan^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\left(\frac{1}{b^2}-\frac{1-a^2+2ax}{b^2(1+x^2)}\right)\,dx,x,a+bx\right)\\
&= -\frac{x}{2b}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{\text{Subst}\left(\int\frac{1-a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2b^2}\\
&= -\frac{x}{2b}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{a\text{Subst}\left(\int\frac{x}{1+x^2}\,dx,x,a+bx\right)}{b^2}+\frac{(1-a^2)\,\text{Subst}\left(\int\frac{1}{1+x^2}\,d\right.}{2b^2}\\
&= -\frac{x}{2b}+\frac{(1-a^2)\tan^{-1}(a+bx)}{2b^2}+\frac{1}{2}x^2\tan^{-1}(a+bx)+\frac{a\log\left(1+(a+bx)^2\right)}{2b^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 90, normalized size = 1.50

$$\frac{-2bx+2b^2x^2\text{ArcTan}(a+bx)+i(-i+a)^2\log(i-a-bx)+i\log(i+a+bx)+2a\log(i+a+bx)-ia^2\log(i+a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[a + b*x],x]
```

```
[Out] (-2*b*x + 2*b^2*x^2*ArcTan[a + b*x] + I*(-I + a)^2*Log[I - a - b*x] + I*Log
[I + a + b*x] + 2*a*Log[I + a + b*x] - I*a^2*Log[I + a + b*x])/(4*b^2)
```

Maple [A]
time = 0.04, size = 63, normalized size = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2}-\arctan(bx+a)(bx+a)a-\frac{bx}{2}-\frac{a}{2}+\frac{a\ln\left(1+(bx+a)^2\right)}{2}+\frac{\arctan(bx+a)}{2}}{b^2}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)^2}{2}-\arctan(bx+a)(bx+a)a-\frac{bx}{2}-\frac{a}{2}+\frac{a\ln\left(1+(bx+a)^2\right)}{2}+\frac{\arctan(bx+a)}{2}}{b^2}$
risch	$-\frac{ix^2\ln(1+i(bx+a))}{4}+\frac{ix^2\ln(1-i(bx+a))}{4}-\frac{a^2\arctan(bx+a)}{2b^2}+\frac{a\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}-\frac{x}{2b}+\frac{\arctan(bx+a}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arctan(b*x+a)*(b*x+a)^2-arctan(b*x+a)*(b*x+a)*a-1/2*b*x-1/2*a+1/
2*a*ln(1+(b*x+a)^2)+1/2*arctan(b*x+a))
```

Maxima [A]
time = 0.47, size = 68, normalized size = 1.13

$$\frac{1}{2}x^2\arctan(bx+a)-\frac{1}{2}b\left(\frac{x}{b^2}+\frac{(a^2-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3}-\frac{a\log(b^2x^2+2abx+a^2+1)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(b*x + a) - 1/2*b*(x/b^2 + (a^2 - 1)*arctan((b^2*x + a*b)/b)/
b^3 - a*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^3)
```

Fricas [A]
time = 2.47, size = 52, normalized size = 0.87

$$-\frac{bx-(b^2x^2-a^2+1)\arctan(bx+a)-a\log(b^2x^2+2abx+a^2+1)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b^2*x^2 - a^2 + 1)*arctan(b*x + a) - a*log(b^2*x^2 + 2*a*b*x +
 a^2 + 1))/b^2
```

Sympy [A]
time = 0.20, size = 78, normalized size = 1.30

$$\begin{cases}-\frac{a^2\operatorname{atan}(a+bx)}{2b^2}+\frac{a\log(a^2+2abx+b^2x^2+1)}{2b^2}+\frac{x^2\operatorname{atan}(a+bx)}{2}-\frac{x}{2b}+\frac{\operatorname{atan}(a+bx)}{2b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{atan}(a)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(b*x+a),x)
```

```
[Out] Piecewise((-a**2*atan(a + b*x)/(2*b**2) + a*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)/(2*b**2) + x**2*atan(a + b*x)/2 - x/(2*b) + atan(a + b*x)/(2*b**2), Ne
(b, 0)), (x**2*atan(a)/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.97, size = 61, normalized size = 1.02

$$\frac{x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{\frac{\mathrm{atan}(a+b\,x)}{2}-\frac{b\,x}{2}-\frac{a^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a + b*x),x)
```

```
[Out] (x^2*atan(a + b*x))/2 + (atan(a + b*x)/2 - (b*x)/2 - (a^2*atan(a + b*x))/2
+ (a*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2)/b^2
```

3.47 $\int \mathbf{ArcTan}(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{(a+bx)\text{ArcTan}(a+bx)}{b}-\frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] (b*x+a)*arctan(b*x+a)/b-1/2*ln(1+(b*x+a)^2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5147, 4930, 266}

$$\frac{(a+bx)\text{ArcTan}(a+bx)}{b}-\frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x])/b - Log[1 + (a + b*x)^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5147

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d,
 Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \tan^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \tan^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\tan^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\tan^{-1}(a+bx)}{b} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.18

$$-\frac{-2(a+bx)\text{ArcTan}(a+bx)+\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x],x]
```

```
[Out] -1/2*(-2*(a + b*x)*ArcTan[a + b*x] + Log[1 + a^2 + 2*a*b*x + b^2*x^2])/b
```

Maple [A]
time = 0.03, size = 30, normalized size = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
default	$\frac{(bx+a)\arctan(bx+a)-\frac{\ln\left(1+(bx+a)^2\right)}{2}}{b}$	30
risch	$-\frac{ix\ln(1+i(bx+a))}{2}+\frac{ix\ln(1-i(bx+a))}{2}+\frac{a\arctan(bx+a)}{b}-\frac{\ln(b^2x^2+2abx+a^2+1)}{2b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arctan(b*x+a)-1/2*ln(1+(b*x+a)^2))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.94

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left((bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

Fricas [A]
time = 2.45, size = 39, normalized size = 1.18

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left(b^{2}x^{2}+2\,abx+a^{2}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log(b^2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [A]
time = 0.14, size = 46, normalized size = 1.39

$$\begin{cases}\frac{a\operatorname{atan}\left(a+bx\right)}{b}+x\operatorname{atan}\left(a+bx\right)-\frac{\log\left(a^{2}+2abx+b^{2}x^{2}+1\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{atan}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a),x)
[Out] Piecewise((a*atan(a + b*x)/b + x*atan(a + b*x) - log(a**2 + 2*a*b*x + b**2*
x**2 + 1)/(2*b), Ne(b, 0)), (x*atan(a), True))
```

Giac [A]
time = 0.39, size = 31, normalized size = 0.94

$$\frac{2\,(bx+a)\arctan\left(bx+a\right)-\log\left((bx+a)^{2}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a),x, algorithm="giac")
[Out] 1/2*(2*(b*x + a)*arctan(b*x + a) - log((b*x + a)^2 + 1))/b
```

Mupad [B]
time = 0.45, size = 42, normalized size = 1.27

$$x\,\mathrm{atan}(a+b\,x)-\frac{\ln\left(a^{2}+2\,a\,b\,x+b^{2}\,x^{2}+1\right)-2\,a\,\mathrm{atan}(a+b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x),x)
[Out] x*atan(a + b*x) - (log(a^2 + b^2*x^2 + 2*a*b*x + 1) - 2*a*atan(a + b*x))/(2
*b)
```

3.48 $\int \frac{\text{ArcTan}(a+bx)}{x} \, dx$

Optimal. Leaf size=120

$$-\text{ArcTan}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\text{ArcTan}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{}{1-}\right.$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))+arctan(b*x+a)*ln(2*b*x/(I-a)/(1-I*(b*x+a
)))+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))-1/2*I*polylog(2,1-2*b*x/(I-a)/(1-I*(
b*x+a)))
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5155, 4966, 2449, 2352, 2497}

$$-\text{ArcTan}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\text{ArcTan}(a+bx)\log\left(\frac{2bx}{(-a+i)(1-i(a+bx))}\right)+\frac{1}{2}i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)-\frac{1}{2}i\text{Li}_2\left(1-\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x,x]
```

```
[Out] -(ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))]) + ArcTan[a + b*x]*Log[(2*b*x)/(
(I - a)*(1 - I*(a + b*x)))] + (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*x))] - (
I/2)*PolyLog[2, 1 - (2*b*x)/((I - a)*(1 - I*(a + b*x)))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

$$= -\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)+\tan^{-1}(a+bx)\log\left(\frac{2bx}{(i-a)(1-i(a+bx))}\right)$$

Mathematica [A]
time = 0.01, size = 171, normalized size = 1.42

$$-\frac{1}{2}i\log(1+i(a+bx))\log\left(\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}-\frac{ia}{b}}\right)+\frac{1}{2}i\log(1-i(a+bx))\log\left(-\frac{i\left(-\frac{a}{b}+\frac{a+bx}{b}\right)}{-\frac{1}{b}+\frac{ia}{b}}\right)+\frac{1}{2}i\text{PolyLog}\left(2,\frac{i(1-i(a+bx))}{i+a}\right)-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{i(1+i(a+bx))}{-i+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x,x]
```

```
[Out] (-1/2*I)*Log[1 + I*(a + b*x)]*Log[(I*(-(a/b) + (a + b*x)/b))/(-b^(-1) - (I*
a)/b)] + (I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*(-(a/b) + (a + b*x)/b))/(-b^(
-1) + (I*a)/b)] + (I/2)*PolyLog[2, (I*(1 - I*(a + b*x)))/(I + a)] - (I/2)*P
olyLog[2, ((-I)*(1 + I*(a + b*x)))/(-I + a)]
```

Maple [A]
time = 0.05, size = 106, normalized size = 0.88

method	result
risch	$\frac{i\,\mathrm{dilog}\left(-\frac{ixb}{ia-1}\right)}{2}+\frac{i\ln(-ibx-ia+1)\ln\left(-\frac{ixb}{ia-1}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{ixb}{-ia-1}\right)}{2}-\frac{i\ln(ibx+ia+1)\ln\left(\frac{ixb}{-ia-1}\right)}{2}$
derivativedivides	$\ln(-bx)\arctan(bx+a)-\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\,\mathrm{dilog}(}{}$
default	$\ln(-bx)\arctan(bx+a)-\frac{i\ln(-bx)\ln\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\ln(-bx)\ln\left(\frac{-bx-a+i}{i-a}\right)}{2}-\frac{i\,\mathrm{dilog}\left(\frac{bx+a+i}{i+a}\right)}{2}+\frac{i\,\mathrm{dilog}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-b*x)*arctan(b*x+a)-1/2*I*ln(-b*x)*ln((I+a+b*x)/(I+a))+1/2*I*ln(-b*x)*ln
((I-a-b*x)/(I-a))-1/2*I*dilog((I+a+b*x)/(I+a))+1/2*I*dilog((I-a-b*x)/(I-a))
```

Maxima [A]
time = 0.51, size = 134, normalized size = 1.12

$$-\frac{1}{2}\arctan\left(\frac{bx}{a^2+1},-\frac{abx}{a^2+1}\right)\log\left(b^2x^2+2\,abx+a^2+1\right)+\frac{1}{2}\arctan(bx+a)\log\left(\frac{b^2x^2}{a^2+1}\right)+\arctan(bx+a)\log(x)-\arctan\left(\frac{b^2x+ab}{b}\right)\log(x)-\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a+1}{i\,a+1}\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{i\,bx+i\,a-1}{i\,a-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*arctan2(b*x/(a^2 + 1), -a*b*x/(a^2 + 1))*log(b^2*x^2 + 2*a*b*x + a^2 +
 1) + 1/2*arctan(b*x + a)*log(b^2*x^2/(a^2 + 1)) + arctan(b*x + a)*log(x) -
 arctan((b^2*x + a*b)/b)*log(x) - 1/2*I*dilog((I*b*x + I*a + 1)/(I*a + 1))
+ 1/2*I*dilog((I*b*x + I*a - 1)/(I*a - 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x,x)
```

```
[Out] int(atan(a + b*x)/x, x)
```

3.49 $\int \frac{\text{ArcTan}(a+bx)}{x^2} \, dx$

Optimal. Leaf size=62

$$-\frac{ab\text{ArcTan}(a+bx)}{1+a^2}-\frac{\text{ArcTan}(a+bx)}{x}+\frac{b\log(x)}{1+a^2}-\frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}$$

```
[Out] -a*b*arctan(b*x+a)/(a^2+1)-arctan(b*x+a)/x+b*ln(x)/(a^2+1)-1/2*b*ln(1+(b*x+
a)^2)/(a^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5153, 378, 720, 31, 649, 209, 266}

$$-\frac{ab\text{ArcTan}(a+bx)}{a^2+1}+\frac{b\log(x)}{a^2+1}-\frac{b\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)}-\frac{\text{ArcTan}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -((a*b*ArcTan[a + b*x])/(1 + a^2)) - ArcTan[a + b*x]/x + (b*Log[x])/(1 + a^
2) - (b*Log[1 + (a + b*x)^2])/(2*(1 + a^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{x^2}\,dx &= -\frac{\tan^{-1}(a+bx)}{x} + b\int \frac{1}{x\left(1+(a+bx)^2\right)}\,dx \\
&= -\frac{\tan^{-1}(a+bx)}{x} + b\text{Subst}\left(\int \frac{1}{(-a+x)\left(1+x^2\right)}\,dx, x, a+bx\right) \\
&= -\frac{\tan^{-1}(a+bx)}{x} + \frac{b\text{Subst}\left(\int \frac{1}{-a+x}\,dx, x, a+bx\right)}{1+a^2} + \frac{b\text{Subst}\left(\int \frac{-a-x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2} \\
&= -\frac{\tan^{-1}(a+bx)}{x} + \frac{b\log(x)}{1+a^2} - \frac{b\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{1+a^2} - \frac{(ab)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, \right.}{1+a^2} \\
&= -\frac{ab\tan^{-1}(a+bx)}{1+a^2} - \frac{\tan^{-1}(a+bx)}{x} + \frac{b\log(x)}{1+a^2} - \frac{b\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 67, normalized size = 1.08

$$
-\frac{\text{ArcTan}(a+bx)}{x} + \frac{b(2\log(x) + i(i+a)\log(i-a-bx) + (-1-ia)\log(i+a+bx))}{2\left(1+a^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^2,x]
```

```
[Out] -(ArcTan[a + b*x]/x) + (b*(2*Log[x] + I*(I + a)*Log[I - a - b*x] + (-1 - I*
a)*Log[I + a + b*x]))/(2*(1 + a^2))
```

Maple [A]
time = 0.06, size = 61, normalized size = 0.98

method	result
derivativedivides	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+\arctan(bx+a)a}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
default	$b\left(-\frac{\arctan(bx+a)}{bx}-\frac{\frac{\ln\left(1+(bx+a)^2\right)}{2}+\arctan(bx+a)a}{a^2+1}+\frac{\ln(-bx)}{a^2+1}\right)$
risch	$\frac{i\ln(1+i(bx+a))}{2x}-\frac{i\left(a^2\ln(1-i(bx+a))+\ln(1-i(bx+a))-i\ln\left(\left(-a^2b+3iab\right)x-3a+2ia^2-a^3\right)bx+\ln\left(\left(-a^2b+3iab\right)x-3a+\right.\right.}{2x(i\cdot}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arctan(b*x+a)/b/x-1/(a^2+1)*(1/2*ln(1+(b*x+a)^2)+arctan(b*x+a)*a)+1/(a^
2+1)*ln(-b*x))
```

Maxima [A]
time = 0.49, size = 77, normalized size = 1.24

$$-\frac{1}{2}\,b\left(\frac{2\,a\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1}+\frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1}-\frac{2\,\log\left(x\right)}{a^2+1}\right)-\frac{\arctan\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*a*arctan((b^2*x + a*b)/b)/(a^2 + 1) + log(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(a^2 + 1) - 2*log(x)/(a^2 + 1)) - arctan(b*x + a)/x
```

Fricas [A]
time = 2.10, size = 57, normalized size = 0.92

$$-\frac{bx\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,bx\log\left(x\right)+2\left(abx+a^2+1\right)\arctan\left(bx+a\right)}{2\left(a^2+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*b*x*log(x) + 2*(a*b*x + a^2
+ 1)*arctan(b*x + a))/((a^2 + 1)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 168, normalized size = 2.71

$$\begin{cases} -\frac{ib\,\mathrm{atan}\,(bx-i)}{2} - \frac{\mathrm{atan}\,(bx-i)}{x} - \frac{i}{2x} & \text{for } a = -i \\ \frac{ib\,\mathrm{atan}\,(bx+i)}{2} - \frac{\mathrm{atan}\,(bx+i)}{x} + \frac{i}{2x} & \text{for } a = i \\ -\frac{2a^2\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} - \frac{2abx\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} + \frac{2bx\log(x)}{2a^2x+2x} - \frac{bx\log(a^2+2abx+b^2x^2+1)}{2a^2x+2x} - \frac{2\,\mathrm{atan}\,(a+bx)}{2a^2x+2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**2,x)
```

```
[Out] Piecewise((-I*b*atan(b*x - I)/2 - atan(b*x - I)/x - I/(2*x), Eq(a, -I)), (I
*b*atan(b*x + I)/2 - atan(b*x + I)/x + I/(2*x), Eq(a, I)), (-2*a**2*atan(a
+ b*x)/(2*a**2*x + 2*x) - 2*a*b*x*atan(a + b*x)/(2*a**2*x + 2*x) + 2*b*x*lo
g(x)/(2*a**2*x + 2*x) - b*x*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a**2*x +
 2*x) - 2*atan(a + b*x)/(2*a**2*x + 2*x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.04, size = 63, normalized size = 1.02

$$-\frac{\mathrm{atan}(a+b\,x)}{x}-\frac{\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2}-b\,x\,\ln\left(x\right)+a\,b\,x\,\mathrm{atan}(a+b\,x)}{x\,\left(a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^2,x)
```

```
[Out] - atan(a + b*x)/x - ((b*x*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/2 - b*x*log(x)
+ a*b*x*atan(a + b*x))/(x*(a^2 + 1))
```

3.50 $\int \frac{\operatorname{ArcTan}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=96

$$-\frac{b}{2\left(1+a^2\right)x}-\frac{\left(1-a^2\right)b^2\operatorname{ArcTan}(a+bx)}{2\left(1+a^2\right)^2}-\frac{\operatorname{ArcTan}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{ab^2\log\left(1+(a+bx)^2\right)}{2\left(1+a^2\right)^2}$$

```
[Out] -1/2*b/(a^2+1)/x-1/2*(-a^2+1)*b^2*arctan(b*x+a)/(a^2+1)^2-1/2*arctan(b*x+a)
/x^2-a*b^2*ln(x)/(a^2+1)^2+1/2*a*b^2*ln(1+(b*x+a)^2)/(a^2+1)^2
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5153, 378, 724, 815, 649, 209, 266}

$$-\frac{\left(1-a^2\right)b^2\operatorname{ArcTan}(a+bx)}{2\left(a^2+1\right)^2}-\frac{ab^2\log(x)}{\left(a^2+1\right)^2}+\frac{ab^2\log\left((a+bx)^2+1\right)}{2\left(a^2+1\right)^2}-\frac{b}{2\left(a^2+1\right)x}-\frac{\operatorname{ArcTan}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^3,x]
```

```
[Out] -1/2*b/((1 + a^2)*x) - ((1 - a^2)*b^2*ArcTan[a + b*x])/(2*(1 + a^2)^2) - Ar
cTan[a + b*x]/(2*x^2) - (a*b^2*Log[x])/(1 + a^2)^2 + (a*b^2*Log[1 + (a + b*
x)^2])/(2*(1 + a^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{x^3}\,dx &= -\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2\left(1+(a+bx)^2\right)}\,dx\\
&= -\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)(1+x^2)}\,dx,x,a+bx\right)}{2\left(1+a^2\right)}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}+\frac{b^2\text{Subst}\left(\int \left(\frac{2a}{(1+a^2)(a-x)}+\frac{-1+a^2+2ax}{(1+a^2)(1+x^2)}\right)\,dx,x,a+b
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{b^2\text{Subst}\left(\int \frac{-1+a^2+2ax}{1+x^2}\,dx,x,a+bx\right)}{2\left(1+a^2\right)^2}\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{\left(ab^2\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,a+bx\right)}{\left(1+a^2\right)^2}-\\
&= -\frac{b}{2\left(1+a^2\right)x}-\frac{\left(1-a^2\right)b^2\tan^{-1}(a+bx)}{2\left(1+a^2\right)^2}-\frac{\tan^{-1}(a+bx)}{2x^2}-\frac{ab^2\log(x)}{\left(1+a^2\right)^2}+\frac{ab^2\log(}{2(}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 92, normalized size = 0.96

$$\frac{-2\text{ArcTan}(a+bx)+\frac{bx(-4abx\log(x)-i(i+a)^2bx\log(i-a-bx)+(-i+a)(-2(i+a)+(1+ia)bx\log(i+a+bx)))}{(1+a^2)^2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^3,x]
```

```
[Out] (-2*ArcTan[a + b*x] + (b*x*(-4*a*b*x*Log[x] - I*(I + a)^2*b*x*Log[I - a - b
*x] + (-I + a)*(-2*(I + a) + (1 + I*a)*b*x*Log[I + a + b*x])))/(1 + a^2)^2)
/(4*x^2)
```

Maple [A]
time = 0.09, size = 84, normalized size = 0.88

method	result
derivativedivides	$b^2\left(-\frac{\arctan(bx+a)}{2b^2x^2}+\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}-\frac{1}{2(a^2+1)bx}-\frac{a\ln(-bx)}{(a^2+1)^2}\right)$
default	$b^2\left(-\frac{\arctan(bx+a)}{2b^2x^2}+\frac{a\ln\left(1+(bx+a)^2\right)+(a^2-1)\arctan(bx+a)}{2(a^2+1)^2}-\frac{1}{2(a^2+1)bx}-\frac{a\ln(-bx)}{(a^2+1)^2}\right)$
risch	$\frac{i\ln(1+i(bx+a))}{4x^2}-\frac{i\left(a^4\ln(1-i(bx+a))+2a^2\ln(1-i(bx+a))+\ln(1-i(bx+a))-\ln\left((a^6b-4ia^5b+9a^4b+8ia^3b-9a^2b-4iab\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arctan(b*x+a)/b^2/x^2+1/2/(a^2+1)^2*(a*ln(1+(b*x+a)^2)+(a^2-1)*ar
ctan(b*x+a))-1/2/(a^2+1)/b/x-1/(a^2+1)^2*a*ln(-b*x))
```

Maxima [A]
time = 0.47, size = 112, normalized size = 1.17

$$\frac{1}{2}\left(\frac{(a^2-1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4+2a^2+1}+\frac{ab\log\left(b^2x^2+2abx+a^2+1\right)}{a^4+2a^2+1}-\frac{2ab\log(x)}{a^4+2a^2+1}-\frac{1}{(a^2+1)x}\right)b-\frac{\arctan(bx+a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + a*b*log(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*a*b*log(x)/(a^4 + 2*a^2 + 1) -
 1/((a^2 + 1)*x))*b - 1/2*arctan(b*x + a)/x^2
```

Fricas [A]
time = 2.30, size = 95, normalized size = 0.99

$$\frac{ab^2x^2\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,ab^2x^2\log\left(x\right)-(a^2+1)bx+\left((a^2-1)b^2x^2-a^4-2\,a^2-1\right)\arctan\left(bx+a\right)}{2\,(a^4+2\,a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a*b^2*x^2*log(x) - (a^2
 + 1)*b*x + ((a^2 - 1)*b^2*x^2 - a^4 - 2*a^2 - 1)*arctan(b*x + a))/((a^4 +
2*a^2 + 1)*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 382, normalized size = 3.98

$$\begin{cases} -\frac{b^2\operatorname{atan}(bx-i)}{8}-\frac{b}{8x}-\frac{\operatorname{atan}(bx-i)}{2x^2}-\frac{i}{8x^2} & \text{for } a=-i \\ -\frac{b^2\operatorname{atan}(bx+i)}{8}-\frac{b}{8x}-\frac{\operatorname{atan}(bx+i)}{2x^2}+\frac{i}{8x^2} & \text{for } a=i \\ -\frac{a^4\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}-\frac{a^2bx}{2a^4x^2+4a^2x^2+2x^2}-\frac{2a^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}-\frac{2ab^2x^2\log(x)}{2a^4x^2+4a^2x^2+2x^2}+\frac{ab^2x^2\log(a^2+2abx+b^2x^2+1)}{2a^4x^2+4a^2x^2+2x^2}-\frac{b^2x^2\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2}-\frac{bx}{2a^4x^2+4a^2x^2+2x^2}-\frac{\operatorname{atan}(a+bx)}{2a^4x^2+4a^2x^2+2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**3,x)
```

```
[Out] Piecewise((-b**2*atan(b*x - I)/8 - b/(8*x) - atan(b*x - I)/(2*x**2) - I/(8*
x**2), Eq(a, -I)), (-b**2*atan(b*x + I)/8 - b/(8*x) - atan(b*x + I)/(2*x**2
) + I/(8*x**2), Eq(a, I)), (-a**4*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2
+ 2*x**2) + a**2*b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**
2) - a**2*b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a**2*atan(a + b*x)/(
2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) + a*b**2*x**2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(2*a
**4*x**2 + 4*a**2*x**2 + 2*x**2) - b**2*x**2*atan(a + b*x)/(2*a**4*x**2 + 4
*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2) - atan(a +
b*x)/(2*a**4*x**2 + 4*a**2*x**2 + 2*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.22, size = 232, normalized size = 2.42

$$\frac{a\,b^2\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}{2\,{\left(a^2+1\right)}^2}-\frac{\frac{b\,x}{2}+\mathrm{atan}(a+b\,x)\,\left(\frac{a^2}{2}+\frac{1}{2}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{2}+\frac{x^3\,\left(b^3-3\,a^2\,b^3\right)}{2\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^4\,x^4}{{\left(a^2+1\right)}^2}+a\,b\,x\,\mathrm{atan}(a+b\,x)}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4+x^2}-\frac{\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(b^3-a^2\,b^3\right)}{\sqrt{b^2}\,\left(2\,a^4+4\,a^2+2\right)}-\frac{a\,b^2\,\ln\left(x\right)}{{\left(a^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^3,x)
```

```
[Out] (a*b^2*log(a^2 + b^2*x^2 + 2*a*b*x + 1))/(2*(a^2 + 1)^2) - ((b*x)/2 + atan(
a + b*x)*(a^2/2 + 1/2) + (b^2*x^2*atan(a + b*x))/2 + (x^3*(b^3 - 3*a^2*b^3)
)/(2*(2*a^2 + a^4 + 1)) - (a*b^4*x^4)/(a^2 + 1)^2 + a*b*x*atan(a + b*x))/(x
^2 + a^2*x^2 + b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2
+ 1) - a^2*b^2)^(1/2)))*(b^3 - a^2*b^3))/((b^2)^(1/2)*(4*a^2 + 2*a^4 + 2))
- (a*b^2*log(x))/(a^2 + 1)^2
```

3.51 $\int \frac{\text{ArcTan}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=129

$$-\frac{b}{6\left(1+a^2\right)x^2}+\frac{2ab^2}{3\left(1+a^2\right)^2x}+\frac{a\left(3-a^2\right)b^3\text{ArcTan}(a+bx)}{3\left(1+a^2\right)^3}-\frac{\text{ArcTan}(a+bx)}{3x^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3}+\frac{(1-3a$$

```
[Out] -1/6*b/(a^2+1)/x^2+2/3*a*b^2/(a^2+1)^2/x+1/3*a*(-a^2+3)*b^3*arctan(b*x+a)/(
a^2+1)^3-1/3*arctan(b*x+a)/x^3-1/3*(-3*a^2+1)*b^3*ln(x)/(a^2+1)^3+1/6*(-3*a
^2+1)*b^3*ln(1+(b*x+a)^2)/(a^2+1)^3
```

Rubi [A]

time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5153, 378, 724, 815, 649, 209, 266}

$$\frac{a\left(3-a^2\right)b^3\text{ArcTan}(a+bx)}{3\left(a^2+1\right)^3}-\frac{\left(1-3a^2\right)b^3\log(x)}{3\left(a^2+1\right)^3}+\frac{\left(1-3a^2\right)b^3\log\left((a+bx)^2+1\right)}{6\left(a^2+1\right)^3}+\frac{2ab^2}{3\left(a^2+1\right)^2x}-\frac{b}{6\left(a^2+1\right)x^2}-\frac{\text{ArcTan}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/x^4,x]
```

```
[Out] -1/6*b/((1 + a^2)*x^2) + (2*a*b^2)/(3*(1 + a^2)^2*x) + (a*(3 - a^2)*b^3*Arc
Tan[a + b*x])/(3*(1 + a^2)^3) - ArcTan[a + b*x]/(3*x^3) - ((1 - 3*a^2)*b^3*
Log[x])/(3*(1 + a^2)^3) + ((1 - 3*a^2)*b^3*Log[1 + (a + b*x)^2])/(6*(1 + a^
2)^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5153

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcTan[c
 + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{x^4}\,dx = -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b\int \frac{1}{x^3\left(1+(a+bx)^2\right)}\,dx$$
$$= -\frac{\tan^{-1}(a+bx)}{3x^3} + \frac{1}{3}b^3\text{Subst}\left(\int \frac{1}{(-a+x)^3\left(1+x^2\right)}\,dx, x, a+bx\right)$$
$$= -\frac{b}{6\left(1+a^2\right)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2(1+x^2)}\,dx, x, a+bx\right)}{3\left(1+a^2\right)}$$
$$= -\frac{b}{6\left(1+a^2\right)x^2} - \frac{\tan^{-1}(a+bx)}{3x^3} + \frac{b^3\text{Subst}\left(\int \left(-\frac{2a}{(1+a^2)(a-x)^2} + \frac{1-3a^2}{(1+a^2)^2(a-x)} + \frac{a(3-a^2)}{(1+a^2}\right.\right.}{3\left(1+a^2\right)}$$
$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3} + \frac{b^3\text{Subst}\left(\int\right.}{}$$
$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2x} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{\left(1-3a^2\right)b^3\log(x)}{3\left(1+a^2\right)^3} + \frac{\left(\left(1-3a^2\right)\right.}{}$$
$$= -\frac{b}{6\left(1+a^2\right)x^2} + \frac{2ab^2}{3\left(1+a^2\right)^2x} + \frac{a\left(3-a^2\right)b^3\tan^{-1}(a+bx)}{3\left(1+a^2\right)^3} - \frac{\tan^{-1}(a+bx)}{3x^3} - \frac{(1}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 128, normalized size = 0.99

$$\frac{-2\left(1+a^2\right)^3\text{ArcTan}(a+bx)+2\left(-1+3a^2\right)b^3x^3\log(x)+i(i+a)^3b^3x^3\log(i-a-bx)-(-i+a)bx\left((i+a)\left(1+a^2-4abx\right)+i(-i+a)^2b^2x^2\log(i+a+bx)\right)}{6\left(1+a^2\right)^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/x^4,x]
[Out] (-2*(1 + a^2)^3*ArcTan[a + b*x] + 2*(-1 + 3*a^2)*b^3*x^3*Log[x] + I*(I + a)
^3*b^3*x^3*Log[I - a - b*x] - (-I + a)*b*x*((I + a)*(1 + a^2 - 4*a*b*x) + I
*(-I + a)^2*b^2*x^2*Log[I + a + b*x]))/(6*(1 + a^2)^3*x^3)
```

Maple [A]
time = 0.12, size = 115, normalized size = 0.89

method	result
derivativedivides	$b^3\left(-\frac{\arctan(bx+a)}{3b^3x^3} - \frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\arctan(bx+a)}{3(a^2+1)^3} - \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} - \frac{1}{6(a^2+1)b^2x^2} +\right.$
default	$b^3\left(-\frac{\arctan(bx+a)}{3b^3x^3} - \frac{\frac{(3a^2-1)\ln\left(1+(bx+a)^2\right)}{2}+(a^3-3a)\arctan(bx+a)}{3(a^2+1)^3} - \frac{(-3a^2+1)\ln(-bx)}{3(a^2+1)^3} - \frac{1}{6(a^2+1)b^2x^2} +\right.$

risch	$\frac{i\ln(1+i(bx+a))}{6x^3} - \frac{i(a^6\ln(1-i(bx+a))+3a^4\ln(1-i(bx+a))+3a^2\ln(1-i(bx+a))+\ln(1-i(bx+a))-\ln((a^{10}b+5ia^9b+24a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*arctan(b*x+a)/b^3/x^3-1/3/(a^2+1)^3*(1/2*(3*a^2-1)*ln(1+(b*x+a)^2
)+(a^3-3*a)*arctan(b*x+a))-1/3*(-3*a^2+1)/(a^2+1)^3*ln(-b*x)-1/6/(a^2+1)/b^
2/x^2+2/3/(a^2+1)^2*a/b/x)
```

Maxima [A]
time = 0.47, size = 165, normalized size = 1.28

$$-\frac{1}{6}\left(\frac{2\,(a^3-3\,a)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1}+\frac{(3\,a^2-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{2\,(3\,a^2-1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{4\,abx-a^2-1}{(a^4+2\,a^2+1)x^2}\right)b-\frac{\arctan\left(bx+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(a^3 - 3*a)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*a^2 + 1) +
 (3*a^2 - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^6 + 3*a^4 + 3*a^2 + 1)
 - 2*(3*a^2 - 1)*b^2*log(x)/(a^6 + 3*a^4 + 3*a^2 + 1) - (4*a*b*x - a^2 - 1)
/((a^4 + 2*a^2 + 1)*x^2))*b - 1/3*arctan(b*x + a)/x^3
```

Fricas [A]
time = 2.22, size = 135, normalized size = 1.05

$$-\frac{(3\,a^2-1)b^3x^3\log\left(b^2x^2+2\,abx+a^2+1\right)-2\,(3\,a^2-1)b^3x^3\log\left(x\right)-4\,(a^3+a)b^2x^2+(a^4+2\,a^2+1)bx+2\,((a^3-3\,a)b^3x^3+a^6+3\,a^4+3\,a^2+1)\arctan\left(bx+a\right)}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((3*a^2 - 1)*b^3*x^3*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*(3*a^2 - 1)*
b^3*x^3*log(x) - 4*(a^3 + a)*b^2*x^2 + (a^4 + 2*a^2 + 1)*b*x + 2*((a^3 - 3*
a)*b^3*x^3 + a^6 + 3*a^4 + 3*a^2 + 1)*arctan(b*x + a))/((a^6 + 3*a^4 + 3*a^
2 + 1)*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 760, normalized size = 5.89

$$\begin{cases} \frac{ib^3\operatorname{atan}(bx-i)}{24}+\frac{ib^2}{24x}-\frac{b}{24x^2}-\frac{\operatorname{atan}(bx-i)}{3x^3}-\frac{i}{18x^3} & \text{for}\: a=-i \\ -\frac{ib^3\operatorname{atan}(bx+i)}{24}-\frac{ib^2}{24x}-\frac{b}{24x^2}-\frac{\operatorname{atan}(bx+i)}{3x^3}+\frac{i}{18x^3} & \text{for}\: a=i \\ -\frac{2a^6\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{a^4bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^4\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^3b^3x^3\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{4a^3b^2x^2}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{6a^2b^3x^3\log(x)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{3a^2b^3x^3\log(a^2+2abx+b^2x^2+1)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2a^2bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{6a^2\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{6ab^3x^3\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{4ab^2x^2}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2b^3x^3\log(x)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}+\frac{b^3x^3\log(a^2+2abx+b^2x^2+1)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{bx}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3}-\frac{2\operatorname{atan}(a+bx)}{6a^6x^3+18a^4x^3+18a^2x^3+6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/x**4,x)
```

```
[Out] Piecewise((I*b**3*atan(b*x - I)/24 + I*b**2/(24*x) - b/(24*x**2) - atan(b*x
 - I)/(3*x**3) - I/(18*x**3), Eq(a, -I)), (-I*b**3*atan(b*x + I)/24 - I*b**
2/(24*x) - b/(24*x**2) - atan(b*x + I)/(3*x**3) + I/(18*x**3), Eq(a, I)), (
-2*a**6*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
- a**4*b*x/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 6*a**4*at
an(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**3*b
**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3)
 + 4*a**3*b**2*x**2/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) +
6*a**2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3
) - 3*a**2*b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*a**2*b*x/(6*a**6*x**3 + 18*a**4*x**3
 + 18*a**2*x**3 + 6*x**3) - 6*a**2*atan(a + b*x)/(6*a**6*x**3 + 18*a**4*x**
3 + 18*a**2*x**3 + 6*x**3) + 6*a*b**3*x**3*atan(a + b*x)/(6*a**6*x**3 + 18*
a**4*x**3 + 18*a**2*x**3 + 6*x**3) + 4*a*b**2*x**2/(6*a**6*x**3 + 18*a**4*x
**3 + 18*a**2*x**3 + 6*x**3) - 2*b**3*x**3*log(x)/(6*a**6*x**3 + 18*a**4*x*
*3 + 18*a**2*x**3 + 6*x**3) + b**3*x**3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)
/(6*a**6*x**3 + 18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - b*x/(6*a**6*x**3 +
18*a**4*x**3 + 18*a**2*x**3 + 6*x**3) - 2*atan(a + b*x)/(6*a**6*x**3 + 18*a
**4*x**3 + 18*a**2*x**3 + 6*x**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 1.05, size = 288, normalized size = 2.23

$$-\frac{\frac{b\,x}{6}+\mathrm{atan}(a+b\,x)\,\left(\frac{a^2}{3}+\frac{1}{3}\right)+\frac{b^2\,x^2\,\mathrm{atan}(a+b\,x)}{3}+\frac{x^3\,\left(b^3-7\,a^2\,b^3\right)}{6\,\left(a^4+2\,a^2+1\right)}-\frac{a\,b^2\,x^2}{3\,\left(a^2+1\right)}-\frac{2\,a\,b^4\,x^4}{3\,{\left(a^2+1\right)}^2}+\frac{2\,a\,b\,x\,\mathrm{atan}(a+b\,x)}{3}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5+x^3}-\frac{\ln\left(x\right)\,\left(\frac{b^3}{3}-a^2\,b^3\right)}{a^6+3\,a^4+3\,a^2+1}-\frac{b^3\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)\,\left(3\,a^2-1\right)}{6\,\left(a^6+3\,a^4+3\,a^2+1\right)}-\frac{a\,\mathrm{atan}\left(\frac{2\,x\,b^2+2\,a\,b}{2\,\sqrt{b^2\,\left(a^2+1\right)-a^2\,b^2}}\right)\,\left(a^2-3\right)\,{\left(b^2\right)}^{3/2}}{3\,\left(a^6+3\,a^4+3\,a^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/x^4,x)
```

```
[Out] - ((b*x)/6 + atan(a + b*x)*(a^2/3 + 1/3) + (b^2*x^2*atan(a + b*x))/3 + (x^3
*(b^3 - 7*a^2*b^3))/(6*(2*a^2 + a^4 + 1)) - (a*b^2*x^2)/(3*(a^2 + 1)) - (2*
a*b^4*x^4)/(3*(a^2 + 1)^2) + (2*a*b*x*atan(a + b*x))/3)/(x^3 + a^2*x^3 + b^
2*x^5 + 2*a*b*x^4) - (log(x)*(b^3/3 - a^2*b^3))/(3*a^2 + 3*a^4 + a^6 + 1) -
 (b^3*log(a^2 + b^2*x^2 + 2*a*b*x + 1)*(3*a^2 - 1))/(6*(3*a^2 + 3*a^4 + a^6
 + 1)) - (a*atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 + 1) - a^2*b^2)^(1/2)))*(a^
2 - 3)*(b^2)^(3/2))/(3*(3*a^2 + 3*a^4 + a^6 + 1))
```

3.52 $\int \frac{\mathbf{ArcTan}(a+bx)}{c+dx^3}\,dx$

Optimal. Leaf size=863

$$-\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}}+\frac{\sqrt[6]{-1}\,\log(1+ia+i}{}$$

```
[Out] -1/6*I*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/
c^(2/3)/d^(1/3)+1/6*I*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+d^(1/3)*x)/(b*c^(1/3)-(
I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*ln(1+I*a+I*b*x)*ln(b*(c^(1/3)
-(-1)^(1/3)*d^(1/3)*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3
)-1/6*(-1)^(1/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)-(-1)^(1/3)*d^(1/3)*x)/(b*c^(
1/3)+(-1)^(1/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(5/6)*ln(1+I*a+I*b
*x)*ln(b*(c^(1/3)+(-1)^(2/3)*d^(1/3)*x)/(b*c^(1/3)+(-1)^(2/3)*(I-a)*d^(1/3)
))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*ln(1-I*a-I*b*x)*ln(b*(c^(1/3)+(-1)^(2/3)*
d^(1/3)*x)/(b*c^(1/3)+(-1)^(1/6)*(1-I*a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*I*po
lylog(2,d^(1/3)*(I-a-b*x)/(b*c^(1/3)+(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-
1)^(5/6)*polylog(2,-(-1)^(1/6)*d^(1/3)*(I-a-b*x)/(I*b*c^(1/3)-(-1)^(1/6)*(I
-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*(-1)^(1/6)*polylog(2,-(-1)^(1/3)*d^(1/3)*
(I-a-b*x)/(b*c^(1/3)-(-1)^(1/3)*(I-a)*d^(1/3)))/c^(2/3)/d^(1/3)+1/6*I*polyl
og(2,-d^(1/3)*(I+a+b*x)/(b*c^(1/3)-(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)
^(1/6)*polylog(2,(-1)^(1/3)*d^(1/3)*(I+a+b*x)/(b*c^(1/3)+(-1)^(1/3)*(I+a)*d
^(1/3)))/c^(2/3)/d^(1/3)-1/6*(-1)^(5/6)*polylog(2,-(-1)^(2/3)*d^(1/3)*(I+a+
b*x)/(b*c^(1/3)-(-1)^(2/3)*(I+a)*d^(1/3)))/c^(2/3)/d^(1/3)
```

Rubi [A]
time = 1.13, antiderivative size = 863, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5159, 2456, 2441, 2440, 2438}

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-1/6*I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) + (
I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((I/6)*Log[1 - I*a - I*b*x]*Log[(b*(c
^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (I + a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((
-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*
c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(1/6)*L
og[1 - I*a - I*b*x]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (
-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(5/6)*Log[1 + I*a
```

```
+ I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(2/3)*(
I - a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - ((-1)^(5/6)*Log[1 - I*a - I*b*x]*Lo
g[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*a)*d^
(1/3))])/(6*c^(2/3)*d^(1/3)) - ((I/6)*PolyLog[2, (d^(1/3)*(I - a - b*x))/(b
*c^(1/3) + (I - a)*d^(1/3))])/(c^(2/3)*d^(1/3)) + ((-1)^(5/6)*PolyLog[2, -(
((-1)^(1/6)*d^(1/3)*(I - a - b*x))/(I*b*c^(1/3) - (-1)^(1/6)*(I - a)*d^(1/3
)))])/(6*c^(2/3)*d^(1/3)) + ((-1)^(1/6)*PolyLog[2, -(((-1)^(1/3)*d^(1/3)*(I
 - a - b*x))/(b*c^(1/3) - (-1)^(1/3)*(I - a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3)
) + ((I/6)*PolyLog[2, -((d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (I + a)*d^(1/3
)))])/(c^(2/3)*d^(1/3)) - ((-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(I + a
 + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3)) - (
(-1)^(5/6)*PolyLog[2, -(((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(b*c^(1/3) - (-1
)^(2/3)*(I + a)*d^(1/3)))])/(6*c^(2/3)*d^(1/3))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^3}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^3}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^3}\,dx$$

$$= \frac{1}{2}i\int \left(-\frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}-\sqrt[3]{d}\,x\right)} - \frac{\log(1-ia-ibx)}{3c^{2/3}\left(-\sqrt[3]{c}+\sqrt[3]{-1}\,\sqrt[3]{d}\,x\right)} - \frac{\log(1-ia\ \cdots}{3c^{2/3}\left(-\sqrt[3]{c}-(-\cdots\right.}\right.$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}+\sqrt[3]{-1}\,\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} - \frac{i\int \frac{\log(1-ia-ibx)}{-\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}\,x}\,dx}{6c^{2/3}} + \cdots$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\cdots\right.}{6c^{2/3}\sqrt[3]{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\cdots\right.}{6c^{2/3}\sqrt[3]{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}+(i-a)\sqrt[3]{d}}\right)}{6c^{2/3}\sqrt[3]{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b\sqrt[3]{c}-(i+a)\sqrt[3]{d}}\cdots\right.}{6c^{2/3}\sqrt[3]{d}}$$

Mathematica [A]
time = 0.64, size = 701, normalized size = 0.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^3),x]
```

```
[Out] ((-I)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*c^(1/3) - (-I +
 a)*d^(1/3))] + I*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) + d^(1/3)*x))/(b*
c^(1/3) - (I + a)*d^(1/3))] + (-1)^(1/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/
3) - (-1)^(1/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] - (-
1)^(1/6)*Log[(-I)*(I + a + b*x)]*Log[(b*(c^(1/3) - (-1)^(1/3)*d^(1/3)*x))/(
b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)*Log[(-I)*(I + a + b*x
)]*Log[(b*(c^(1/3) + (-1)^(2/3)*d^(1/3)*x))/(b*c^(1/3) + (-1)^(1/6)*(1 - I*
a)*d^(1/3))] + (-1)^(5/6)*Log[1 + I*a + I*b*x]*Log[(b*(c^(1/3) + (-1)^(2/3)
*d^(1/3)*x))/(b*c^(1/3) - (-1)^(2/3)*(-I + a)*d^(1/3))] - I*PolyLog[2, (d^(
1/3)*(-I + a + b*x))/(-(b*c^(1/3)) + (-I + a)*d^(1/3))] + (-1)^(5/6)*PolyLo
```

```
g[2, ((-1)^(1/6)*d^(1/3)*(-I + a + b*x))/(I*b*c^(1/3) + (-1)^(1/6)*(-I + a)
*d^(1/3))] + (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^(1/3)*(-I + a + b*x))/(b*c
^(1/3) + (-1)^(1/3)*(-I + a)*d^(1/3))] + I*PolyLog[2, (d^(1/3)*(I + a + b*x
))/(-(b*c^(1/3)) + (I + a)*d^(1/3))] - (-1)^(1/6)*PolyLog[2, ((-1)^(1/3)*d^
(1/3)*(I + a + b*x))/(b*c^(1/3) + (-1)^(1/3)*(I + a)*d^(1/3))] - (-1)^(5/6)
*PolyLog[2, ((-1)^(2/3)*d^(1/3)*(I + a + b*x))/(-(b*c^(1/3)) + (-1)^(2/3)*(
I + a)*d^(1/3))])/(6*c^(2/3)*d^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 787, normalized size = 0.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/3*b^3/d*sum(1/(-_R^2+2*_R*a-a^2)*ln(b*x-_R+a),_R=RootOf(_Z^3*d-3*_Z
^2*a*d+3*_Z*a^2*d-a^3*d+b^3*c))*arctan(b*x+a)+1/3*b^3/d*(arctan(b*x+a)*sum(
1/(-_R^2+2*_R*a-a^2)*ln(b*x-_R+a),_R=RootOf(_Z^3*d-3*_Z^2*a*d+3*_Z*a^2*d-a^
3*d+b^3*c))-3*d*(-2/3*sum(1/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^3*c*_R1^4+2*a^3*
d*_R1^2+2*I*a^2*d*_R1^2-3*a*d*_R1^4-2*b^3*c*_R1^2-I*d*_R1^4+a^3*d-I*a^2*d+2
*a*d*_R1^2-b^3*c+2*I*d*_R1^2+a*d-I*d)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a)
)/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_
R1)),_R1=RootOf((3*I*a^2*d+a^3*d-b^3*c-I*d-3*a*d)*_Z^6+(3*I*a^2*d+3*a^3*d-3
*b^3*c+3*I*d+3*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*b^3*c-3*I*d+3*a*d)*_Z^2-3*I*
a^2*d+a^3*d-b^3*c+I*d-3*a*d))-2/3*sum(_R1^2/(a^3*d*_R1^4+3*I*a^2*d*_R1^4-b^
3*c*_R1^4+2*a^3*d*_R1^2+2*I*a^2*d*_R1^2-3*a*d*_R1^4-2*b^3*c*_R1^2-I*d*_R1^4
+a^3*d-I*a^2*d+2*a*d*_R1^2-b^3*c+2*I*d*_R1^2+a*d-I*d)*(I*arctan(b*x+a)*ln((
_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))/(1+(b*
x+a)^2)^(1/2))/_R1)),_R1=RootOf((3*I*a^2*d+a^3*d-b^3*c-I*d-3*a*d)*_Z^6+(3*I
*a^2*d+3*a^3*d-3*b^3*c+3*I*d+3*a*d)*_Z^4+(-3*I*a^2*d+3*a^3*d-3*b^3*c-3*I*d+
3*a*d)*_Z^2-3*I*a^2*d+a^3*d-b^3*c+I*d-3*a*d)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*x^3 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^3 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a+b\,x)}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^3),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x^3), x)
```

3.53 $\int \frac{\text{ArcTan}(a+bx)}{c+dx^2} \, dx$

Optimal. Leaf size=543

$$-\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\log(1+ia+i}{}$$

```
[Out] -1/4*I*ln(1+I*a+I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/(b*(-c)^(1/2)-(I-a)*d^(1
/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)-x*d^(1/2))/
(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*ln(1+I*a+I*b*x)*ln(b
*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/
4*I*ln(1-I*a-I*b*x)*ln(b*((-c)^(1/2)+x*d^(1/2))/(b*(-c)^(1/2)-(I+a)*d^(1/2)
))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I-a-b*x)*d^(1/2)/(b*(-c)^(1/2)-(I-a
)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I-a-b*x)*d^(1/2)/(b*(-c)^(1
/2)+(I-a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*I*polylog(2,-(I+a+b*x)*d^(1/2)/(
b*(-c)^(1/2)-(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*I*polylog(2,(I+a+b*x)*d
^(1/2)/(b*(-c)^(1/2)+(I+a)*d^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5159, 2456, 2441, 2440, 2438}

$$-\frac{i\text{Li}_2\left(-\frac{\sqrt{d}\,(-a-bx+i)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{\sqrt{d}\,(-a-bx+i)}{\sqrt{d}\,(i-a)+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{i\text{Li}_2\left(-\frac{\sqrt{d}\,(a+bx+i)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{\sqrt{d}\,(a+bx+i)}{\sqrt{d}\,(a+i)+b\sqrt{-c}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{i\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}+(-a+i)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{i\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(a+i)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] -
 (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*Log[1 - I*a - I*b*x]*Log[(b
*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]
) + ((I/4)*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c]
+ (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*Log[1 - I*a - I*b*x]*Log[(
b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d
]) - ((I/4)*PolyLog[2, -((Sqrt[d]*(I - a - b*x))/(b*Sqrt[-c] - (I - a)*Sqrt
[d]))])/(Sqrt[-c]*Sqrt[d]) + ((I/4)*PolyLog[2, (Sqrt[d]*(I - a - b*x))/(b*S
qrt[-c] + (I - a)*Sqrt[d])])/(Sqrt[-c]*Sqrt[d]) - ((I/4)*PolyLog[2, -((Sqrt
[d]*(I + a + b*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d]))])/(Sqrt[-c]*Sqrt[d]) + (
(I/4)*PolyLog[2, (Sqrt[d]*(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])])/(
Sqrt[-c]*Sqrt[d])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx^2}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+dx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+dx^2}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\sqrt{-c}\,\log(1-ia-ibx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)} + \frac{\sqrt{-c}\,\log(1-ia-ibx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-c}\,\log}{2c\left(\sqrt{-\right.}}\right.$$

$$= -\frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} - \frac{i\int \frac{\log(1-ia-ibx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4\sqrt{-c}} + \frac{i\int \frac{\log(1+ia+ibx)}{\sqrt{-c}+\sqrt{d}\,x}}{4\sqrt{-c}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{\right.}}{b\sqrt{-c}+(i+a}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{\right.}}{b\sqrt{-c}+(i+a}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

$$= -\frac{i\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}} + \frac{i\log(1-ia-ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{\right.}}{b\sqrt{-c}+(i+a}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

Mathematica [A]
time = 0.25, size = 409, normalized size = 0.75

$$-\frac{i\left(\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)-\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)-\log(1+ia+ibx)\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(-i+a)\sqrt{d}}\right)+\log(-i(i+a+bx))\log\left(\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}\,(-i+a+bx)}{-b\sqrt{-c}+(-i+a)\sqrt{d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}\,(-i+a+bx)}{b\sqrt{-c}+(-i+a)\sqrt{d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}\,(i+a+bx)}{-b\sqrt{-c}+(i+a)\sqrt{d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}\,(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right)\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*(Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c]
+ (-I + a)*Sqrt[d])] - Log[(-I)*(I + a + b*x)]*Log[(b*(Sqrt[-c] - Sqrt[d]*x
))/(b*Sqrt[-c] + (I + a)*Sqrt[d])] - Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[-c]
+ Sqrt[d]*x))/(b*Sqrt[-c] - (-I + a)*Sqrt[d])] + Log[(-I)*(I + a + b*x)]*Lo
g[(b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (I + a)*Sqrt[d])] - PolyLog[2, (
Sqrt[d]*(-I + a + b*x))/(-(b*Sqrt[-c]) + (-I + a)*Sqrt[d])] + PolyLog[2, (S
qrt[d]*(-I + a + b*x))/(b*Sqrt[-c] + (-I + a)*Sqrt[d])] + PolyLog[2, (Sqrt[
d]*(I + a + b*x))/(-(b*Sqrt[-c]) + (I + a)*Sqrt[d])] - PolyLog[2, (Sqrt[d]*
(I + a + b*x))/(b*Sqrt[-c] + (I + a)*Sqrt[d])]))/(Sqrt[-c]*Sqrt[d])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(411) = 822$.
time = 0.44, size = 2184, normalized size = 4.02

method	result
risch	$\frac{\ln(-ibx-ia+1)\ln\left(\frac{iad-b\sqrt{cd}+(-ibx-ia+1)d-d}{iad-b\sqrt{cd}-d}\right)}{4\sqrt{cd}}-\frac{\ln(-ibx-ia+1)\ln\left(\frac{iad+b\sqrt{cd}+(-ibx-ia+1)d-d}{iad+b\sqrt{cd}-d}\right)}{4\sqrt{cd}}+\frac{\mathrm{dilog}\left(\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a
*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(
1/2)-d))*arctan(b*x+a)-1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/
2)+d)*ln(1-(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^
2*c+2*(b^2*c*d)^(1/2)-d))*a^2*arctan(b*x+a)-1/2*I*b^2/d*(b^2*c*d)^(1/2)/(a^
2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2
/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1/2)-d))*arctan(b*x+a)+1/2*I*(b^2
*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*d+a^2*d+b^2*c-d
)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c-2*(b^2*c*d)^(1/2)-d))*arctan(
b*x+a)-1/2*b^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arctan(b
*x+a)^2+b^2/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arctan(b*x+a)^2-1/2*(b^2*c*d)
^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*arctan(b*x+a)^2-1/2*(b^2*c*d)^(1
/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*a^2*arctan(b*x+a)^2-1/4*b^2/d*(b^2*
c*d)^(1/2)/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c
-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1/2)-d))+1/2*b
^2/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c-d)*(1+I
*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1/2)-d))-1/4*(b^2*c*d)
^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c-d
)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1/2)-d))-1/4*(b^
2*c*d)^(1/2)/c/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*polylog(2,(2*I*a*d+a^2*d+b
^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1/2)-d))*a
^2+1/2*I*b^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*
a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c-2*(b^2*c*d)^
(1/2)-d))*arctan(b*x+a)+I*b^2/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a
*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c-2*(b^2*c*d)^(
1/2)-d))*arctan(b*x+a)+I*b^2/(a^2*d+b^2*c-2*(b^2*c*d)^(1/2)+d)*ln(1-(2*I*a*
d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c+2*(b^2*c*d)^(1
/2)-d))*arctan(b*x+a)+1/2*I*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2
)+d)*ln(1-(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2
*c-2*(b^2*c*d)^(1/2)-d))*a^2*arctan(b*x+a)+1/2*b^2/d*(b^2*c*d)^(1/2)/(a^2*d
```

```
+b^2*c+2*(b^2*c*d)^(1/2)+d)*arctan(b*x+a)^2+b^2/(a^2*d+b^2*c+2*(b^2*c*d)^(1
/2)+d)*arctan(b*x+a)^2+1/2*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)
+d)*arctan(b*x+a)^2+1/2*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)
*a^2*arctan(b*x+a)^2+1/4*b^2/d*(b^2*c*d)^(1/2)/(a^2*d+b^2*c+2*(b^2*c*d)^(1/
2)+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2
*d-b^2*c-2*(b^2*c*d)^(1/2)-d))+1/2*b^2/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)+d)*po
lylog(2,(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d-b^2*c
-2*(b^2*c*d)^(1/2)-d))+1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)^(1/2)
+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(-a^2*d
-b^2*c-2*(b^2*c*d)^(1/2)-d))+1/4*(b^2*c*d)^(1/2)/c/(a^2*d+b^2*c+2*(b^2*c*d)
^(1/2)+d)*polylog(2,(2*I*a*d+a^2*d+b^2*c-d)*(1+I*(b*x+a))^2/(1+(b*x+a)^2)/(
-a^2*d-b^2*c-2*(b^2*c*d)^(1/2)-d))*a^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8520 vs. 2(369) = 738.
time = 5.42, size = 8520, normalized size = 15.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(d*x/sqrt(c*d))*arctan((b^2*x + a*b)/b)/b - (4*arctan(sqrt(d
)*x/sqrt(c))*arctan2((2*a*b^2*c*d + (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^
2 + 3)*b^2*d)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^
2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a^2 + 1)*b*d)
*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + (2*a*b^2*d*
x + b^3*c + 3*(a^2 + 1)*b*d)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2
)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 + 4*(b^3*c + (a
^2 + 1)*b*d)*sqrt(c)*sqrt(d))) + 4*arctan(sqrt(d)*x/sqrt(c))*arctan2((2*a*b
^2*c*d - (a*b^3*c + (a^3 + a)*b*d + (b^4*c + (a^2 + 3)*b^2*d)*x)*sqrt(c)*sq
rt(d) + (3*b^3*c*d + (a^2 + 1)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)*b^2*c*d + (
a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d)), ((a^2 +
3)*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - (2*a*b^2*d*x + b^3*c + 3*(a^2 + 1)*b*d
)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^3 + a)*b*d^2)*x)/(b^4*c^2 + 2*(a^2 + 3)
*b^2*c*d + (a^4 + 2*a^2 + 1)*d^2 - 4*(b^3*c + (a^2 + 1)*b*d)*sqrt(c)*sqrt(d
))) + log(d*x^2 + c)*log(((a^2 + 1)*b^22*c^11*d + 11*(a^4 + 22*a^2 + 21)*b^
20*c^10*d^2 + 55*(a^6 + 39*a^4 + 171*a^2 + 133)*b^18*c^9*d^3 + 33*(5*a^8 +
260*a^6 + 1870*a^4 + 3876*a^2 + 2261)*b^16*c^8*d^4 + 330*(a^10 + 61*a^8 + 5
70*a^6 + 1802*a^4 + 2261*a^2 + 969)*b^14*c^7*d^5 + 22*(21*a^12 + 1386*a^10
+ 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22
*(21*a^14 + 1407*a^12 + 16401*a^10 + 75075*a^8 + 169455*a^6 + 201773*a^4 +
121771*a^2 + 29393)*b^10*c^5*d^7 + 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^
10 + 9438*a^8 + 13728*a^6 + 11492*a^4 + 5168*a^2 + 969)*b^8*c^4*d^8 + 33*(5
*a^18 + 285*a^16 + 3220*a^14 + 15876*a^12 + 42966*a^10 + 70070*a^8 + 70980*
```

```
a^6 + 43860*a^4 + 15181*a^2 + 2261)*b^6*c^3*d^9 + 55*(a^20 + 46*a^18 + 465*
a^16 + 2184*a^14 + 5922*a^12 + 10164*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4
 + 1102*a^2 + 133)*b^4*c^2*d^10 + 11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16
 + 2730*a^14 + 4662*a^12 + 5502*a^10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*
a^2 + 21)*b^2*c*d^11 + (a^24 + 12*a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 79
2*a^14 + 924*a^12 + 792*a^10 + 495*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*d^1
2 + (b^24*c^11*d + 11*(a^2 + 21)*b^22*c^10*d^2 + 55*(a^4 + 38*a^2 + 133)*b^
20*c^9*d^3 + 33*(5*a^6 + 255*a^4 + 1615*a^2 + 2261)*b^18*c^8*d^4 + 330*(a^8
 + 60*a^6 + 510*a^4 + 1292*a^2 + 969)*b^16*c^7*d^5 + 22*(21*a^10 + 1365*a^8
 + 13650*a^6 + 46410*a^4 + 62985*a^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 +
1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a^4 + 92378*a^2 + 29393)*b^12*c^
5*d^7 + 330*(a^14 + 63*a^12 + 693*a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4
199*a^2 + 969)*b^10*c^4*d^8 + 33*(5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^1
0 + 30030*a^8 + 40040*a^6 + 30940*a^4 + 12920*a^2 + 2261)*b^8*c^3*d^9 + 55*
(a^18 + 45*a^16 + 420*a^14 + 1764*a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 +
3060*a^4 + 969*a^2 + 133)*b^6*c^2*d^10 + 11*(a^20 + 30*a^18 + 225*a^16 + 84
0*a^14 + 1890*a^12 + 2772*a^10 + 2730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 +
21)*b^4*c*d^11 + (a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12
 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b^2*d^12)*x^2 + 2*(1
1*(a^2 + 1)*b^21*c^10*d + 110*(a^4 + 8*a^2 + 7)*b^19*c^9*d^2 + 33*(15*a^6 +
 205*a^4 + 589*a^2 + 399)*b^17*c^8*d^3 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 64
6*a^2 + 323)*b^15*c^7*d^4 + 110*(21*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 +
6137*a^2 + 2261)*b^13*c^6*d^5 + 4*(693*a^12 + 15708*a^10 + 105105*a^8 + 308
880*a^6 + 449735*a^4 + 319124*a^2 + 88179)*b^11*c^5*d^6 + 110*(21*a^14 + 48
3*a^12 + 3465*a^10 + 11583*a^8 + 20735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*
b^9*c^4*d^7 + 264*(5*a^16 + 110*a^14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 68
90*a^6 + 4930*a^4 + 1938*a^2 + 323)*b^7*c^3*d^8 + 33*(15*a^18 + 295*a^16 +
2044*a^14 + 7308*a^12 + 15554*a^10 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 298
3*a^2 + 399)*b^5*c^2*d^9 + 110*(a^20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a
^12 + 1008*a^10 + 966*a^8 + 624*a^6 + 261*a^4 + 64*a^2 + 7)*b^3*c*d^10 + 11
*(a^22 + 11*a^20 + 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 33
0*a^8 + 165*a^6 + 55*a^4 + 11*a^2 + 1)*b*d^11 + (11*b^23*c^10*d + 110*(a^2
+ 7)*b^21*c^9*d^2 + 33*(15*a^4 + 190*a^2 + 399)*b^19*c^8*d^3 + 264*(5*a^6 +
 85*a^4 + 323*a^2 + 323)*b^17*c^7*d^4 + 110*(21*a^8 + 420*a^6 + 2142*a^4 +
3876*a^2 + 2261)*b^15*c^6*d^5 + 4*(693*a^10 + 15015*a^8 + 90090*a^6 + 21879
0*a^4 + 230945*a^2 + 88179)*b^13*c^5*d^6 + 110*(21*a^12 + 462*a^10 + 3003*a
^8 + 8580*a^6 + 12155*a^4 + 8398*a^2 + 2261)*b^11*c^4*d^7 + 264*(5*a^14 + 1
05*a^12 + 693*a^10 + 2145*a^8 + 3575*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c
^3*d^8 + 33*(15*a^16 + 280*a^14 + 1764*a^12 + 5544*a^10 + 10010*a^8 + 10920
*a^6 + 7140*a^4 + 2584*a^2 + 399)*b^7*c^2*d^9 + 110*(a^18 + 15*a^16 + 84*a^
14 + 252*a^12 + 462*a^10 + 546*a^8 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c*
d^10 + 11*(a^20 + 10*a^18 + 45*a^16 + 120*a^14 + 210*a^12 + 252*a^10 + 210*
a^8 + 120*a^6 + 45*a^4 + 10*a^2 + 1)*b^3*d^11)*x^2 + 2*(11*a*b^22*c^10*d +
110*(a^3 + 7*a)*b^20*c^9*d^2 + 33*(15*a^5 + 190...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x^2 + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x^2), x)
```

3.54 $\int \frac{\text{ArcTan}(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=152

$$-\frac{\text{ArcTan}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}+\frac{\text{ArcTan}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d}+\frac{i\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bx)}\right)}{2d}-$$

```
[Out] -arctan(b*x+a)*ln(2/(1-I*(b*x+a)))/d+arctan(b*x+a)*ln(2*b*(d*x+c)/(b*c+I*d-
a*d)/(1-I*(b*x+a)))/d+1/2*I*polylog(2,1-2/(1-I*(b*x+a)))/d-1/2*I*polylog(2,
1-2*b*(d*x+c)/(b*c+I*d-a*d)/(1-I*(b*x+a)))/d
```

Rubi [A]
time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5155, 4966, 2449, 2352, 2497}

$$\frac{\text{ArcTan}(a+bx)\log\left(\frac{2b(c+dx)}{(1-i(a+bx))(-ad+bc+id)}\right)}{d}-\frac{\text{ArcTan}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d}-\frac{i\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc-ad+id)(1-i(a+bx))}\right)}{2d}+\frac{i\text{Li}_2\left(1-\frac{2}{1-i(a+bx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcTan[a + b*x]*Log[2/(1 - I*(a + b*x))])/d) + (ArcTan[a + b*x]*Log[(2*b
*(c + d*x))/((b*c + I*d - a*d)*(1 - I*(a + b*x)))])/d + ((I/2)*PolyLog[2, 1
 - 2/(1 - I*(a + b*x))])/d - ((I/2)*PolyLog[2, 1 - (2*b*(c + d*x))/((b*c +
I*d - a*d)*(1 - I*(a + b*x)))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + \text{Subst}\Big($$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} - i\text{Li}_2\Big(1$$

$$= -\frac{\tan^{-1}(a+bx)\log\left(\frac{2}{1-i(a+bx)}\right)}{d} + \frac{\tan^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+id-ad)(1-i(a+bx))}\right)}{d} + i\text{Li}_2\Big(1$$

Mathematica [A]
time = 0.02, size = 231, normalized size = 1.52

$$\frac{i\log(1-i(a+bx))\log\left(-\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}-\frac{i(bc-ad)}{b}}\right)}{2d} - \frac{i\log(1+i(a+bx))\log\left(\frac{i\left(\frac{bc-ad}{b}+\frac{d(a+bx)}{b}\right)}{-\frac{d}{b}+\frac{i(bc-ad)}{b}}\right)}{2d} + \frac{i\text{PolyLog}\left(2,-\frac{id(1-i(a+bx))}{bc-id-ad}\right)}{2d} - \frac{i\text{PolyLog}\left(2,\frac{id(1+i(a+bx))}{bc+id-ad}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*x),x]
```

```
[Out] ((I/2)*Log[1 - I*(a + b*x)]*Log[((-I)*((b*c - a*d)/b + (d*(a + b*x))/b))/(-
(d/b) - (I*(b*c - a*d))/b)])/d - ((I/2)*Log[1 + I*(a + b*x)]*Log[(I*((b*c -
 a*d)/b + (d*(a + b*x))/b))/(-(d/b) + (I*(b*c - a*d))/b)])/d + ((I/2)*PolyL
```

```
og[2, ((-I)*d*(1 - I*(a + b*x)))/(b*c - I*d - a*d)])/d - ((I/2)*PolyLog[2,
(I*d*(1 + I*(a + b*x)))/(b*c + I*d - a*d)])/d
```

Maple [A]
time = 0.08, size = 186, normalized size = 1.22

method	result
derivativedivides	$\frac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\mathrm{di}\cdots\right)}{2d}\right.}{b}$
default	$\frac{\frac{b\ln(ad-bc-d(bx+a))\arctan(bx+a)}{d}+b\left(-\frac{i\ln(ad-bc-d(bx+a))\left(\ln\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\ln\left(\frac{id-d(bx+a)}{-ad+bc+id}\right)\right)}{2d}-\frac{i\left(\operatorname{dilog}\left(\frac{id+d(bx+a)}{ad-bc+id}\right)-\mathrm{di}\cdots\right)}{2d}\right.}{b}$
risch	$\frac{i\operatorname{dilog}\left(\frac{iad-icb+(-ibx-ia+1)d-d}{iad-icb-d}\right)}{2d}+\frac{i\ln(-ibx-ia+1)\ln\left(\frac{iad-icb+(-ibx-ia+1)d-d}{iad-icb-d}\right)}{2d}-\frac{i\operatorname{dilog}\left(\frac{-iad+icb+(ibx+ia+1)d-\cdots}{-iad+icb-d}\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(a*d-b*c-d*(b*x+a))/d*arctan(b*x+a)+b*(-1/2*I*ln(a*d-b*c-d*(b*x+a)
)*(ln((I*d+d*(b*x+a))/(a*d-b*c+I*d))-ln((I*d-d*(b*x+a))/(b*c+I*d-a*d)))/d-1
/2*I*(dilog((I*d+d*(b*x+a))/(a*d-b*c+I*d))-dilog((I*d-d*(b*x+a))/(b*c+I*d-a
*d)))/d))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(130) = 260.
time = 0.56, size = 284, normalized size = 1.87

$$\frac{\arctan(bx+a)\log(dx+c)}{d}-\frac{\arctan\left(\frac{b^2x+ab}{b}\right)\log(dx+c)}{d}-\frac{\arctan\left(\frac{bd^2x+bcd}{b^2c^2-2\,abcd+(a^2+1)d^2},\frac{b^2c^2-abcd+(b^2cd-abd^2)x}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)\log(b^2x^2+2\,abx+a^2+1)-\arctan(bx+a)\log\left(\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2}{b^2c^2-2\,abcd+(a^2+1)d^2}\right)+i\,\mathrm{Li}_2\left(\frac{i\,bdx+(i\,a+1)d}{-i\,bc+(i\,a+1)d}\right)-i\,\mathrm{Li}_2\left(\frac{i\,bdx+(i\,a-1)d}{-i\,bc+(i\,a-1)d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] arctan(b*x + a)*log(d*x + c)/d - arctan((b^2*x + a*b)/b)*log(d*x + c)/d - 1
/2*(arctan2((b*d^2*x + b*c*d)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2), (b^2*c
^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1) - arctan(b*x + a)*log((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)/(b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)) + I*dilog((I*b*d
*x + (I*a + 1)*d)/(-I*b*c + (I*a + 1)*d)) - I*dilog((I*b*d*x + (I*a - 1)*d)
/(-I*b*c + (I*a - 1)*d)))/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(atan(a + b*x)/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x),x)
```

```
[Out] int(atan(a + b*x)/(c + d*x), x)
```

3.55 $\int \frac{\mathbf{ArcTan}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=244

$$-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}-\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c-1/2*I*d*ln(1-I*a-I*b*x)*ln(-b*(c*x+d)/((I+a)*c-b*d))/c^2+1/2*I*d*ln(1+I*
a+I*b*x)*ln(b*(c*x+d)/((I-a)*c+b*d))/c^2+1/2*I*d*polylog(2,c*(I-a-b*x)/(I*c
-a*c+b*d))/c^2-1/2*I*d*polylog(2,c*(I+a+b*x)/((I+a)*c-b*d))/c^2
```

Rubi [A]
time = 0.20, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5159, 2456, 2436, 2332, 2441, 2440, 2438}

$$\frac{id\text{Li}_2\left(\frac{c(-a-bx+i)}{-ac+ic+bd}\right)}{2c^2}-\frac{id\text{Li}_2\left(\frac{c(a+bx+i)}{(a+i)c-bd}\right)}{2c^2}+\frac{id\log(ia+ibx+1)\log\left(\frac{b(cx+d)}{bd+(-a+i)c}\right)}{2c^2}-\frac{id\log(-ia-ibx+1)\log\left(-\frac{b(cx+d)}{-bd+(a+i)c}\right)}{2c^2}-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc}-\frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Lo
g[(-I)*(I + a + b*x)])/(2*b*c) - ((I/2)*d*Log[1 - I*a - I*b*x]*Log[-((b*(d
+ c*x))/((I + a)*c - b*d))])/c^2 + ((I/2)*d*Log[1 + I*a + I*b*x]*Log[(b*(d
+ c*x))/((I - a)*c + b*d)])/c^2 + ((I/2)*d*PolyLog[2, (c*(I - a - b*x))/(I*
c - a*c + b*d)])/c^2 - ((I/2)*d*PolyLog[2, (c*(I + a + b*x))/((I + a)*c - b
*d)])/c^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x}}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c}-\frac{d\log(1-ia-ibx)}{c(d+cx)}\right)dx-\frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c}-\right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c}-\frac{i\int \log(1+ia+ibx)\,dx}{2c}-\frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx}\,dx}{2c}+\frac{(id)\int}{}$$

$$= -\frac{id\log(1-ia-ibx)\log\left(-\frac{b(d+cx)}{(i+a)c-bd}\right)}{2c^2}+\frac{id\log(1+ia+ibx)\log\left(\frac{b(d+cx)}{(i-a)c+bd}\right)}{2c^2}-\frac{\text{Sub}}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}-\frac{id\log(1-}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}-\frac{id\log(1-}{}$$

Mathematica [A]

time = 7.67, size = 405, normalized size = 1.66

$$\frac{2c(a+bx)\text{ArcTan}(a+bx)+\frac{bcd\,\text{ArcTan}(a+bx)^2}{ac-bd}+2c\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)-ibd\left(\text{ArcTan}(a+bx)\left(\text{ArcTan}(a+bx)+2i\log\left(1+e^{2i\text{ArcTan}(a+bx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(a+bx)}\right)\right)+\frac{bd\left(c\sqrt{1+a^2-\frac{2abd}{c}+\frac{b^2d^2}{c^2}}\,e^{-i\text{ArcTan}\left(a-\frac{bd}{c}\right)}\text{ArcTan}(a+bx)^2+(ac-bd)\text{ArcTan}(a+bx)\left(i\pi+2i\text{ArcTan}\left(a-\frac{bd}{c}\right)+2\log\left(1-e^{2i\left(-\text{ArcTan}\left(a-\frac{bd}{c}\right)+\text{ArcTan}(a+bx)\right)}\right)\right)+(ac-bd)\left(\pi\left(\log\left(1+e^{-2i\text{ArcTan}(a+bx)}\right)-\log\left(\frac{1}{\sqrt{1+(a+bx)^2}}\right)\right)-2\text{ArcTan}\left(a-\frac{bd}{c}\right)\left(\log\left(1-e^{2i\left(-\text{ArcTan}\left(a-\frac{bd}{c}\right)+\text{ArcTan}(a+bx)\right)}\right)-\log\left(-\sin\left(\text{ArcTan}\left(a-\frac{bd}{c}\right)-\text{ArcTan}(a+bx)\right)\right)\right)\right)-i(ac-bd)\text{PolyLog}\left(2,e^{2i\left(-\text{ArcTan}\left(a-\frac{bd}{c}\right)+\text{ArcTan}(a+bx)\right)}\right)\right)}{-ac+bd}}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x),x]
```

```
[Out] (2*c*(a + b*x)*ArcTan[a + b*x] + (b*c*d*ArcTan[a + b*x]^2)/(a*c - b*d) + 2*
c*Log[1/Sqrt[1 + (a + b*x)^2]] - I*b*d*(ArcTan[a + b*x]*(ArcTan[a + b*x] +
(2*I)*Log[1 + E^((2*I)*ArcTan[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcTan[a +
 b*x])]) + (b*d*((c*Sqrt[1 + a^2 - (2*a*b*d)/c + (b^2*d^2)/c^2]*ArcTan[a +
b*x]^2)/E^(I*ArcTan[a - (b*d)/c]) + (a*c - b*d)*ArcTan[a + b*x]*(I*Pi + (2*
I)*ArcTan[a - (b*d)/c] + 2*Log[1 - E^((2*I)*(-ArcTan[a - (b*d)/c] + ArcTan[
a + b*x]))]) + (a*c - b*d)*(Pi*(Log[1 + E^((-2*I)*ArcTan[a + b*x])] - Log[1
/Sqrt[1 + (a + b*x)^2]]) - 2*ArcTan[a - (b*d)/c]*(Log[1 - E^((2*I)*(-ArcTan
[a - (b*d)/c] + ArcTan[a + b*x]))] - Log[-Sin[ArcTan[a - (b*d)/c] - ArcTan[
a + b*x]]])) - I*(a*c - b*d)*PolyLog[2, E^((2*I)*(-ArcTan[a - (b*d)/c] + Ar
cTan[a + b*x]))]))/(-(a*c) + b*d))/(2*b*c^2)
```

Maple [A]

time = 0.06, size = 321, normalized size = 1.32

method	result
risch	$\frac{i\ln(-ibx-ia+1)x}{2c}+\frac{a\arctan(bx+a)}{bc}-\frac{\ln(b^2x^2+2abx+a^2+1)}{2bc}+\frac{1}{bc}-\frac{id\,\text{dilog}\left(\frac{iac-ibd+(-ibx-ia+1)c-c}{iac-ibd-c}\right)}{2c^2}-\frac{id\ln}{}$
derivativedivides	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$
default	$\frac{\frac{\arctan(bx+a)(bx+a)}{c}-\frac{\arctan(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}+\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))+c\right.}{2}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctan(b*x+a)/c*(b*x+a)-arctan(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))+1/
c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c-b
*d-c*(b*x+a))+c^2+(a*c-b*d-c*(b*x+a))^2)+1/2*I*b*d/c*ln(a*c-b*d-c*(b*x+a))*
ln((I*c+c*(b*x+a))/(a*c-b*d+I*c))-1/2*I*b*d/c*ln(a*c-b*d-c*(b*x+a))*ln((I*c
-c*(b*x+a))/(I*c-a*c+b*d))+1/2*I*b*d/c*dilog((I*c+c*(b*x+a))/(a*c-b*d+I*c))
-1/2*I*b*d/c*dilog((I*c-c*(b*x+a))/(I*c-a*c+b*d))))
```

Maxima [A]

time = 0.57, size = 284, normalized size = 1.16

$$-\frac{bd\arctan(bx+a)\log\left(-\frac{b^2c^2x^2+2b^2cdx+b^2d^2}{2abcd-b^2d^2-(a^2+1)c^2}\right)+i\,bd\,\text{Li}_2\left(-\frac{i\,bcx+(i\,a-1)c}{(-i\,a+1)c+i\,bd}\right)-i\,bd\,\text{Li}_2\left(-\frac{i\,bcx+(i\,a+1)c}{(-i\,a-1)c+i\,bd}\right)-2(bcx+ac)\arctan(bx+a)-\left(bd\arctan\left(-\frac{bc^2x+bcd}{2abcd-b^2d^2-(a^2+1)c^2},\frac{abcd-b^2d^2+(abc^2-b^2cd)x}{2abcd-b^2d^2-(a^2+1)c^2}\right)-c\right)\log(b^2x^2+2abx+a^2+1)}{2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] -1/2*(b*d*arctan(b*x + a)*log(-(b^2*c^2*x^2 + 2*b^2*c*d*x + b^2*d^2)/(2*a*b
*c*d - b^2*d^2 - (a^2 + 1)*c^2)) + I*b*d*dilog(-(I*b*c*x + (I*a - 1)*c)/((-
I*a + 1)*c + I*b*d)) - I*b*d*dilog(-(I*b*c*x + (I*a + 1)*c)/((-I*a - 1)*c +
 I*b*d)) - 2*(b*c*x + a*c)*arctan(b*x + a) - (b*d*arctan2(-(b*c^2*x + b*c*d
)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2), (a*b*c*d - b^2*d^2 + (a*b*c^2 - b^
2*c*d)*x)/(2*a*b*c*d - b^2*d^2 - (a^2 + 1)*c^2)) - c)*log(b^2*x^2 + 2*a*b*x
 + a^2 + 1))/(b*c^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="fricas")
```

```
[Out] integral(x*arctan(b*x + a)/(c*x + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x), x)
```

3.56 $\int \frac{\mathbf{ArcTan}(a+bx)}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=668

$$-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}+\frac{i\sqrt{d}\,\log(1+ia+ibx)\log\left(-\frac{b(\sqrt{\cdots}}{i\sqrt{-c}\cdots}\right.}{4(-c)^{3/2}}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c+1/4*I*ln(1+I*a+I*b*x)*ln(-b*(-x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)
^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*ln(1-I*a-I*b*x)*ln(-b*(x*(-c)^(
1/2)+d^(1/2))/((I+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1+I
*a+I*b*x)*ln(b*(x*(-c)^(1/2)+d^(1/2))/(I*(-c)^(1/2)-a*(-c)^(1/2)+b*d^(1/2))
)*d^(1/2)/(-c)^(3/2)-1/4*I*ln(1-I*a-I*b*x)*ln(b*(-x*(-c)^(1/2)+d^(1/2))/(I*
(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*I*polylog(2,(I-a
-b*x)*(-c)^(1/2)/(I*(-c)^(1/2)-a*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+
1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)/(I*(-c)^(1/2)+a*(-c)^(1/2)-b*d^(1/2)))
*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(1+I*a+I*b*x)*(-c)^(1/2)/((1+I*a)*(-c)^
(1/2)-I*b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*I*polylog(2,(I+a+b*x)*(-c)^(1/2)
/(I*(-c)^(1/2)+a*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)
```

Rubi [A]
time = 0.81, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5159, 2456, 2436, 2332, 2441, 2440, 2438}

$$\frac{i\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(-a-bx+i)}{-\sqrt{-c}\,a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(ia+ibx+1)}{(ia+1)\sqrt{-c}-ib\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+i)}{\sqrt{-c}\,a+i\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}\,(a+bx+i)}{\sqrt{-c}\,a+i\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\,\log(ia+ibx+1)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{a\left(-\sqrt{-c}\right)-b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\log(ia+ibx+1)\log\left(\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{a\left(-\sqrt{-c}\right)+b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{i\sqrt{d}\,\log(-ia-ibx+1)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{a\sqrt{-c}+b\sqrt{d}+i\sqrt{-c}}\right)}{4(-c)^{3/2}}+\frac{i\sqrt{d}\,\log(-ia-ibx+1)\log\left(-\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{-b\sqrt{d}+(a+i)\sqrt{-c}}\right)}{4(-c)^{3/2}}-\frac{(ia+ibx+1)\log(ia+ibx+1)}{2bc}-\frac{(-ia-ibx+1)\log(-i(a+bx+i))}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Lo
g[(-I)*(I + a + b*x)])/(2*b*c) + ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[-(
(b*(Sqrt[d] - Sqrt[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d]))])/(-c)^(3
/2) - ((I/4)*Sqrt[d]*Log[1 - I*a - I*b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/(I
*Sqrt[-c] + a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*Log[1 - I
*a - I*b*x]*Log[-((b*(Sqrt[d] + Sqrt[-c]*x))/((I + a)*Sqrt[-c] - b*Sqrt[d])
)])/(-c)^(3/2) - ((I/4)*Sqrt[d]*Log[1 + I*a + I*b*x]*Log[(b*(Sqrt[d] + Sqrt
[-c]*x))/(I*Sqrt[-c] - a*Sqrt[-c] + b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d
]*PolyLog[2, (Sqrt[-c]*(I - a - b*x))/(I*Sqrt[-c] - a*Sqrt[-c] - b*Sqrt[d])
])/(-c)^(3/2) - ((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + I*a + I*b*x))/((1
+ I*a)*Sqrt[-c] - I*b*Sqrt[d])])/(-c)^(3/2) + ((I/4)*Sqrt[d]*PolyLog[2, (Sq
rt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c] - b*Sqrt[d])])/(-c)^(3/2) -
```

```
((I/4)*Sqrt[d]*PolyLog[2, (Sqrt[-c]*(I + a + b*x))/(I*Sqrt[-c] + a*Sqrt[-c]
 + b*Sqrt[d])])/(-c)^(3/2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^2}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^2}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\,(d+cx^2)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c} - d\right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c} - \frac{i\int \log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx^2}\,dx}{2c} + \frac{(id)\int \frac{\mathrm{lo}}{}}{}$$

$$= -\frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1+ia+ibx)}{2bc} - \frac{(id)\int}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{\left(i\sqrt{d}\right)\int}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{i\sqrt{d}\,\log(1}{}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1536 vs. 2(668) = 1336.
time = 16.76, size = 1536, normalized size = 2.30

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]])/(b*c) - (Sqrt[d]
*(-2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]*x)/Sqrt
[d]] - 2*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt[c]
```

```
*x)/Sqrt[d]] + 2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTan[(Sqrt
[c]*x)/Sqrt[d]] + 2*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*ArcTa
n[(Sqrt[c]*x)/Sqrt[d]] - 2*b*Sqrt[d]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2 + (b*Sqr
t[d]*Sqrt[((-I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^
(I*ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]) - (I*a*b*Sqrt[d]*Sqrt[((-I + a)^
2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((-I + a)*
Sqrt[c])/(b*Sqrt[d])]) + (b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + (
I*a*b*Sqrt[d]*Sqrt[((I + a)^2*c + b^2*d)/(b^2*d)]*ArcTan[(Sqrt[c]*x)/Sqrt[d
]]^2)/E^(I*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]) + 4*(1 + a^2)*Sqrt[c]*Arc
Tan[(Sqrt[c]*x)/Sqrt[d]]*ArcTan[a + b*x] + (2*I)*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*S
qrt[c])/(b*Sqrt[d])]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d
])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqr
t[d]]*Log[1 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(S
qrt[c]*x)/Sqrt[d]]))] + (2*I)*a^2*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1
 - E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/S
qrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^
((-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]
]))] - (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[1 - E^((
-2*I)*(ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]])
)] - (2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((
I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*a^2*S
qrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[d]]*Log[1 - E^((-2*I)*(ArcTan[((I + a)*Sqrt[
c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] - (2*I)*Sqrt[c]*ArcTan[((
-I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d]
)] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (2*I)*a^2*Sqrt[c]*ArcTan[((-I + a)*Sqr
t[c])/(b*Sqrt[d])]*Log[-Sin[ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan
[(Sqrt[c]*x)/Sqrt[d]]]] + (2*I)*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d]
)]*Log[-Sin[ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt
[d]]]] + (2*I)*a^2*Sqrt[c]*ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])]*Log[-Sin[A
rcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]]] - (1 +
 a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(ArcTan[((-I + a)*Sqrt[c])/(b*Sqrt[d])]
+ ArcTan[(Sqrt[c]*x)/Sqrt[d]]))] + (1 + a^2)*Sqrt[c]*PolyLog[2, E^((-2*I)*(
ArcTan[((I + a)*Sqrt[c])/(b*Sqrt[d])] + ArcTan[(Sqrt[c]*x)/Sqrt[d]]))]))/(4
*(1 + a^2)*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 14058, normalized size = 21.04

method	result
risch	$-\frac{i\ln(ibx+ia+1)x}{2c}+\frac{a\arctan(bx+a)}{bc}+\frac{i\ln(-ibx-ia+1)x}{2c}-\frac{\ln(b^2x^2+2abx+a^2+1)}{2bc}+\frac{1}{bc}-\frac{d\,\mathrm{dilog}\left(\frac{-iac+b\sqrt{}}{-}\right.}{4c}$

derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8518 vs. $2(466) = 932$.
time = 1.20, size = 8518, normalized size = 12.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arctan(b*x + a) + 1/8*(8*a*c
*arctan(b*x + a) + (4*b*arctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d + (a
*b^3*d + (a^3 + a)*b*c + (b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*
b^3*c*d + (a^2 + 1)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2
 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d
+ (a^4 + 2*a^2 + 1)*c^2 + (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*s
qrt(d) + (a*b^3*c*d + (a^3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d +
(a^4 + 2*a^2 + 1)*c^2 + 4*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + 4*b*a
rctan(sqrt(c)*x/sqrt(d))*arctan2((2*a*b^2*c*d - (a*b^3*d + (a^3 + a)*b*c +
(b^4*d + (a^2 + 3)*b^2*c)*x)*sqrt(c)*sqrt(d) + (3*b^3*c*d + (a^2 + 1)*b*c^2
)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4*(b^3*d + (a
^2 + 1)*b*c)*sqrt(c)*sqrt(d)), ((a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 -
 (2*a*b^2*c*x + b^3*d + 3*(a^2 + 1)*b*c)*sqrt(c)*sqrt(d) + (a*b^3*c*d + (a^
3 + a)*b*c^2)*x)/(b^4*d^2 + 2*(a^2 + 3)*b^2*c*d + (a^4 + 2*a^2 + 1)*c^2 - 4
*(b^3*d + (a^2 + 1)*b*c)*sqrt(c)*sqrt(d))) + b*log(c*x^2 + d)*log(((a^2 + 1
)*b^22*c*d^11 + 11*(a^4 + 22*a^2 + 21)*b^20*c^2*d^10 + 55*(a^6 + 39*a^4 + 1
71*a^2 + 133)*b^18*c^3*d^9 + 33*(5*a^8 + 260*a^6 + 1870*a^4 + 3876*a^2 + 22
61)*b^16*c^4*d^8 + 330*(a^10 + 61*a^8 + 570*a^6 + 1802*a^4 + 2261*a^2 + 969
)*b^14*c^5*d^7 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6 + 109395*a
^4 + 92378*a^2 + 29393)*b^12*c^6*d^6 + 22*(21*a^14 + 1407*a^12 + 16401*a^10
 + 75075*a^8 + 169455*a^6 + 201773*a^4 + 121771*a^2 + 29393)*b^10*c^7*d^5 +
 330*(a^16 + 64*a^14 + 756*a^12 + 3696*a^10 + 9438*a^8 + 13728*a^6 + 11492*
a^4 + 5168*a^2 + 969)*b^8*c^8*d^4 + 33*(5*a^18 + 285*a^16 + 3220*a^14 + 158
76*a^12 + 42966*a^10 + 70070*a^8 + 70980*a^6 + 43860*a^4 + 15181*a^2 + 2261
)*b^6*c^9*d^3 + 55*(a^20 + 46*a^18 + 465*a^16 + 2184*a^14 + 5922*a^12 + 101
64*a^10 + 11466*a^8 + 8520*a^6 + 4029*a^4 + 1102*a^2 + 133)*b^4*c^10*d^2 +
11*(a^22 + 31*a^20 + 255*a^18 + 1065*a^16 + 2730*a^14 + 4662*a^12 + 5502*a^
```

```
10 + 4530*a^8 + 2565*a^6 + 955*a^4 + 211*a^2 + 21)*b^2*c^11*d + (a^24 + 12*
a^22 + 66*a^20 + 220*a^18 + 495*a^16 + 792*a^14 + 924*a^12 + 792*a^10 + 495
*a^8 + 220*a^6 + 66*a^4 + 12*a^2 + 1)*c^12 + (b^24*c*d^11 + 11*(a^2 + 21)*b
^22*c^2*d^10 + 55*(a^4 + 38*a^2 + 133)*b^20*c^3*d^9 + 33*(5*a^6 + 255*a^4 +
 1615*a^2 + 2261)*b^18*c^4*d^8 + 330*(a^8 + 60*a^6 + 510*a^4 + 1292*a^2 + 9
69)*b^16*c^5*d^7 + 22*(21*a^10 + 1365*a^8 + 13650*a^6 + 46410*a^4 + 62985*a
^2 + 29393)*b^14*c^6*d^6 + 22*(21*a^12 + 1386*a^10 + 15015*a^8 + 60060*a^6
+ 109395*a^4 + 92378*a^2 + 29393)*b^12*c^7*d^5 + 330*(a^14 + 63*a^12 + 693*
a^10 + 3003*a^8 + 6435*a^6 + 7293*a^4 + 4199*a^2 + 969)*b^10*c^8*d^4 + 33*(
5*a^16 + 280*a^14 + 2940*a^12 + 12936*a^10 + 30030*a^8 + 40040*a^6 + 30940*
a^4 + 12920*a^2 + 2261)*b^8*c^9*d^3 + 55*(a^18 + 45*a^16 + 420*a^14 + 1764*
a^12 + 4158*a^10 + 6006*a^8 + 5460*a^6 + 3060*a^4 + 969*a^2 + 133)*b^6*c^10
*d^2 + 11*(a^20 + 30*a^18 + 225*a^16 + 840*a^14 + 1890*a^12 + 2772*a^10 + 2
730*a^8 + 1800*a^6 + 765*a^4 + 190*a^2 + 21)*b^4*c^11*d + (a^22 + 11*a^20 +
 55*a^18 + 165*a^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 +
55*a^4 + 11*a^2 + 1)*b^2*c^12)*x^2 + 2*(11*(a^2 + 1)*b^21*c*d^10 + 110*(a^4
 + 8*a^2 + 7)*b^19*c^2*d^9 + 33*(15*a^6 + 205*a^4 + 589*a^2 + 399)*b^17*c^3
*d^8 + 264*(5*a^8 + 90*a^6 + 408*a^4 + 646*a^2 + 323)*b^15*c^4*d^7 + 110*(2
1*a^10 + 441*a^8 + 2562*a^6 + 6018*a^4 + 6137*a^2 + 2261)*b^13*c^5*d^6 + 4*
(693*a^12 + 15708*a^10 + 105105*a^8 + 308880*a^6 + 449735*a^4 + 319124*a^2
+ 88179)*b^11*c^6*d^5 + 110*(21*a^14 + 483*a^12 + 3465*a^10 + 11583*a^8 + 2
0735*a^6 + 20553*a^4 + 10659*a^2 + 2261)*b^9*c^7*d^4 + 264*(5*a^16 + 110*a^
14 + 798*a^12 + 2838*a^10 + 5720*a^8 + 6890*a^6 + 4930*a^4 + 1938*a^2 + 323
)*b^7*c^8*d^3 + 33*(15*a^18 + 295*a^16 + 2044*a^14 + 7308*a^12 + 15554*a^10
 + 20930*a^8 + 18060*a^6 + 9724*a^4 + 2983*a^2 + 399)*b^5*c^9*d^2 + 110*(a^
20 + 16*a^18 + 99*a^16 + 336*a^14 + 714*a^12 + 1008*a^10 + 966*a^8 + 624*a^
6 + 261*a^4 + 64*a^2 + 7)*b^3*c^10*d + 11*(a^22 + 11*a^20 + 55*a^18 + 165*a
^16 + 330*a^14 + 462*a^12 + 462*a^10 + 330*a^8 + 165*a^6 + 55*a^4 + 11*a^2
+ 1)*b*c^11 + (11*b^23*c*d^10 + 110*(a^2 + 7)*b^21*c^2*d^9 + 33*(15*a^4 + 1
90*a^2 + 399)*b^19*c^3*d^8 + 264*(5*a^6 + 85*a^4 + 323*a^2 + 323)*b^17*c^4*
d^7 + 110*(21*a^8 + 420*a^6 + 2142*a^4 + 3876*a^2 + 2261)*b^15*c^5*d^6 + 4*
(693*a^10 + 15015*a^8 + 90090*a^6 + 218790*a^4 + 230945*a^2 + 88179)*b^13*c
^6*d^5 + 110*(21*a^12 + 462*a^10 + 3003*a^8 + 8580*a^6 + 12155*a^4 + 8398*a
^2 + 2261)*b^11*c^7*d^4 + 264*(5*a^14 + 105*a^12 + 693*a^10 + 2145*a^8 + 35
75*a^6 + 3315*a^4 + 1615*a^2 + 323)*b^9*c^8*d^3 + 33*(15*a^16 + 280*a^14 +
1764*a^12 + 5544*a^10 + 10010*a^8 + 10920*a^6 + 7140*a^4 + 2584*a^2 + 399)*
b^7*c^9*d^2 + 110*(a^18 + 15*a^16 + 84*a^14 + 252*a^12 + 462*a^10 + 546*a^8
 + 420*a^6 + 204*a^4 + 57*a^2 + 7)*b^5*c^10*d + 11*(a^20 + 10*a^18 + 45*a^1
6 + 120*a^14 + 210*a^12 + 252*a^10 + 210*a^8 + 120*a^6 + 45*a^4 + 10*a^2 +
1)*b^3*c^11)*x^2 + 2*(11*a*b^22*c*d^10 + 110*(a...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctan(b*x + a)/(c*x^2 + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^2),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^2), x)
```

3.57 $\int \frac{\textbf{ArcTan}(a+bx)}{c+\frac{d}{x^3}}\,dx$

Optimal. Leaf size=933

$$-\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc}-\frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc}-\frac{i\sqrt[3]{d}\,\log(1-ia-ibx)\log\left(-\frac{b\left(\sqrt[3]{\cdots}\right.}{(i+a)\cdots}\right.}{6c^{4/3}}$$

[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b/c-1/6*I*d^(1/3)*ln(1-I*a-I*b*x)*ln(-b*(d^(1/3)+c^(1/3)*x)/((I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*I*d^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+c^(1/3)*x)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/6)*d^(1/3)*ln(1+I*a+I*b*x)*ln(-b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(I-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)-(-1)^(1/3)*c^(1/3)*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*ln(1+I*a+I*b*x)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(2/3)*(I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*ln(1-I*a-I*b*x)*ln(b*(d^(1/3)+(-1)^(2/3)*c^(1/3)*x)/((-1)^(1/6)*(1-I*a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(I-a-b*x)/((-1)^(1/3)*(I-a)*c^(1/3)-b*d^(1/3)))/c^(4/3)-1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(-1)^(1/6)*c^(1/3)*(I-a-b*x)/((-1)^(1/6)*(I-a)*c^(1/3)-I*b*d^(1/3)))/c^(4/3)+1/6*I*d^(1/3)*polylog(2,c^(1/3)*(I-a-b*x)/((I-a)*c^(1/3)+b*d^(1/3)))/c^(4/3)-1/6*I*d^(1/3)*polylog(2,c^(1/3)*(I+a+b*x)/((I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(5/6)*d^(1/3)*polylog(2,(-1)^(2/3)*c^(1/3)*(I+a+b*x)/((-1)^(2/3)*(I+a)*c^(1/3)-b*d^(1/3)))/c^(4/3)+1/6*(-1)^(1/6)*d^(1/3)*polylog(2,(-1)^(1/3)*c^(1/3)*(I+a+b*x)/((-1)^(1/3)*(I+a)*c^(1/3)+b*d^(1/3)))/c^(4/3)

Rubi [A]

time = 1.26, antiderivative size = 933, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5159, 2456, 2436, 2332, 2441, 2440, 2438}

[illegible]

Antiderivative was successfully verified.

[In] Int[ArcTan[a + b*x]/(c + d/x^3),x]

[Out] -1/2*((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(b*c) - ((1 - I*a - I*b*x)*Log[(-I)*(I + a + b*x)])/(2*b*c) - ((I/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[-((b*(d^(1/3) + c^(1/3)*x))/((I + a)*c^(1/3) - b*d^(1/3)))])/c^(4/3) + ((I/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + c^(1/3)*x))/((I - a)*c^(1/3) + b*d^(1/3))])/c^(4/3) - ((-1)^(1/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[-((b*(d^(1/3) - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I - a)*c^(1/3) - b*d^(1/3)

```
))])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3)
 - (-1)^(1/3)*c^(1/3)*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(
4/3)) - ((-1)^(5/6)*d^(1/3)*Log[1 + I*a + I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/
3)*c^(1/3)*x))/((-1)^(2/3)*(I - a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) + ((-
1)^(5/6)*d^(1/3)*Log[1 - I*a - I*b*x]*Log[(b*(d^(1/3) + (-1)^(2/3)*c^(1/3)*
x))/((-1)^(1/6)*(1 - I*a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(1/6)*
d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I - a - b*x))/((-1)^(1/3)*(I - a)*c
^(1/3) - b*d^(1/3))])/(6*c^(4/3)) - ((-1)^(5/6)*d^(1/3)*PolyLog[2, ((-1)^(1
/6)*c^(1/3)*(I - a - b*x))/((-1)^(1/6)*(I - a)*c^(1/3) - I*b*d^(1/3))])/(6*
c^(4/3)) + ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I - a - b*x))/((I - a)*c^(1/
3) + b*d^(1/3))])/c^(4/3) - ((I/6)*d^(1/3)*PolyLog[2, (c^(1/3)*(I + a + b*x
))/((I + a)*c^(1/3) - b*d^(1/3))])/c^(4/3) + ((-1)^(5/6)*d^(1/3)*PolyLog[2,
 ((-1)^(2/3)*c^(1/3)*(I + a + b*x))/((-1)^(2/3)*(I + a)*c^(1/3) - b*d^(1/3)
)])/(6*c^(4/3)) + ((-1)^(1/6)*d^(1/3)*PolyLog[2, ((-1)^(1/3)*c^(1/3)*(I + a
 + b*x))/((-1)^(1/3)*(I + a)*c^(1/3) + b*d^(1/3))])/(6*c^(4/3))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{x^3}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{x^3}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{x^3}}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\log(1-ia-ibx)}{c} - \frac{d\log(1-ia-ibx)}{c\,(d+cx^3)}\right)dx - \frac{1}{2}i\int \left(\frac{\log(1+ia+ibx)}{c} - \right.$$

$$= \frac{i\int \log(1-ia-ibx)\,dx}{2c} - \frac{i\int \log(1+ia+ibx)\,dx}{2c} - \frac{(id)\int \frac{\log(1-ia-ibx)}{d+cx^3}\,dx}{2c} + \frac{(id)\int}{}$$

$$= -\frac{\text{Subst}(\int \log(x)\,dx, x, 1-ia-ibx)}{2bc} - \frac{\text{Subst}(\int \log(x)\,dx, x, 1+ia+ibx)}{2bc} - \frac{(id)\int}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} + \frac{\left(i\sqrt[3]{d}\right)\int}{}$$

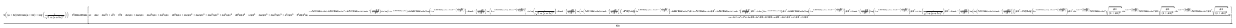
$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(}{}$$

$$= -\frac{(1+ia+ibx)\log(1+ia+ibx)}{2bc} - \frac{(1-ia-ibx)\log(-i(i+a+bx))}{2bc} - \frac{i\sqrt[3]{d}\,\log(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order

4 in optimal.
time = 4.14, size = 933, normalized size = 1.00



Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/x^3),x]
```

```
[Out] (6*((a + b*x)*ArcTan[a + b*x] + Log[1/Sqrt[1 + (a + b*x)^2]]) - b^3*d*RootS
um[I*c - 3*a*c - (3*I)*a^2*c + a^3*c - b^3*d - (3*I)*c*#1 + 3*a*c*#1 - (3*I
)*a^2*c*#1 + 3*a^3*c*#1 - 3*b^3*d*#1 + (3*I)*c*#1^2 + 3*a*c*#1^2 + (3*I)*a^
2*c*#1^2 + 3*a^3*c*#1^2 - 3*b^3*d*#1^2 - I*c*#1^3 - 3*a*c*#1^3 + (3*I)*a^2*
c*#1^3 + a^3*c*#1^3 - b^3*d*#1^3 & , (-(Pi*ArcTan[a + b*x]) - 2*ArcTan[a +
b*x]^2 + (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)] + I*Pi*Log[1 + E
^((-2*I)*ArcTan[a + b*x])] + (2*I)*ArcTan[a + b*x]*Log[1 - E^((2*I)*ArcTan[
a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTanh[(-1 + #1)/(1 + #1)]*
Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - I*Pi*L
og[1/Sqrt[1 + (a + b*x)^2]] + 2*ArcTanh[(-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[
a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]] + PolyLog[2, E^((2*I)*ArcTan[a +
 b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])] - 2*ArcTan[a + b*x]^2*#1 + Pi*ArcTa
n[a + b*x]*#1^2 - (2*I)*ArcTan[a + b*x]*ArcTanh[(-1 + #1)/(1 + #1)]*#1^2 -
I*Pi*Log[1 + E^((-2*I)*ArcTan[a + b*x])]*#1^2 - (2*I)*ArcTan[a + b*x]*Log[1
 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)])]*#1^2 + 2*ArcT
anh[(-1 + #1)/(1 + #1)]*Log[1 - E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 +
#1)/(1 + #1)])]*#1^2 + I*Pi*Log[1/Sqrt[1 + (a + b*x)^2]]*#1^2 - 2*ArcTanh[(
-1 + #1)/(1 + #1)]*Log[Sin[ArcTan[a + b*x] + I*ArcTanh[(-1 + #1)/(1 + #1)]]
]*#1^2 - PolyLog[2, E^((2*I)*ArcTan[a + b*x] - 2*ArcTanh[(-1 + #1)/(1 + #1)
])]*#1^2 + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*Sqrt[#1/(1 + #1
)^2] + 4*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1*Sqrt[#1/(1 + #1)
^2] + 2*E^ArcTanh[(1 - #1)/(1 + #1)]*ArcTan[a + b*x]^2*#1^2*Sqrt[#1/(1 + #1
)^2])/(-(a*c) - (2*I)*a^2*c + a^3*c - b^3*d + 2*a*c*#1 + 2*a^3*c*#1 - 2*b^3
*d*#1 - a*c*#1^2 + (2*I)*a^2*c*#1^2 + a^3*c*#1^2 - b^3*d*#1^2) & ])/(6*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 673, normalized size = 0.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctan(b*x+a)/c*(b*x+a)-2/3/c*d*b^3*sum(_R1^2/(a^3*c*_R1^4+3*I*a^2*c*_
R1^4-b^3*d*_R1^4+2*a^3*c*_R1^2+2*I*a^2*c*_R1^2-3*_R1^4*a*c-2*b^3*d*_R1^2-I*
c*_R1^4+a^3*c-I*a^2*c+2*a*c*_R1^2-b^3*d+2*I*c*_R1^2+a*c-I*c)*(I*arctan(b*x+
a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilog((_R1-(1+I*(b*x+a))
/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((3*I*a^2*c+a^3*c-b^3*d-I*c-3*a*c)*_Z
^6+(3*I*a^2*c+3*a^3*c-3*b^3*d+3*I*c+3*a*c)*_Z^4+(-3*I*a^2*c+3*a^3*c-3*b^3*d
-3*I*c+3*a*c)*_Z^2-3*I*a^2*c+a^3*c-b^3*d+I*c-3*a*c))-2/3/c*d*b^3*sum(1/(a^3
```

```
*c*_R1^4+3*I*a^2*c*_R1^4-b^3*d*_R1^4+2*a^3*c*_R1^2+2*I*a^2*c*_R1^2-3*_R1^4*
a*c-2*b^3*d*_R1^2-I*c*_R1^4+a^3*c-I*a^2*c+2*a*c*_R1^2-b^3*d+2*I*c*_R1^2+a*c
-I*c)*(I*arctan(b*x+a)*ln((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)+dilo
g((_R1-(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))/_R1)),_R1=RootOf((3*I*a^2*c+a^3*c
-b^3*d-I*c-3*a*c)*_Z^6+(3*I*a^2*c+3*a^3*c-3*b^3*d+3*I*c+3*a*c)*_Z^4+(-3*I*a
^2*c+3*a^3*c-3*b^3*d-3*I*c+3*a*c)*_Z^2-3*I*a^2*c+a^3*c-b^3*d+I*c-3*a*c))-1/
2/c*ln(1+(b*x+a)^2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="maxima")
[Out] integrate(arctan(b*x + a)/(c + d/x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="fricas")
[Out] integral(x^3*arctan(b*x + a)/(c*x^3 + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**3),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{c+\frac{d}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^3),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^3), x)
```

3.58 $\int \frac{\text{ArcTan}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=673

$$\frac{2i\sqrt{i+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d}-\frac{2i\sqrt{i-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d}+\frac{ic\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+a\ldots\right.}{d^2}\ldots$$

```
[Out] -I*c*ln(1-I*a-I*b*x)*ln(c+d*x^(1/2))/d^2+I*c*ln(1+I*a+I*b*x)*ln(c+d*x^(1/2)
)/d^2+I*c*ln(c+d*x^(1/2))*ln(d*((-I-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-I-a)^(1/
2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(d*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*ln(c+d*x^(1/2))*ln(-d*((-I-a)^(1/2)+b^(1/
2)*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*ln(c+d*x^(1/2))*ln(-d*((I-
a)^(1/2)+b^(1/2)*x^(1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(
1/2)*(c+d*x^(1/2))/(-d*(-I-a)^(1/2)+c*b^(1/2)))/d^2+I*c*polylog(2,b^(1/2)*(
c+d*x^(1/2))/(d*(-I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(
1/2))/(-d*(I-a)^(1/2)+c*b^(1/2)))/d^2-I*c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(
d*(I-a)^(1/2)+c*b^(1/2)))/d^2-2*I*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a
)^(1/2)/d/b^(1/2)+2*I*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/d/b^(
1/2)+I*ln(1-I*a-I*b*x)*x^(1/2)/d-I*ln(1+I*a+I*b*x)*x^(1/2)/d
```

Rubi [A]
time = 0.73, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {5159, 2455, 2516, 2498, 327, 211, 2512, 266, 2463, 2441, 2440, 2438, 214}

$$\frac{2i\sqrt{a+i}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+i}}\right)}{\sqrt{b}\,d}+\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-i}\,d}\right)}{d^2}+\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-a-i}\,d}\right)}{d^2}-\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right)}{d^2}-\frac{ic\text{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)}{d^2}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\,\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}\,c+\sqrt{-a-i}\,d}\right)}{d^2}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(-\sqrt{b}\,\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}\,c+\sqrt{-a+i}\,d}\right)}{d^2}+\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\,\sqrt{x}+\sqrt{-a-i}\right)}{\sqrt{b}\,c-\sqrt{-a-i}\,d}\right)}{d^2}-\frac{ic\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{b}\,\sqrt{x}+\sqrt{-a+i}\right)}{\sqrt{b}\,c-\sqrt{-a+i}\,d}\right)}{d^2}-\frac{ic\log(-ia-ibx+1)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{ic\log(ia+ibx+1)\log\left(c+d\sqrt{x}\right)}{d^2}+\frac{i\sqrt{x}\,\log(-ia-ibx+1)}{d}-\frac{i\sqrt{x}\,\log(ia+ibx+1)}{d}-\frac{2i\sqrt{-a+i}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{-a+i}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*I)*Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*d) - ((2
*I)*Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*d) + (I*c*
Log[(d*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[
c + d*Sqrt[x]])/d^2 - (I*c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]
*c + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (I*c*Log[-((d*(Sqrt[-I - a]
+ Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d))]*Log[c + d*Sqrt[x]])/d^2
- (I*c*Log[-((d*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d
))]*Log[c + d*Sqrt[x]])/d^2 + (I*Sqrt[x]*Log[1 - I*a - I*b*x])/d - (I*c*Log
[c + d*Sqrt[x]]*Log[1 - I*a - I*b*x])/d^2 - (I*Sqrt[x]*Log[1 + I*a + I*b*x]
)/d + (I*c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x])/d^2 + (I*c*PolyLog[2, (
Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)])/d^2 + (I*c*PolyLog[
2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)])/d^2 - (I*c*Poly
```

```
Log[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)])/d^2 - (I*c*P
olyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+d\sqrt{x}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+d\sqrt{x}}\,dx$$

$$= i\text{Subst}\left(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+dx}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+dx}\,dx, x,\right.$$

$$= i\text{Subst}\left(\int \left(\frac{\log\left(1-ia-ibx^2\right)}{d} - \frac{c\log\left(1-ia-ibx^2\right)}{d(c+dx)}\right)dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \left(\right.\right.$$

$$= \frac{i\text{Subst}\left(\int \log\left(1-ia-ibx^2\right)dx, x, \sqrt{x}\right)}{d} - \frac{i\text{Subst}\left(\int \log\left(1+ia+ibx^2\right)dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ib}{d}$$

$$= \frac{i\sqrt{x}\log(1-ia-ibx)}{d} - \frac{ic\log\left(c+d\sqrt{x}\right)\log(1-ia-ibx)}{d^2} - \frac{i\sqrt{x}\log(1+ia+ib}{d}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{i\sqrt{x}\log(1-ia-i}{d}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

$$= \frac{2i\sqrt{i+a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,d} - \frac{2i\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,d} + \frac{ic\log\left(\frac{d\left(\sqrt{-i-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

Mathematica [A]
time = 0.42, size = 604, normalized size = 0.90

$$\frac{i\left(\frac{2\sqrt{i+a}\,d\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}} - \frac{2\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}} + c\log\left(\frac{d\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\log\left(c+d\sqrt{x}\right) + c\log\left(\frac{d\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{-i-a}\,d}\right)\log\left(c+d\sqrt{x}\right) - c\log\left(\frac{d\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\log\left(c+d\sqrt{x}\right) - d\sqrt{x}\log(1+ia+ibx) + c\log\left(c+d\sqrt{x}\right)\log(1+ia+ibx) + d\sqrt{x}\log(-i(i+a+bx)) - c\log\left(c+d\sqrt{x}\right)\log(-i(i+a+bx)) + c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-i-a}\,d}\right) + c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-i-a}\,d}\right) - c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{i-a}\,d}\right) - c\text{PolyLog}\left(2, \frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{i-a}\,d}\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (I*((2*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/Sqrt[b] - (2*Sq
rt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/Sqrt[b] + c*Log[(d*(Sqr
t[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[
x]] - c*Log[(d*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)
]*Log[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b
]*c) + Sqrt[-I - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[I - a] + Sqrt[b
]*Sqrt[x]))/(-(Sqrt[b]*c) + Sqrt[I - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*
Log[1 + I*a + I*b*x] + c*Log[c + d*Sqrt[x]]*Log[1 + I*a + I*b*x] + d*Sqrt[x
]*Log[(-I)*(I + a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(-I)*(I + a + b*x)] +
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-I - a]*d)] + c*Po
lyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-I - a]*d)] - c*PolyLo
g[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[I - a]*d)] - c*PolyLog[2,
(Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[I - a]*d)]))/d^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 364, normalized size = 0.54

method	result
derivativedivides	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(d^2\left(\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2abd^2+6b^2c^2\right)\_Z\right.\right.\right.}{}$
default	$\frac{2\arctan(bx+a)\sqrt{x}}{d}-\frac{2\arctan(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}-\frac{4b\left(d^2\left(\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2c\_Z^3+\left(2abd^2+6b^2c^2\right)\_Z\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(b*x+a)/d*x^(1/2)-2*arctan(b*x+a)*c/d^2*ln(c+d*x^(1/2))-4*b/d^2*(1/
4*d^2/b*sum((_R^2-2*_R*c+c^2)/(_R^3*b-3*_R^2*b*c+_R*a*d^2+3*_R*b*c^2-a*c*d^
2-b*c^3)*ln(d*x^(1/2)-_R+c),_R=RootOf(b^2*_Z^4-4*b^2*c*_Z^3+(2*a*b*d^2+6*b^
2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4+d^4))
```

```
-1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*c+a*d^2+b*c^2)*(ln(c+d*x^(1/2))*ln((-d*
x^(1/2)+_R1-c)/_R1)+dilog((-d*x^(1/2)+_R1-c)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^
2*c*_Z^3+(2*a*b*d^2+6*b^2*c^2)*_Z^2+(-4*a*b*c*d^2-4*b^2*c^3)*_Z+a^2*d^4+2*a
*b*c^2*d^2+b^2*c^4+d^4)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arctan(b*x + a) - c*arctan(b*x + a))/(d^2*x - c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d*x^(1/2)),x)
[Out] int(atan(a + b*x)/(c + d*x^(1/2)), x)
```

3.59 $\int \frac{\text{ArcTan}(a+bx)}{c+\frac{d}{\sqrt{x}}} \, dx$

Optimal. Leaf size=770

$$-\frac{2i\sqrt{i+a}\, d\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\, c^2}+\frac{2i\sqrt{i-a}\, d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\, c^2}-\frac{id^2\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\log(d}{c^3}$$

```
[Out] -1/2*(1+I*a+I*b*x)*ln(1+I*a+I*b*x)/b/c-1/2*(1-I*a-I*b*x)*ln(-I*(I+a+b*x))/b
/c+I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(I-a)^(1/2)-d*b^(1/2)))/c^3-I*
d^2*ln(1+I*a+I*b*x)*ln(d+c*x^(1/2))/c^3+I*d^2*polylog(2,b^(1/2)*(d+c*x^(1/2
))/(c*(I-a)^(1/2)+d*b^(1/2)))/c^3-I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)-
b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3-I*d^2*polylog(2,b^(1/2)*(d
+c*x^(1/2))/(c*(-I-a)^(1/2)+d*b^(1/2)))/c^3+I*d^2*ln(1-I*a-I*b*x)*ln(d+c*x^
(1/2))/c^3-I*d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-I-a)^(1/2)-d*b^(1/2)
))/c^3+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(I-a)^(1
/2)-d*b^(1/2)))/c^3+I*d*ln(1+I*a+I*b*x)*x^(1/2)/c^2-I*d*ln(1-I*a-I*b*x)*x^(
1/2)/c^2-2*I*d*arctan(b^(1/2)*x^(1/2)/(I+a)^(1/2))*(I+a)^(1/2)/c^2/b^(1/2)-
I*d^2*ln(d+c*x^(1/2))*ln(c*((-I-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-I-a)^(1/2)-d
*b^(1/2)))/c^3+2*I*d*arctanh(b^(1/2)*x^(1/2)/(I-a)^(1/2))*(I-a)^(1/2)/c^2/b
^(1/2)+I*d^2*ln(d+c*x^(1/2))*ln(c*((I-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(I-a)^(1
/2)+d*b^(1/2)))/c^3
```

Rubi [A]
time = 0.81, antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5159, 2455, 2526, 2498, 327, 211, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438, 214}

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((-2*I)*Sqrt[I + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]])/(Sqrt[b]*c^2)
+ ((2*I)*Sqrt[I - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I - a]])/(Sqrt[b]*c^2
) - (I*d^2*Log[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[
b]*d)]*Log[d + c*Sqrt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[
x]))/(Sqrt[I - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d^2*Log[(c*(
Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)]*Log[d + c*Sq
rt[x]])/c^3 + (I*d^2*Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c
 - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (I*d*Sqrt[x]*Log[1 - I*a - I*b*x])
```

```
/c^2 + (I*d^2*Log[d + c*Sqrt[x]]*Log[1 - I*a - I*b*x])/c^3 + (I*d*Sqrt[x]*L
og[1 + I*a + I*b*x])/c^2 - ((1 + I*a + I*b*x)*Log[1 + I*a + I*b*x])/(2*b*c)
 - (I*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x])/c^3 - ((1 - I*a - I*b*x)
*Log[(-I)*(I + a + b*x)])/(2*b*c) - (I*d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqr
t[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d))])/c^3 + (I*d^2*PolyLog[2, -((Sqrt[b]*(
d + c*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d))])/c^3 - (I*d^2*PolyLog[2, (Sqr
t[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)])/c^3 + (I*d^2*PolyLog[2
, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[b]*d)])/c^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_)^(r_))^(q_.), x_Symbol] :> With[{k = Denominator[r]}, Dist[k, Subst[
Int[x^(k - 1)*(f + g*x^(k*r))^q*(a + b*Log[c*(d + e*x^k)^n])^p, x], x, x^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && FractionQ[r] && IG
tQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
```

```
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{c+\frac{d}{\sqrt{x}}}\,dx - \frac{1}{2}i\int \frac{\log(1+ia+ibx)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= i\text{Subst}\Bigg(\int \frac{x\log\left(1-ia-ibx^2\right)}{c+\frac{d}{x}}\,dx, x, \sqrt{x}\Bigg) - i\text{Subst}\Bigg(\int \frac{x\log\left(1+ia+ibx^2\right)}{c+\frac{d}{x}}\,dx, x,$$

$$= i\text{Subst}\Bigg(\int \Bigg(-\frac{d\log\left(1-ia-ibx^2\right)}{c^2}+\frac{x\log\left(1-ia-ibx^2\right)}{c}+\frac{d^2\log\left(1-ia-ibx^2\right)}{c^2(d+cx)}\Bigg)$$

$$= \frac{i\text{Subst}\left(\int x\log\left(1-ia-ibx^2\right)\,dx, x, \sqrt{x}\right)}{c} - \frac{i\text{Subst}\big(\int x\log\left(1+ia+ibx^2\right)\,dx, x, \sqrt{}}{c}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2}+\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3}+\frac{id\sqrt{x}\log(1+ia}{c^2}$$

$$= -\frac{id\sqrt{x}\log(1-ia-ibx)}{c^2}+\frac{id^2\log\left(d+c\sqrt{x}\right)\log(1-ia-ibx)}{c^3}+\frac{id\sqrt{x}\log(1+ia}{c^2}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2}+\frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2}-\frac{id\sqrt{x}\log(1-i}{c^2}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2}+\frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2}-\frac{id^2\log\Bigg(\frac{c\big(\sqrt{-}}{\sqrt{-}}}{}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2}+\frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2}-\frac{id^2\log\Bigg(\frac{c\big(\sqrt{-}}{\sqrt{-}}}{}$$

$$= -\frac{2i\sqrt{i+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}\,c^2}+\frac{2i\sqrt{i-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}\,c^2}-\frac{id^2\log\Bigg(\frac{c\big(\sqrt{-}}{\sqrt{-}}}{}$$

Mathematica [A]
time = 0.60, size = 666, normalized size = 0.86

$$\frac{i\left(4cd\left(\sqrt{x}-\frac{\sqrt{i+a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i+a}}\right)}{\sqrt{b}}\right)-4cd\left(\sqrt{x}-\frac{\sqrt{i-a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{i-a}}\right)}{\sqrt{b}}\right)+2cd\sqrt{x}\log(1+ia+ibx)-\frac{c^2(-i+a+bx)\log(1+ia+ibx)}{b}-2d^2\log\left(d+c\sqrt{x}\right)\log(1+ia+ibx)-2cd\sqrt{x}\log(-i(i+a+bx))+\frac{c^2(i+a+bx)\log(-i(i+a+bx))}{b}+2d^2\log\left(d+c\sqrt{x}\right)\log(-i(i+a+bx))-2d^2\left(\left(\log\left(\frac{c\left(\sqrt{-i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)+\log\left(\frac{c\left(\sqrt{-i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-i-a}\,c-\sqrt{b}\,d}\right)\right)\log\left(d+c\sqrt{x}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-i-a}\,c+\sqrt{b}\,d}\right)\right)+2d^2\left(\left(\log\left(\frac{c\left(\sqrt{i-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right)+\log\left(\frac{c\left(\sqrt{i-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{i-a}\,c-\sqrt{b}\,d}\right)\right)\log\left(d+c\sqrt{x}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{i-a}\,c+\sqrt{b}\,d}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{i-a}\,c+\sqrt{b}\,d}\right)\right)\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] ((I/2)*(4*c*d*(Sqrt[x] - (Sqrt[I + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[I + a]]
)/Sqrt[b]) - 4*c*d*(Sqrt[x] - (Sqrt[I - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[I
 - a]])/Sqrt[b]) + 2*c*d*Sqrt[x]*Log[1 + I*a + I*b*x] - (c^2*(-I + a + b*x)
*Log[1 + I*a + I*b*x])/b - 2*d^2*Log[d + c*Sqrt[x]]*Log[1 + I*a + I*b*x] -
2*c*d*Sqrt[x]*Log[(-I)*(I + a + b*x)] + (c^2*(I + a + b*x)*Log[(-I)*(I + a
+ b*x)])/b + 2*d^2*Log[d + c*Sqrt[x]]*Log[(-I)*(I + a + b*x)] - 2*d^2*((Log
[(c*(Sqrt[-I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)] + Log[(
c*(Sqrt[-I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[-I - a]*c - Sqrt[b]*d)])*Log[d +
c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-I - a]*c) + Sqrt
[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-I - a]*c + Sqrt[b]*d)
]) + 2*d^2*((Log[(c*(Sqrt[I - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c + Sqrt[
b]*d)] + Log[(c*(Sqrt[I - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[I - a]*c - Sqrt[b]*d
)])*Log[d + c*Sqrt[x]] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[I - a
]*c) + Sqrt[b]*d)] + PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[I - a]*c +
Sqrt[b]*d)])))/c^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 388, normalized size = 0.50

method	result
derivativedivides	$\frac{\arctan(bx+a)x}{c} - \frac{2\arctan(bx+a)d\sqrt{x}}{c^2} + \frac{2\arctan(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3} - \frac{4b\left(-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\ldots\right.}\right.}{}\right.}{}$
default	$\frac{\arctan(bx+a)x}{c} - \frac{2\arctan(bx+a)d\sqrt{x}}{c^2} + \frac{2\arctan(bx+a)d^2\ln\left(d+c\sqrt{x}\right)}{c^3} - \frac{4b\left(-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^4-4b^2d\ldots\right.}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(b*x+a)/c*x-2*arctan(b*x+a)/c^2*d*x^(1/2)+2*arctan(b*x+a)*d^2/c^3*ln(
d+c*x^(1/2))-4*b/c^2*(-1/8*c/b*sum((-_R^3+5*_R^2*d-7*_R*d^2+3*d^3)/(_R^3*b-
```

```
3*_R^2*b*d+_R*a*c^2+3*_R*b*d^2-a*c^2*d-b*d^3)*ln(c*x^(1/2)-_R+d),_R=RootOf(
b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-4*a*b*c^2*d-4*b^2*d^3)*_
Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4))+1/4*c*d^2/b*sum(1/(_R1^2*b-2*_R1*b*d+
a*c^2+b*d^2)*(ln(d+c*x^(1/2))*ln((-c*x^(1/2)+_R1-d)/_R1)+dilog((-c*x^(1/2)+
_R1-d)/_R1)),_R1=RootOf(b^2*_Z^4-4*b^2*d*_Z^3+(2*a*b*c^2+6*b^2*d^2)*_Z^2+(-
4*a*b*c^2*d-4*b^2*d^3)*_Z+a^2*c^4+2*a*b*c^2*d^2+b^2*d^4+c^4)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arctan(b*x + a)/(c + d/sqrt(x)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arctan(b*x + a) - d*sqrt(x)*arctan(b*x + a))/(c^2*x - d^2), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c + d/x^(1/2)),x)
```

```
[Out] int(atan(a + b*x)/(c + d/x^(1/2)), x)
```

3.60 $\int \frac{\mathbf{ArcTan}(a+bx)}{1+x^2}\,dx$

Optimal. Leaf size=274

$$\frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+i$$

```
[Out] 1/4*ln(b*(I-x)/(a+I*(1+b)))*ln(1-I*a-I*b*x)-1/4*ln(-b*(I+x)/(a+I*(1-b)))*ln
(1-I*a-I*b*x)-1/4*ln(b*(I-x)/(a-I*(1-b)))*ln(1+I*a+I*b*x)+1/4*ln(-b*(I+x)/(
a-I*(1+b)))*ln(1+I*a+I*b*x)-1/4*polylog(2,(-I+a+b*x)/(a-I*(1-b)))+1/4*polyl
og(2,(-I+a+b*x)/(a-I*(1+b)))-1/4*polylog(2,(I+a+b*x)/(I+a-I*b))+1/4*polylog
(2,(I+a+b*x)/(a+I*(1+b)))
```

Rubi [A]

time = 0.22, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5159, 2456, 2441, 2440, 2438}

$$-\frac{1}{4}\text{Li}_2\left(-\frac{-a-bx+i}{a-i(1-b)}\right)+\frac{1}{4}\text{Li}_2\left(-\frac{-a-bx+i}{a-i(b+1)}\right)-\frac{1}{4}\text{Li}_2\left(\frac{a+bx+i}{a-ib+i}\right)+\frac{1}{4}\text{Li}_2\left(\frac{a+bx+i}{a+i(b+1)}\right)+\frac{1}{4}\log\left(\frac{b(-x+i)}{a+i(b+1)}\right)\log(-ia-ibx+1)-\frac{1}{4}\log\left(-\frac{b(x+i)}{a+i(1-b)}\right)\log(-ia-ibx+1)-\frac{1}{4}\log\left(\frac{b(-x+i)}{a-i(1-b)}\right)\log(ia+ibx+1)+\frac{1}{4}\log\left(-\frac{b(x+i)}{a-i(b+1)}\right)\log(ia+ibx+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, -((I - a - b*x)/(a - I*(1 - b)))]/4 + PolyL
og[2, -((I - a - b*x)/(a - I*(1 + b)))]/4 - PolyLog[2, (I + a + b*x)/(I + a
 - I*b)]/4 + PolyLog[2, (I + a + b*x)/(a + I*(1 + b))]/4
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(a+bx)}{1+x^2}\,dx &= \frac{1}{2}i\int \frac{\log(1-ia-ibx)}{1+x^2}\,dx-\frac{1}{2}i\int \frac{\log(1+ia+ibx)}{1+x^2}\,dx\\
&= \frac{1}{2}i\int \left(\frac{i\log(1-ia-ibx)}{2(i-x)}+\frac{i\log(1-ia-ibx)}{2(i+x)}\right)dx-\frac{1}{2}i\int \left(\frac{i\log(1+ia+ibx)}{2(i-x)}+\right.\\
&= -\left(\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-ia-ibx)}{i+x}\,dx+\frac{1}{4}\int \frac{\log(1+ia+ibx)}{i-x}\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\\
&= \frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 283, normalized size = 1.03

$$\frac{1}{4}\log\left(\frac{b(i-x)}{a+i(1+b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(-\frac{b(i+x)}{a+i(1-b)}\right)\log(1-ia-ibx)-\frac{1}{4}\log\left(\frac{b(i-x)}{a-i(1-b)}\right)\log(1+ia+ibx)+\frac{1}{4}\log\left(-\frac{b(i+x)}{a-i(1+b)}\right)\log(1+ia+ibx)-\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{1-ia-ibx}{1-ia-b}\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{1-ia-ibx}{1-ia+b}\right)-\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{1+ia+ibx}{1+ia-b}\right)+\frac{1}{4}\mathrm{PolyLog}\left(2,\frac{1+ia+ibx}{1+ia+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/(1 + x^2),x]
```

```
[Out] (Log[(b*(I - x))/(a + I*(1 + b))]*Log[1 - I*a - I*b*x])/4 - (Log[-((b*(I +
x))/(a + I*(1 - b)))]*Log[1 - I*a - I*b*x])/4 - (Log[(b*(I - x))/(a - I*(1
```

```
- b))]*Log[1 + I*a + I*b*x])/4 + (Log[-((b*(I + x))/(a - I*(1 + b)))]*Log[1
 + I*a + I*b*x])/4 - PolyLog[2, (1 - I*a - I*b*x)/(1 - I*a - b)]/4 + PolyLo
g[2, (1 - I*a - I*b*x)/(1 - I*a + b)]/4 - PolyLog[2, (1 + I*a + I*b*x)/(1 +
 I*a - b)]/4 + PolyLog[2, (1 + I*a + I*b*x)/(1 + I*a + b)]/4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(225) = 450$.
time = 0.31, size = 501, normalized size = 1.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)*arctan(b*x+a)-b*(-1/2*I/b*arctan(x)*ln(1-(-I*b+a-I)*(1+I*x)^2/(x^
2+1)/(-I*b+I-a))-1/2/b*arctan(x)^2-1/4/b*polylog(2,(-I*b+a-I)*(1+I*x)^2/(x^
2+1)/(-I*b+I-a))-1/2/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*x)^2/(x^2+1)/(-I*b-I-a))
*arctan(x)-1/2/b/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*x)^2/(x^2+1)/(-I*b-I-a))*arc
tan(x)+1/2*I/b/(I*b+I+a)*ln(1-(I+a-I*b)*(1+I*x)^2/(x^2+1)/(-I*b-I-a))*a*arc
tan(x)+1/2*I/(I*b+I+a)*arctan(x)^2+1/4*I/(I*b+I+a)*polylog(2,(I+a-I*b)*(1+I
*x)^2/(x^2+1)/(-I*b-I-a))+1/2*I/b/(I*b+I+a)*arctan(x)^2+1/2/b/(I*b+I+a)*a*a
rctan(x)^2+1/4*I/b/(I*b+I+a)*polylog(2,(I+a-I*b)*(1+I*x)^2/(x^2+1)/(-I*b-I-
a))+1/4/b/(I*b+I+a)*polylog(2,(I+a-I*b)*(1+I*x)^2/(x^2+1)/(-I*b-I-a))*a)
```

Maxima [A]
time = 0.51, size = 328, normalized size = 1.20

$$\frac{1}{8}b\left(\frac{8\arctan\left(x\right)\arctan\left(\frac{b^2x+ab}{b}\right)}{b}-\frac{4\arctan\left(x\right)\arctan\left(\frac{ab+(b^2+b)x}{a^2+b^2+2b+1},\frac{abx+a^2+b+1}{a^2+b^2+2b+1}\right)-4\arctan\left(x\right)\arctan\left(\frac{ab+(b^2-b)x}{a^2+b^2-2b+1},\frac{abx+a^2-b+1}{a^2+b^2-2b+1}\right)+\log\left(x^2+1\right)\log\left(\frac{b^2x^2+2abx+a^2+1}{a^2+b^2+2b+1}\right)-\log\left(x^2+1\right)\log\left(\frac{b^2x^2+2abx+a^2+1}{a^2+b^2-2b+1}\right)+2\,\mathrm{Li}_2\left(-\frac{ibx-b}{ia+b+1}\right)-2\,\mathrm{Li}_2\left(-\frac{ibx-b}{ia+b-1}\right)+2\,\mathrm{Li}_2\left(\frac{ibx+b}{-ia+b+1}\right)-2\,\mathrm{Li}_2\left(\frac{ibx+b}{-ia+b-1}\right)}{b}\right)+\arctan\left(bx+a\right)\arctan\left(x\right)-\arctan\left(x\right)\arctan\left(\frac{b^2x+ab}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*b*(8*arctan(x)*arctan((b^2*x + a*b)/b)/b - (4*arctan(x)*arctan2((a*b +
(b^2 + b)*x)/(a^2 + b^2 + 2*b + 1), (a*b*x + a^2 + b + 1)/(a^2 + b^2 + 2*b
+ 1)) - 4*arctan(x)*arctan2((a*b + (b^2 - b)*x)/(a^2 + b^2 - 2*b + 1), (a*b
*x + a^2 - b + 1)/(a^2 + b^2 - 2*b + 1)) + log(x^2 + 1)*log((b^2*x^2 + 2*a*
b*x + a^2 + 1)/(a^2 + b^2 + 2*b + 1)) - log(x^2 + 1)*log((b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(a^2 + b^2 - 2*b + 1)) + 2*dilog(-(I*b*x - b)/(I*a + b + 1)) -
2*dilog(-(I*b*x - b)/(I*a + b - 1)) + 2*dilog((I*b*x + b)/(-I*a + b + 1)) -
 2*dilog((I*b*x + b)/(-I*a + b - 1)))/b) + arctan(b*x + a)*arctan(x) - arct
an(x)*arctan((b^2*x + a*b)/b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(arctan(b*x + a)/(x^2 + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(a+bx)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(x**2+1),x)
```

```
[Out] Integral(atan(a + b*x)/(x**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(x^2 + 1),x)
```

```
[Out] int(atan(a + b*x)/(x^2 + 1), x)
```

3.61 $\int \frac{\mathbf{ArcTan}(d+ex)}{a+bx^2}\,dx$

Optimal. Leaf size=543

$$\frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(1-id-iex)}{4\sqrt{-a}\,\sqrt{b}}-\frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right)\log(1-id-iex)}{4\sqrt{-a}\,\sqrt{b}}-\frac{i\log\left(-\frac{e\left(\sqrt{-a}\right.}{\sqrt{b}\,(i-}\right.}{}$$

```
[Out] -1/4*I*ln(1+I*d+I*e*x)*ln(-e*((-a)^(1/2)-x*b^(1/2))/(-e*(-a)^(1/2)+(I-d)*b^
(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1-I*d-I*e*x)*ln(e*((-a)^(1/2)-x*b^(1/2)
)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*ln(1+I*d+I*e*x)*ln
(e*((-a)^(1/2)+x*b^(1/2))/(e*(-a)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-
1/4*I*ln(1-I*d-I*e*x)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+(I+d)*b^(
1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(-e*(-a)^(1/2)+
(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I-d-e*x)*b^(1/2)/(e*(-a
)^(1/2)+(I-d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)-1/4*I*polylog(2,(I+d+e*x)*b^(1/2
)/(-e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)+1/4*I*polylog(2,(I+d+e*
x)*b^(1/2)/(e*(-a)^(1/2)+(I+d)*b^(1/2)))/(-a)^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5159, 2456, 2441, 2440, 2438}

$$-\frac{i\text{Li}_2\left(\frac{\sqrt{b}\,(-d-ex+i)}{\sqrt{b}\,(i-d)-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{i\text{Li}_2\left(\frac{\sqrt{b}\,(-d-ex+i)}{\sqrt{b}\,(i-d)+\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{i\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex+i)}{\sqrt{b}\,(d+i)-\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{i\text{Li}_2\left(\frac{\sqrt{b}\,(d+ex+i)}{\sqrt{b}\,(d+i)+\sqrt{-a}\,e}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{i\log(-id-iex+1)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,(d+i)}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{i\log(-id-iex+1)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{-a}\,e+\sqrt{b}\,(d+i)}\right)}{4\sqrt{-a}\,\sqrt{b}}-\frac{i\log(id+iex+1)\log\left(-\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{-\sqrt{-a}\,e+\sqrt{b}\,(-d+i)}\right)}{4\sqrt{-a}\,\sqrt{b}}+\frac{i\log(id+iex+1)\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{-a}\,e+\sqrt{b}\,(-d+i)}\right)}{4\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]*Log[1
 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*Log[-((e*(Sqrt[-a] + Sqrt[b]*x
))/(Sqrt[b]*(I + d) - Sqrt[-a]*e))]*Log[1 - I*d - I*e*x])/(Sqrt[-a]*Sqrt[b]
) - ((I/4)*Log[-((e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(I - d) - Sqrt[-a]*e))
]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(Sqrt[b]*(I - d) + Sqrt[-a]*e)]*Log[1 + I*d + I*e*x])/(Sqrt[-a]*Sqr
t[b]) - ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqrt[b]*(I - d) - Sqrt[-
a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/4)*PolyLog[2, (Sqrt[b]*(I - d - e*x))/(Sqr
t[b]*(I - d) + Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) - ((I/4)*PolyLog[2, (Sqrt[b
]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)])/(Sqrt[-a]*Sqrt[b]) + ((I/
4)*PolyLog[2, (Sqrt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)])/(Sqr
t[-a]*Sqrt[b])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5159

```
Int[ArcTan[(a_) + (b_.)*(x_)]/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Dist[
I/2, Int[Log[1 - I*a - I*b*x]/(c + d*x^n), x], x] - Dist[I/2, Int[Log[1 + I
*a + I*b*x]/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[n]
```

Rubi steps

$$\int \frac{\tan^{-1}(d+ex)}{a+bx^2}\,dx = \frac{1}{2}i\int \frac{\log(1-id-iex)}{a+bx^2}\,dx - \frac{1}{2}i\int \frac{\log(1+id+iex)}{a+bx^2}\,dx$$

$$= \frac{1}{2}i\int \left(\frac{\sqrt{-a}\,\log(1-id-iex)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\log(1-id-iex)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx - \frac{1}{2}i\int \left(\frac{\sqrt{-a}\,\log}{2a\left(\sqrt{-\right.}\right.}\right.$$

$$= -\frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}} - \frac{i\int \frac{\log(1-id-iex)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}} + \frac{i\int \frac{\log(1+id+iex)}{\sqrt{-a}+\sqrt{b}\,x}}{4\sqrt{-a}}$$

$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(1-id-iex)}{4\sqrt{-a}\,\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right)\log(1-id}{4\sqrt{-a}\,\sqrt{b}}$$

$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(1-id-iex)}{4\sqrt{-a}\,\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right)\log(1-id}{4\sqrt{-a}\,\sqrt{b}}$$

$$= \frac{i\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(1-id-iex)}{4\sqrt{-a}\,\sqrt{b}} - \frac{i\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right)\log(1-id}{4\sqrt{-a}\,\sqrt{b}}$$

<u>Mathematica</u> [A]
time = 0.29, size = 409, normalized size = 0.75

$$\frac{i\left(-\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(-i+d)+\sqrt{-a}\,e}\right)\log(1+id+iex)+\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,(-i+d)+\sqrt{-a}\,e}\right)\log(1+id+iex)+\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}\,x\right)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(-i(i+d+ex))-\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}\,x\right)}{-\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\log(-i(i+d+ex))+\text{PolyLog}\left(2,\frac{\sqrt{b}\,(-i+d+ex)}{\sqrt{b}\,(-i+d)-\sqrt{-a}\,e}\right)-\text{PolyLog}\left(2,\frac{\sqrt{b}\,(-i+d+ex)}{\sqrt{b}\,(-i+d)+\sqrt{-a}\,e}\right)-\text{PolyLog}\left(2,\frac{\sqrt{b}\,(i+d+ex)}{\sqrt{b}\,(i+d)-\sqrt{-a}\,e}\right)+\text{PolyLog}\left(2,\frac{\sqrt{b}\,(i+d+ex)}{\sqrt{b}\,(i+d)+\sqrt{-a}\,e}\right)\right)}{4\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x^2),x]
```

```
[Out] ((I/4)*(-(Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)]*L
og[1 + I*d + I*e*x]) + Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*(-I + d))
+ Sqrt[-a]*e)]*Log[1 + I*d + I*e*x] + Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[
b]*(I + d) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] - Log[(e*(Sqrt[-a] + Sqrt
[b]*x))/(-(Sqrt[b]*(I + d)) + Sqrt[-a]*e)]*Log[(-I)*(I + d + e*x)] + PolyLo
g[2, (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) - Sqrt[-a]*e)] - PolyLog[2,
 (Sqrt[b]*(-I + d + e*x))/(Sqrt[b]*(-I + d) + Sqrt[-a]*e)] - PolyLog[2, (Sq
rt[b]*(I + d + e*x))/(Sqrt[b]*(I + d) - Sqrt[-a]*e)] + PolyLog[2, (Sqrt[b]*
(I + d + e*x))/(Sqrt[b]*(I + d) + Sqrt[-a]*e)]))/(Sqrt[-a]*Sqrt[b])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(411) = 822$.

time = 0.47, size = 2184, normalized size = 4.02

method	result
risch	$\frac{\ln(-iex-id+1)\ln\left(\frac{ibd-e\sqrt{ab}+b(-iex-id+1)-b}{ibd-e\sqrt{ab}-b}\right)}{4\sqrt{ab}}-\frac{\ln(-iex-id+1)\ln\left(\frac{ibd+e\sqrt{ab}+b(-iex-id+1)-b}{ibd+e\sqrt{ab}-b}\right)}{4\sqrt{ab}}+\frac{\mathrm{dilog}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x+d)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(I*e^2/(a*e^2+d^2*b-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(
1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2*b+2*(a*b*e^2)^(1/2)-b))*arctan(e*x
+d)+1/2*I/b*e^2*(a*b*e^2)^(1/2)/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I
*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2*b-2*(a*b*e^2)
^(1/2)-b))*arctan(e*x+d)+1/2*I*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b+2*(a*b*e^2)^(
1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-
d^2*b-2*(a*b*e^2)^(1/2)-b))*d^2*arctan(e*x+d)+I*e^2/(a*e^2+d^2*b+2*(a*b*e^2
)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e
^2-d^2*b-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)-1/2/b*e^2*(a*b*e^2)^(1/2)/(a*e
^2+d^2*b-2*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e^2/(a*e^2+d^2*b-2*(a*b*e^2)^
(1/2)+b)*arctan(e*x+d)^2-1/2*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b-2*(a*b*e^2)^(1/
2)+b)*arctan(e*x+d)^2-1/2*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b-2*(a*b*e^2)^(1/2)+
b)*d^2*arctan(e*x+d)^2-1/4/b*e^2*(a*b*e^2)^(1/2)/(a*e^2+d^2*b-2*(a*b*e^2)^(
1/2)+b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a
*e^2-d^2*b+2*(a*b*e^2)^(1/2)-b))+1/2*e^2/(a*e^2+d^2*b-2*(a*b*e^2)^(1/2)+b)*
polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2
*b+2*(a*b*e^2)^(1/2)-b))-1/4*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b-2*(a*b*e^2)^(1/
2)+b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e
^2-d^2*b+2*(a*b*e^2)^(1/2)-b))-1/4*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b-2*(a*b*e^
2)^(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)
/(-a*e^2-d^2*b+2*(a*b*e^2)^(1/2)-b))*d^2+1/2*I*(a*b*e^2)^(1/2)/a/(a*e^2+d^2
*b+2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+
d)^2+1)/(-a*e^2-d^2*b-2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)-1/2*I*(a*b*e^2)^(
1/2)/a/(a*e^2+d^2*b-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(1+I*
(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2*b+2*(a*b*e^2)^(1/2)-b))*d^2*arctan(e*x
+d)-1/2*I/b*e^2*(a*b*e^2)^(1/2)/(a*e^2+d^2*b-2*(a*b*e^2)^(1/2)+b)*ln(1-(2*I
*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2*b+2*(a*b*e^2)
^(1/2)-b))*arctan(e*x+d)-1/2*I*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b-2*(a*b*e^2)^(
1/2)+b)*ln(1-(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-
d^2*b+2*(a*b*e^2)^(1/2)-b))*arctan(e*x+d)+1/2/b*e^2*(a*b*e^2)^(1/2)/(a*e^2+
```

```
d^2*b+2*(a*b*e^2)^(1/2)+b)*arctan(e*x+d)^2+e^2/(a*e^2+d^2*b+2*(a*b*e^2)^(1/
2)+b)*arctan(e*x+d)^2+1/2*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2)+
b)*arctan(e*x+d)^2+1/2*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2)+b)*
d^2*arctan(e*x+d)^2+1/4/b*e^2*(a*b*e^2)^(1/2)/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2
)+b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^
2-d^2*b-2*(a*b*e^2)^(1/2)-b))+1/2*e^2/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2)+b)*pol
ylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-d^2*b-
2*(a*b*e^2)^(1/2)-b))+1/4*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b+2*(a*b*e^2)^(1/2)+
b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2-
d^2*b-2*(a*b*e^2)^(1/2)-b))+1/4*(a*b*e^2)^(1/2)/a/(a*e^2+d^2*b+2*(a*b*e^2)^
(1/2)+b)*polylog(2,(2*I*b*d+a*e^2+d^2*b-b)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-
a*e^2-d^2*b-2*(a*b*e^2)^(1/2)-b))*d^2)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 13320 vs. 2(389) = 778.
time = 1.16, size = 13320, normalized size = 24.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(x*e + d)*arctan(b*x/sqrt(a*b))/sqrt(a*b) - arctan((x*e^2 + d*e)*e^(-
1))*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/8*(8*arctan((x*e^2 + d*e)*e^(-1))*a
rctan(b*x/sqrt(a*b))*e^(-1) - (4*arctan(sqrt(b)*x/sqrt(a))*arctan2((2*a*b*d
*e^2 + (b*d^3*e + (a*e^3 + b*e)*d + (b*d^2*e^2 + a*e^4 + 3*b*e^2)*x)*sqrt(a
)*sqrt(b) + (b^2*d^2*e + 3*a*b*e^3 + b^2*e)*x)/(b^2*d^4 + 2*(a*b*e^2 + b^2)
*d^2 + a^2*e^4 + 6*a*b*e^2 + 4*(b*d^2*e + a*e^3 + b*e)*sqrt(a)*sqrt(b) + b^
2), (b^2*d^4 + (a*b*e^2 + 2*b^2)*d^2 + 3*a*b*e^2 + (2*b*d*x*e^2 + 3*b*d^2*e
 + a*e^3 + 3*b*e)*sqrt(a)*sqrt(b) + b^2 + (b^2*d^3*e + (a*b*e^3 + b^2*e)*d)
*x)/(b^2*d^4 + 2*(a*b*e^2 + b^2)*d^2 + a^2*e^4 + 6*a*b*e^2 + 4*(b*d^2*e + a
*e^3 + b*e)*sqrt(a)*sqrt(b) + b^2)) + 4*arctan(sqrt(b)*x/sqrt(a))*arctan2((
2*a*b*d*e^2 - (b*d^3*e + (a*e^3 + b*e)*d + (b*d^2*e^2 + a*e^4 + 3*b*e^2)*x)
*sqrt(a)*sqrt(b) + (b^2*d^2*e + 3*a*b*e^3 + b^2*e)*x)/(b^2*d^4 + 2*(a*b*e^2
 + b^2)*d^2 + a^2*e^4 + 6*a*b*e^2 - 4*(b*d^2*e + a*e^3 + b*e)*sqrt(a)*sqrt(
b) + b^2), (b^2*d^4 + (a*b*e^2 + 2*b^2)*d^2 + 3*a*b*e^2 - (2*b*d*x*e^2 + 3*
b*d^2*e + a*e^3 + 3*b*e)*sqrt(a)*sqrt(b) + b^2 + (b^2*d^3*e + (a*b*e^3 + b^
2*e)*d)*x)/(b^2*d^4 + 2*(a*b*e^2 + b^2)*d^2 + a^2*e^4 + 6*a*b*e^2 - 4*(b*d^
2*e + a*e^3 + b*e)*sqrt(a)*sqrt(b) + b^2)) + log(b*x^2 + a)*log((b^12*d^24
+ (11*a*b^11*e^2 + 12*b^12)*d^22 + 11*(5*a^2*b^10*e^4 + 31*a*b^11*e^2 + 6*b
^12)*d^20 + 55*(3*a^3*b^9*e^6 + 46*a^2*b^10*e^4 + 51*a*b^11*e^2 + 4*b^12)*d
^18 + 165*(2*a^4*b^8*e^8 + 57*a^3*b^9*e^6 + 155*a^2*b^10*e^4 + 71*a*b^11*e^
2 + 3*b^12)*d^16 + 66*(7*a^5*b^7*e^10 + 320*a^4*b^8*e^8 + 1610*a^3*b^9*e^6
+ 1820*a^2*b^10*e^4 + 455*a*b^11*e^2 + 12*b^12)*d^14 + 462*(a^6*b^6*e^12 +
67*a^5*b^7*e^10 + 540*a^4*b^8*e^8 + 1134*a^3*b^9*e^6 + 705*a^2*b^10*e^4 + 1
```

11*a*b^11*e^2 + 2*b^12)*d^12 + a^11*b*e^22 + 231*a^10*b^2*e^20 + 7315*a^9*b
^3*e^18 + 74613*a^8*b^4*e^16 + 319770*a^7*b^5*e^14 + 646646*a^6*b^6*e^12 +
646646*a^5*b^7*e^10 + 319770*a^4*b^8*e^8 + 74613*a^3*b^9*e^6 + 7315*a^2*b^1
0*e^4 + 231*a*b^11*e^2 + b^12 + 66*(5*a^7*b^5*e^14 + 462*a^6*b^6*e^12 + 546
7*a^5*b^7*e^10 + 18480*a^4*b^8*e^8 + 21483*a^3*b^9*e^6 + 8470*a^2*b^10*e^4
+ 917*a*b^11*e^2 + 12*b^12)*d^10 + 165*(a^8*b^4*e^16 + 122*a^7*b^5*e^14 + 2
002*a^6*b^6*e^12 + 10010*a^5*b^7*e^10 + 18876*a^4*b^8*e^8 + 14014*a^3*b^9*e
^6 + 3822*a^2*b^10*e^4 + 302*a*b^11*e^2 + 3*b^12)*d^8 + 55*(a^9*b^3*e^18 +
156*a^8*b^4*e^16 + 3420*a^7*b^5*e^14 + 24024*a^6*b^6*e^12 + 67782*a^5*b^7*e
^10 + 82368*a^4*b^8*e^8 + 42588*a^3*b^9*e^6 + 8520*a^2*b^10*e^4 + 513*a*b^1
1*e^2 + 4*b^12)*d^6 + 11*(a^10*b^2*e^20 + 195*a^9*b^3*e^18 + 5610*a^8*b^4*e
^16 + 54060*a^7*b^5*e^14 + 218790*a^6*b^6*e^12 + 403546*a^5*b^7*e^10 + 3447
60*a^4*b^8*e^8 + 131580*a^3*b^9*e^6 + 20145*a^2*b^10*e^4 + 955*a*b^11*e^2 +
6*b^12)*d^4 + (a^11*b*e^22 + 242*a^10*b^2*e^20 + 9405*a^9*b^3*e^18 + 12790
8*a^8*b^4*e^16 + 746130*a^7*b^5*e^14 + 2032316*a^6*b^6*e^12 + 2678962*a^5*b
^7*e^10 + 1705440*a^4*b^8*e^8 + 500973*a^3*b^9*e^6 + 60610*a^2*b^10*e^4 + 2
321*a*b^11*e^2 + 12*b^12)*d^2 + (b^12*d^22*e^2 + 11*(a*b^11*e^4 + b^12*e^2)
d^20 + 55(a^2*b^10*e^6 + 6*a*b^11*e^4 + b^12*e^2)*d^18 + 165*(a^3*b^9*e^8
+ 15*a^2*b^10*e^6 + 15*a*b^11*e^4 + b^12*e^2)*d^16 + 330*(a^4*b^8*e^10 + 2
8*a^3*b^9*e^8 + 70*a^2*b^10*e^6 + 28*a*b^11*e^4 + b^12*e^2)*d^14 + 462*(a^5
*b^7*e^12 + 45*a^4*b^8*e^10 + 210*a^3*b^9*e^8 + 210*a^2*b^10*e^6 + 45*a*b^1
1*e^4 + b^12*e^2)*d^12 + a^11*b*e^24 + 231*a^10*b^2*e^22 + 7315*a^9*b^3*e^2
0 + 74613*a^8*b^4*e^18 + 319770*a^7*b^5*e^16 + 646646*a^6*b^6*e^14 + 646646
*a^5*b^7*e^12 + 319770*a^4*b^8*e^10 + 74613*a^3*b^9*e^8 + 7315*a^2*b^10*e^6
+ 231*a*b^11*e^4 + b^12*e^2 + 462*(a^6*b^6*e^14 + 66*a^5*b^7*e^12 + 495*a^
4*b^8*e^10 + 924*a^3*b^9*e^8 + 495*a^2*b^10*e^6 + 66*a*b^11*e^4 + b^12*e^2)
d^10 + 330(a^7*b^5*e^16 + 91*a^6*b^6*e^14 + 1001*a^5*b^7*e^12 + 3003*a^4*
b^8*e^10 + 3003*a^3*b^9*e^8 + 1001*a^2*b^10*e^6 + 91*a*b^11*e^4 + b^12*e^2)
d^8 + 165(a^8*b^4*e^18 + 120*a^7*b^5*e^16 + 1820*a^6*b^6*e^14 + 8008*a^5*
b^7*e^12 + 12870*a^4*b^8*e^10 + 8008*a^3*b^9*e^8 + 1820*a^2*b^10*e^6 + 120*
a*b^11*e^4 + b^12*e^2)*d^6 + 55*(a^9*b^3*e^20 + 153*a^8*b^4*e^18 + 3060*a^7
*b^5*e^16 + 18564*a^6*b^6*e^14 + 43758*a^5*b^7*e^12 + 43758*a^4*b^8*e^10 +
18564*a^3*b^9*e^8 + 3060*a^2*b^10*e^6 + 153*a*b^11*e^4 + b^12*e^2)*d^4 + 11
*(a^10*b^2*e^22 + 190*a^9*b^3*e^20 + 4845*a^8*b^4*e^18 + 38760*a^7*b^5*e^16
+ 125970*a^6*b^6*e^14 + 184756*a^5*b^7*e^12 + 125970*a^4*b^8*e^10 + 38760*
a^3*b^9*e^8 + 4845*a^2*b^10*e^6 + 190*a*b^11*e^4 + b^12*e^2)*d^2)*x^2 + 2*(
11*b^11*d^22*e + 11*(10*a*b^10*e^3 + 11*b^11*e)*d^20 + 55*(9*a^2*b^9*e^5 +
32*a*b^10*e^3 + 11*b^11*e)*d^18 + 165*(8*a^3*b^8*e^7 + 59*a^2*b^9*e^5 + 66*
a*b^10*e^3 + 11*b^11*e)*d^16 + 66*(35*a^4*b^7*e^9 + 440*a^3*b^8*e^7 + 1022*
a^2*b^9*e^5 + 560*a*b^10*e^3 + 55*b^11*e)*d^14 + 462*(6*a^5*b^6*e^11 + 115*
a^4*b^7*e^9 + 456*a^3*b^8*e^7 + 522*a^2*b^9*e^5 + 170*a*b^10*e^3 + 11*b^11*
e)*d^12 + 11*a^10*b*e^21 + 770*a^9*b^2*e^19 + 13167*a^8*b^3*e^17 + 85272*a^
7*b^4*e^15 + 248710*a^6*b^5*e^13 + 352716*a^5*b^6*e^11 + 248710*a^4*b^7*e^9
+ 85272*a^3*b^8*e^7 + 13167*a^2*b^9*e^5 + 770*...

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(arctan(x*e + d)/(b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(b*x^2+a),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(d+e\,x)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(d + e*x)/(a + b*x^2),x)
[Out] int(atan(d + e*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\textbf{ArcTan}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=367

$$\frac{\text{ArcTan}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(i-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}}-\frac{\text{ArcTan}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}\right.}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right.\right.}\right.}{\sqrt{b^2-4ac}}$$

```
[Out] arctan(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+
e*(b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arctan(e*x+d)*ln(2*e*(b+2*c*x
+(-4*a*c+b^2)^(1/2))/(1-I*(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-
4*a*c+b^2)^(1/2)-1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(
1/2)))/(1-I*(e*x+d))/(2*I*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(
1/2)+1/2*I*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(1-I*
(e*x+d))/(2*c*(I-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {632, 212, 6860, 5155, 4966, 2449, 2352, 2497}

$$\frac{\text{ArcTan}(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\text{ArcTan}(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(1-i(d+ex))\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(-d+i)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2ic+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}+\frac{i\text{Li}_2\left(\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(i-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1-i(d+ex))}+1\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcTan[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) +
(b - Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - (ArcTan
[d + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(I - d) + (b + Sq
rt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c] - ((I/2)*PolyLog
[2, 1 + (2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/(((2*I)*c -
 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)*(1 - I*(d + e*x)))])/Sqrt[b^2 - 4*a*c]
+ ((I/2)*PolyLog[2, 1 + (2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*
x)))/((2*c*(I - d) + (b + Sqrt[b^2 - 4*a*c])*e)*(1 - I*(d + e*x)))])/Sqrt[b
^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5155

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tan^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)$$

$$= \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tan^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2c)\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\frac{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}{e}+\ldots}\right.}{\sqrt{b^2\ldots}}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{\ldots}{(2c(\ldots}\right.}{\ldots}$$

$$= \frac{\tan^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2ic-2cd+be-\sqrt{b^2-4ac}\,e\right)(1-i(d+ex))}\right)}{\sqrt{b^2-4ac}} - \frac{\tan^{-1}(d+ex)\log\left(\frac{\ldots}{(2c(\ldots}\right.}{\ldots}$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] $Aborted
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4752 vs. 2(329) = 658.
time = 0.68, size = 4753, normalized size = 12.95

method	result
risch	$\frac{e\,\mathrm{dilog}\left(\frac{ibe-2icd-2(-iex-id+1)c+\sqrt{4ac\,e^2-b^2e^2}+2c}{ibe-2icd+2c+\sqrt{4ac\,e^2-b^2e^2}}\right)}{2\sqrt{4ac\,e^2-b^2e^2}}-\frac{e\,\mathrm{dilog}\left(\frac{ibe-2icd-2(-iex-id+1)c-\sqrt{4ac\,e^2-b^2e^2}}{ibe-2icd+2c-\sqrt{4ac\,e^2-b^2e^2}}\right.}{2\sqrt{4ac\,e^2-b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*
(4*a*c-b^2))^(1/2)+c)*b^2*arctan(e*x+d)^2-1/2*e*(e^2*(4*a*c-b^2))^(1/2)/(4*
a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*
I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^
2-(e^2*(4*a*c-b^2))^(1/2)-c))*b*d-1/8*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-
b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*
d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e
^2*(4*a*c-b^2))^(1/2)-c))*b^2+1/4*I*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*
e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2
-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/
2)-c))*arctan(e*x+d)-1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b
*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*b^2*arctan(e*x+d)^2+1/8*e^2*(e^2*(4*a
*c-b^2))^(1/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*
polylog(2,(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1
)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*b^2-e*(e^2*(4*a*c-b^2))^(
1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*b*d*arctan(e
*x+d)^2+1/2*e*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(
4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e
*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*b*d+
e*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^
(1/2)+c)*b*d*arctan(e*x+d)^2-I*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2
-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*
d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^
(1/2)-c))*arctan(e*x+d)-1/4*I*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*
d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1
+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))
*arctan(e*x+d)+I*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(
e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e
*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arct
an(e*x+d)-1/2*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2
*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*
(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))-(e
^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1
```

```
/2)+c)*d^2*arctan(e*x+d)^2-1/2*(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2
-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*d+a*e^2-b*e
*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b
^2))^(1/2)-c))*d^2+(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2
+(e^2*(4*a*c-b^2))^(1/2)+c)*d^2*arctan(e*x+d)^2+1/2*(e^2*(4*a*c-b^2))^(1/2)
*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*
b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*
d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*d^2+I*e*(e^2*(4*a*c-b^2))^(1/2)/(4*a*c-
b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e
^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4
*a*c-b^2))^(1/2)-c))*b*d*arctan(e*x+d)+1/4*I*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/
(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*
c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-
(e^2*(4*a*c-b^2))^(1/2)-c))*b^2*arctan(e*x+d)-I*e*(e^2*(4*a*c-b^2))^(1/2)/(
4*a*c-b^2)/(a*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c
*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(
e^2*(4*a*c-b^2))^(1/2)-c))*b*d*arctan(e*x+d)-1/4*I*e^2*(e^2*(4*a*c-b^2))^(1
/2)/c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*
e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-
c*d^2+(e^2*(4*a*c-b^2))^(1/2)-c))*b^2*arctan(e*x+d)+I*e^2/(a*e^2-b*e*d+c*d^
2+(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1+I
*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*a
rctan(e*x+d)-(e^2*(4*a*c-b^2))^(1/2)*c/(4*a*c-b^2)/(a*e^2-b*e*d+c*d^2-(e^2*
(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+1/8*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*
e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*d+a*e^2-
b*e*d+c*d^2-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2-(e^2*(4*a*
c-b^2))^(1/2)-c))-1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(e^2
*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+1/4*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a
*e^2-b*e*d+c*d^2+(e^2*(4*a*c-b^2))^(1/2)+c)*arctan(e*x+d)^2+I*e^2/(a*e^2-b*
e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)+c)*ln(1-(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2
-c)*(1+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2+(e^2*(4*a*c-b^2))^(1/
2)-c))*arctan(e*x+d)-1/8*e^2*(e^2*(4*a*c-b^2))^(1/2)/c/(a*e^2-b*e*d+c*d^2-(
e^2*(4*a*c-b^2))^(1/2)+c)*polylog(2,(-I*b*e+2*I*c*d+a*e^2-b*e*d+c*d^2-c)*(1
+I*(e*x+d))^2/((e*x+d)^2+1)/(-a*e^2+b*e*d-c*d^2...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] integral(arctan(x*e + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(e*x+d)/(c*x**2+b*x+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(d + e*x)/(a + b*x + c*x^2),x)
[Out] int(atan(d + e*x)/(a + b*x + c*x^2), x)
```

3.63 $\int \frac{\text{ArcTan}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{2i\text{ArcTan}(a+bx)\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1+}}{\sqrt{1-}}\right.}{b}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+I*poly
log(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-I*polylog(2,I*(1+I*(b*x
+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5163, 5006}

$$-\frac{2i\text{ArcTan}(a+bx)\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}+\frac{i\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}-\frac{i\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ((-2*I)*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]
)/b + (I*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
- (I*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/b
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5163

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{2i\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b} + \frac{i\text{Li}_2\left(-\frac{i\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

Mathematica [A]
time = 0.08, size = 97, normalized size = 0.73

$$\frac{\text{ArcTan}(a+bx)\left(\log\left(1-ie^{i\text{ArcTan}(a+bx)}\right)-\log\left(1+ie^{i\text{ArcTan}(a+bx)}\right)\right)+i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(a+bx)}\right)-i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] (ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] - Log[1 + I*E^(I*ArcTan[
a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, I*E^(
I*ArcTan[a + b*x])])/b
```

Maple [A]
time = 0.16, size = 135, normalized size = 1.02

method	result
default	$\frac{-\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+i\,\text{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+arctan(b*x+a)
*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+I*dilog(1+I*(1+I*(b*x+a))/(1+(b*
x+a)^2)^(1/2))-I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(a + bx \right)}}{\sqrt{a^{2} + 2abx + b^{2}x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a+b\,x)}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.64 $\int \frac{\mathbf{ArcTan}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2i\sqrt{1+(a+bx)^2}\,\mathrm{ArcTan}(a+bx)\mathrm{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}}+\frac{i\sqrt{1+(a+bx)^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+i(}}{\sqrt{1-i(}}\right.}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] -2*I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+
a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I
*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-I*polylog(2,I*
(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)
^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5163, 5010, 5006}

$$-\frac{2i\sqrt{(a+bx)^2+1}\,\mathrm{ArcTan}(a+bx)\mathrm{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}-\frac{i\sqrt{(a+bx)^2+1}\,\mathrm{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c(a+bx)^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] ((-2*I)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + (I*Sqrt[1 + (a + b*x)
^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt
[c + c*(a + b*x)^2]) - (I*Sqrt[1 + (a + b*x)^2]*PolyLog[2, (I*Sqrt[1 + I*(a
 + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2])
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
```

```
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5163

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((A_.) + (B_.)*(x_) + (
C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)^
q*(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C, p
, q}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\sqrt{1+(a+bx)^2}\,\text{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b\sqrt{c+c(a+bx)^2}}$$

$$= -\frac{2i\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)\tan^{-1}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b\sqrt{c+c(a+bx)^2}} + \frac{i\sqrt{}}{}$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 0.58

$$\frac{\sqrt{1+(a+bx)^2}\left(\text{ArcTan}(a+bx)\left(\log\left(1-ie^{i\text{ArcTan}(a+bx)}\right)-\log\left(1+ie^{i\text{ArcTan}(a+bx)}\right)\right)+i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(a+bx)}\right)-i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(a+bx)}\right)\right)}{b\sqrt{c\left(1+(a+bx)^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a + b*x]/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2],x]
```

```
[Out] (Sqrt[1 + (a + b*x)^2]*(ArcTan[a + b*x]*(Log[1 - I*E^(I*ArcTan[a + b*x])] -
 Log[1 + I*E^(I*ArcTan[a + b*x])]) + I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x]
)] - I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(b*Sqrt[c*(1 + (a + b*x)^2)])
```

Maple [A]
time = 0.20, size = 176, normalized size = 0.81

method	result

default	$\frac{i\left(i\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-i\arctan(bx+a)\ln\left(1-\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)+\mathrm{dilog}\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right.\right.}{\sqrt{b^2x^2+2abx+a^2+1}\,bc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan(b*x+a
)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(1+(b*x
+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+b*x)*(
I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(a+bx)}{\sqrt{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),x)
```

```
[Out] Integral(atan(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{atan}(a+b\,x)}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2),x)
```

```
[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2), x)
```

3.65 $\int \frac{\textbf{ArcTan}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\operatorname{ArcTan}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{ArcTan}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(23) = 46.
time = 0.31, size = 163, normalized size = 7.09

$$\frac{6\operatorname{Gamma}\left(\frac{11}{6}\right)\operatorname{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\operatorname{ArcTan}(a+bx)+\frac{4(a+bx)\operatorname{ArcTan}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+(a+bx)^2}\right)}{1+(a+bx)^2}\right)+\frac{5\sqrt[3]{2}\,\sqrt{\pi}\operatorname{Gamma}\left(\frac{5}{3}\right)\,{}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{1+(a+bx)^2}}{20b\sqrt[3]{1+a^2+2abx+b^2x^2}\operatorname{Gamma}\left(\frac{11}{6}\right)\operatorname{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(1
+ a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(bx+a\right)}{\left(b^{2}x^{2}+2abx+a^{2}+1\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
[Out] int(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="maxima"
)
[Out] integrate(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="fricas"
)
[Out] integral(arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(a+bx \right)}}{\sqrt[3]{a^{2}+2abx+b^{2}x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
[Out] Integral(atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a+b\,x)}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

```
[Out] int(atan(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.66 $\int \frac{\mathbf{ArcTan}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\mathrm{ArcTan}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable(arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\mathrm{ArcTan}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][ArcTan[x]/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\tan^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(25) = 50.
time = 0.12, size = 165, normalized size = 6.60

$$\frac{6\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)\left(15+10(a+bx)\mathrm{ArcTan}(a+bx)+\frac{4(a+bx)\mathrm{ArcTan}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+(a+bx)^2}\right)}{1+(a+bx)^2}\right)+\frac{5\sqrt[3]{2}\,\sqrt{\pi}\,\mathrm{Gamma}\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{1+(a+bx)^2}}{20b\sqrt[3]{c\,(1+a^2+2abx+b^2x^2)}\,\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
```

```
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(c*
(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arctan \left(b x +a \right)}{\left(\left(a^{2}+1\right) c +2 a b c x +b^{2} c \,x^{2}\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
[Out] int(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"maxima")
[Out] integrate(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"fricas")
[Out] integral(arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}{\left(a + b x \right)}}{\sqrt[3]{c \left(a^{2} + 2 a b x + b^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),x)
[Out] Integral(atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x, algorithm=
"giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atan}(a+b\,x)}{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3),x)
```

```
[Out] int(atan(a + b*x)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3), x)
```

3.67 $\int \frac{(a+bx)^2 \textbf{ArcTan}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{\sqrt{1+(a+bx)^2}}{2b}+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{ArcTan}(a+bx)}{2b}+\frac{i\text{ArcTan}(a+bx)\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}{b}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*I*pol
ylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b+1/2*I*polylog(2,I*(1+I
*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))/b-1/2*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a
)*arctan(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5165, 5072, 267, 5006}

$$\frac{i\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\text{ArcTan}(a+bx)}{b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\text{ArcTan}(a+bx)}{2b}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}+\frac{i\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b}-\frac{\sqrt{(a+bx)^2+1}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + (a + b*x)^2]/b + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a +
b*x])/(2*b) + (I*ArcTan[a + b*x]*ArcTan[Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a
 + b*x)]])/b - ((I/2)*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a
 + b*x)]])/b + ((I/2)*PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a +
b*x)]])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+b\right.}{2b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\tan^{-1}(a+bx)}{2b} + \frac{i\tan^{-1}(a}{}$$

Mathematica [A]
time = 0.47, size = 145, normalized size = 0.78

$$\frac{-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\,\text{ArcTan}(a+bx)-\text{ArcTan}(a+bx)\log\left(1-ie^{i\text{ArcTan}(a+bx)}\right)+\text{ArcTan}(a+bx)\log\left(1+ie^{i\text{ArcTan}(a+bx)}\right)-i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(a+bx)}\right)+i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x
]
```

```
[Out] (-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x] -
 ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a + b*x])] + ArcTan[a + b*x]*Log[1 +
 I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a + b*x])] + I*Po
lyLog[2, I*E^(I*ArcTan[a + b*x])])/(2*b)
```

Maple [A]
time = 0.46, size = 180, normalized size = 0.96

method	result
default	$\frac{(\arctan(bx+a)bx+\arctan(bx+a)a-1)\sqrt{b^2x^2+2abx+a^2+1}}{2b}+\frac{\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)-\arctan(bx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2*(arctan(b*x+a)*b*x+arctan(b*x+a)*a-1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b+1
/2*(arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-arctan(b*x+a)*l
n(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*dilog(1+I*(1+I*(b*x+a))/(1+(b*x+
a)^2)^(1/2))+I*dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atan}(a+b\,x)\,{(a+b\,x)}^2}{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.68 $\int \frac{(a+bx)^2 \text{ArcTan}(a+bx)}{\sqrt{(1+a^2)c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=281

$$-\frac{\sqrt{c+c(a+bx)^2}}{2bc}+\frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\text{ArcTan}(a+bx)}{2bc}+\frac{i\sqrt{1+(a+bx)^2}\,\text{ArcTan}(a+bx)\text{ArcTan}\Big(}{b\sqrt{c+c(a+bx)^2}}$$

```
[Out] I*arctan(b*x+a)*arctan((1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^
2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)-1/2*I*polylog(2,-I*(1+I*(b*x+a))^(1/2)/(1-
I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b*x+a)^2)^(1/2)+1/2*I*polylog
(2,I*(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2))*(1+(b*x+a)^2)^(1/2)/b/(c+c*(b
*x+a)^2)^(1/2)-1/2*(c+c*(b*x+a)^2)^(1/2)/b/c+1/2*(b*x+a)*arctan(b*x+a)*(c+c
*(b*x+a)^2)^(1/2)/b/c
```

Rubi [A]
time = 0.24, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5165, 5072, 267, 5010, 5006}

$$\frac{i\sqrt{(a+bx)^2+1}\,\text{ArcTan}\left(\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)\text{ArcTan}(a+bx)}{b\sqrt{c(a+bx)^2+c}}+\frac{(a+bx)\text{ArcTan}(a+bx)\sqrt{c(a+bx)^2+c}}{2bc}-\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(-\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}+\frac{i\sqrt{(a+bx)^2+1}\,\text{Li}_2\left(\frac{i\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}\right)}{2b\sqrt{c(a+bx)^2+c}}-\frac{\sqrt{c(a+bx)^2+c}}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2]
,x]
```

```
[Out] -1/2*Sqrt[c + c*(a + b*x)^2]/(b*c) + ((a + b*x)*Sqrt[c + c*(a + b*x)^2]*Arc
Tan[a + b*x])/(2*b*c) + (I*Sqrt[1 + (a + b*x)^2]*ArcTan[a + b*x]*ArcTan[Sqr
t[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) - ((
I/2)*Sqrt[1 + (a + b*x)^2]*PolyLog[2, ((-I)*Sqrt[1 + I*(a + b*x)])/Sqrt[1 -
 I*(a + b*x)]])/(b*Sqrt[c + c*(a + b*x)^2]) + ((I/2)*Sqrt[1 + (a + b*x)^2]*
PolyLog[2, (I*Sqrt[1 + I*(a + b*x)])/Sqrt[1 - I*(a + b*x)]])/(b*Sqrt[c + c*
(a + b*x)^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5006

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
```

```
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

Rule 5010

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTan[c*x])^p
/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
 IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5072

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Dist[b*f*(p/(c*m)), Int[(f*x)^(m - 1)*((a
 + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Dist[f^2*((m - 1)/(c^2
*m)), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5165

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subst
[Int[((d*e - c*f)/d + f*(x/d))^m*(C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTan[x])^
p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x] &&
 EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2 \tan^{-1}(a+bx)}{\sqrt{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt{c+cx^2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+cx^2}}\,dx, x\right.}{2b}$$

$$= -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc} - \frac{\sqrt{}}{}$$

$$= -\frac{\sqrt{c+c(a+bx)^2}}{2bc} + \frac{(a+bx)\sqrt{c+c(a+bx)^2}\,\tan^{-1}(a+bx)}{2bc} + \frac{i\sqrt{}}{}$$

Mathematica [A]
time = 0.10, size = 189, normalized size = 0.67

$$\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(-\sqrt{1+(a+bx)^2}+(a+bx)\sqrt{1+(a+bx)^2}\,\text{ArcTan}(a+bx)-\text{ArcTan}(a+bx)\log\left(1-ie^{i\text{ArcTan}(a+bx)}\right)+\text{ArcTan}(a+bx)\log\left(1+ie^{i\text{ArcTan}(a+bx)}\right)-i\text{PolyLog}\left(2,-ie^{i\text{ArcTan}(a+bx)}\right)+i\text{PolyLog}\left(2,ie^{i\text{ArcTan}(a+bx)}\right)\right)}{2b\sqrt{c\,(1+a^2+2abx+b^2x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/Sqrt[(1 + a^2)*c + 2*a*b*c*x + b^2*
c*x^2],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-Sqrt[1 + (a + b*x)^2] + (a + b*x)*Sqrt
[1 + (a + b*x)^2]*ArcTan[a + b*x] - ArcTan[a + b*x]*Log[1 - I*E^(I*ArcTan[a
 + b*x])] + ArcTan[a + b*x]*Log[1 + I*E^(I*ArcTan[a + b*x])] - I*PolyLog[2,
 (-I)*E^(I*ArcTan[a + b*x])] + I*PolyLog[2, I*E^(I*ArcTan[a + b*x])]))/(2*b
*Sqrt[c*(1 + a^2 + 2*a*b*x + b^2*x^2)])
```

Maple [A]
time = 0.48, size = 222, normalized size = 0.79

method	result
default	$\frac{(\arctan(bx+a)bx+\arctan(bx+a)a-1)\sqrt{c\,(bx+a-i)\,(bx+a+i)}}{2bc} - \frac{i\left(i\arctan(bx+a)\ln\left(1+\frac{i(1+i(bx+a))}{\sqrt{1+(bx+a)^2}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(b*x+a)*b*x+arctan(b*x+a)*a-1)*(c*(-I+a+b*x)*(I+a+b*x))^(1/2)/b/
c-1/2*I*(I*arctan(b*x+a)*ln(1+I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))-I*arctan
(b*x+a)*ln(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))+dilog(1+I*(1+I*(b*x+a))/(
1+(b*x+a)^2)^(1/2))-dilog(1-I*(1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)))*(c*(-I+a+
b*x)*(I+a+b*x))^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1
)*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/sqrt(b^2*c*x^2 + 2*a*b*c
*x + (a^2 + 1)*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt{c(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/2),
x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/sqrt(c*(a**2 + 2*a*b*x + b**2*x**2 + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/2),x,
algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atan}(a+b\,x)\,{\left(a+b\,x\right)}^2}{\sqrt{c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/2)
, x)
```

3.69 $\int \frac{(a+bx)^2 \mathbf{ArcTan}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=30

$$\operatorname{Int}\left(\frac{(a+bx)^2\operatorname{ArcTan}(a+bx)}{\sqrt[3]{1+(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(1+(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2\operatorname{ArcTan}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(1 + x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt[3]{1+x^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(30) = 60.
time = 4.93, size = 181, normalized size = 6.03

$$-\frac{3(1+(a+bx)^2)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\mathrm{Gamma}\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)\left(15+\frac{90}{1+(a+bx)^2}+\frac{24(a+bx)\operatorname{ArcTan}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+5\operatorname{ArcTan}(a+bx)(-4(a+bx)+6\sin(2\operatorname{ArcTan}(a+bx)))\right)\right)}{140b\,\mathrm{Gamma}\left(\frac{11}{6}\right)\mathrm{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricP
FQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2
 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan
```

```
[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a
+ b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]])))
)/(140*b*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{(b^2x^2+2abx+a^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(1/3),
x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2
 + 1)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt[3]{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/3),x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(1/3)
, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/3),x, algorith
m="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a+b\,x)\,\left(a+b\,x\right)^2}{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3),x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/3), x)
```

3.70 $\int \frac{(a+bx)^2 \mathbf{ArcTan}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+bx)^2\text{ArcTan}(a+bx)}{\sqrt[3]{c+c(a+bx)^2}},x\right)$$

```
[Out] Unintegrable((b*x+a)^2*arctan(b*x+a)/(c+c*(b*x+a)^2)^(1/3),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx)^2\text{ArcTan}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] Defer[Subst][Defer[Int][(x^2*ArcTan[x])/(c + c*x^2)^(1/3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{(1+a^2)\,c+2abcx+b^2cx^2}}\,dx=\frac{\text{Subst}\left(\int \frac{x^2\tan^{-1}(x)}{\sqrt[3]{c+cx^2}}\,dx,x,a+bx\right)}{b}$$

Mathematica [A] Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(32) = 64.
time = 0.63, size = 225, normalized size = 7.03

$$-\frac{3\sqrt[3]{1+a^2+2abx+b^2x^2}\,(1+(a+bx)^2)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\,\text{Gamma}\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)\left(15+\frac{90}{1+(a+bx)^2}+\frac{24(a+bx)\text{ArcTan}(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{1+(a+bx)^2}\right)}{(1+(a+bx)^2)^2}+5\text{ArcTan}(a+bx)(-4(a+bx)+6\sin(2\text{ArcTan}(a+bx)))\right)\right)}{140b\sqrt[3]{c\,(1+a^2+2abx+b^2x^2)}\,\text{Gamma}\left(\frac{11}{6}\right)\text{Gamma}\left(\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)
*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a
+ b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 +
 (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/
6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a
+ b*x) + 6*Sin[2*ArcTan[a + b*x]]))))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^
2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Maple [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 \arctan(bx+a)}{\left((a^2+1)\,c+2abcx+b^2c\,x^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

```
[Out] int((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="maxima")
```

```
[Out] integrate((b*x + a)^2*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x + (a^2 + 1)*c)
^(1/3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*arctan(b*x + a)/(b^2*c*x^2 + 2*a*b*c*x +
 (a^2 + 1)*c)^(1/3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \operatorname{atan}(a+bx)}{\sqrt[3]{c\,(a^2+2abx+b^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*atan(b*x+a)/((a**2+1)*c+2*a*b*c*x+b**2*c*x**2)**(1/3),
x)
```

```
[Out] Integral((a + b*x)**2*atan(a + b*x)/(c*(a**2 + 2*a*b*x + b**2*x**2 + 1))**(
1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arctan(b*x+a)/((a^2+1)*c+2*a*b*c*x+b^2*c*x^2)^(1/3),x,
algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(a+b\,x)\,{\left(a+b\,x\right)}^2}{{\left(c\,b^2\,x^2+2\,a\,c\,b\,x+c\,\left(a^2+1\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
,x)
```

```
[Out] int((atan(a + b*x)*(a + b*x)^2)/(c*(a^2 + 1) + b^2*c*x^2 + 2*a*b*c*x)^(1/3)
, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```